From Vicious Circles to Cycles of Poverty*

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1 Introduction

In recent years, a theoretical framework, dating back to Ragnar Nurkse seminal book published in 1953, gained more and more interest in development economics, namely the "vicious circle model". Nurkse (1953, citet in Ascher and Healy, 1990) interpreted the term "vicious circle of poverty" as "circular constellation of forces tending to act and react upon one another in such a way as to keep a poor country in a state of poverty" (Ascher and Healy, 1990). In the remainder of the paper, we follow the more specific definition by Lutz and Scherbov (2000): "[The vicious circle] model is based on the assumption that high fertility, poverty, low education and low status of women and children are bound up in a web of interactions with environmental degradation and declining food production, in such a way that stress from one of these sources can trap certain rural societies, especially those living in marginal lands, into a vicious circle of increasing destructive responses." In the spirit

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of this definition, Lutz and Scherbov developed the so-called PEDA-model, where PEDA stands for Population, Environment, Development and Agriculture. Its aim is, among other things, to "help quantify assumed vicious circle dynamics at the level of individual African countries" (Lutz and Scherbov, 2000). PEDA is being coordinated by the UN Economic Commission for Africa and has been applied by so far to several African countries (Burkina Faso, Mali, Madagaskar, Uganda, Cameroon, Zambia, etc.).

In the next paragraph, we want to elaborate the idea of the vicious circle more in detail and give some impression of the size of the problem in African countries today.

"In the early 1980s, about 100 million people in sub-Saharan Africa were unable to secure sufficient food to ensure an adequate level of nutrition for themselves" (Cleaver and Schreiber, 1994). By the end of the 1990s, this number already rose up to about 186 million people (Food and Agriculture Organization, 2000). "Poverty induces a variety of behaviors that can promote environmental degradation.[1] For example, a farmer living in poverty can easily let the immediate need to produce food outweigh the long-term benefits of conserving his land. Overplanting—planting too many crops within a growing season—is a serious problem in many areas" Livernash (1998). As a consequence, crop yields decline due to soil degradation and erosion on cropland. The livestock carrying capacity of pastures and rangeland decline due to proceeding vegetative degradation and desertifcation. High population growth increases the pressure on the poor farmers and hence amplifies environmental degradation.[2] Consequently, food production declines and the vicious circle of poverty, population growth, environmental degradation and agricultural stagnation closes.

As noted above, the vicious circles is bounded in a web of intervening factors, where some of them are not explicitly mentioned in the description of this vicious circle. The most important of this underlying factors are the institutional settings. In particular, Scruggs (1998) finds on testing the 'equality hypothesis' based on the discussion in Boyce (1994) that "distributional issues do not systematically explain variations in environmen-

[1] Land degradation is by no means caused only by the poor. Commercial logging companies contribute a great deal to deforestation in tropical countries. Nevertheless, the focus of this paper is on poverty-driven land degradation, since in contrast to the rich, the poor lack alternative sources of incomes or are constraint in the access to the credit market. Boyce (1994) discusses incentives of both the poor and the rich to degrade the environment, where he concludes that greater inequalities of power and wealth lead to more environmental degradation.

[2] Unlike other authors (e.g. Nerlove and Meyer, 1997) Lutz and Scherbov (2000) and ourselves do not assume that environmental degradation itself increases fertility. We only postulate a higher fertility rate for the food-insecure than for the food-secure population.

tal quality." Nevertheless, Scruggs (1998) highlights the "complex interplay of individual and group preferences and the institutional situations." The importance of the relationship between individual preferences and the institutional settings on land degradation is further discussed by Beaumont and Walker (1996), who investigate the optimal choice of intensity of use of soil in peasant agriculture under three different property rights regime. Beaumont and Walker (1996) demonstrate that "agents may have an incentive to drive soil quality lower under a private property regime than under a common property regime" and secondly that "access to a goods market outside the home is a much more important than the agent's discount rate." The former result clearly contradicts the popular 'tragedy of the commons' by Hardin (1968), which argues that users of common property resources have no incentive to invest or conserve the resources because of lack of personal titles. However, the 'tragedy of the commons' confuses common property with open access and therefore should be rather named 'tragedy of the open access' (**?**). The second result of Beaumont and Walker (1996) challenges the analysis of Boyce (1994), where the latter explains poverty-driven land degradation through higher rates of time preference of the poor.

In this paper we embed the idea of the vicious circle in a more structured population model as proposed by Lutz and Scherbov (2000) in order to focus on specific mechanism. While Lutz and Scherbov (2000) refer to eight different groups of populations (the combination of food secure vs. food insecure, rural vs. urban, literate vs. illiterate) we consider a more aggregate model by only distinguishing between two types of populations: the food-secure, P_S, and the food-insecure population P_I. Similar to Lutz and Scherbov (2000), we impose an exogenously given Lorenz curve (which plots the cumulative share of income versus the cumulative share of the population, if population is ranked in increasing order of income) to determine the share of the food-insecure and food-secure population, respectively.[3] More specific, we postulate an exogenously given subsistence level of food y^* and calculate the percentage of the total population that falls short of this subsistence level each period. This fraction of the total population constitutes the food-insecure population as opposed to the food-secure population which is characterized by income above the subsistence level.[4]

[3] For a detailed discussion of the Lorenz curve please refer to Dworak (2002) and Prskawetz et al. (2000).

[4] The interested reader may observe that our definition of food-insecure population is slightly different from Lutz and Scherbov (2000). While we consider the share of the population, which falls short of the subsistence requirement, to be food insecure, Lutz and Scherbov (2000) establish the border line between the food security states where the remaining food supply falls below the subsistence requirement times the remaining

Total output within the economy Y is produced by the food-secure and food-insecure population together with the stock of environmental resources, R. Each time period, total food production is distributed according to the Lorenz curve determining the share of food-secure and food-insecure population in the following period.

Since the food-insecure population is characterized by low levels of food entitlement, people in this group are likely to overuse their resources (planting too frequently, overgrazing). Furthermore, "the resources of the poor are degraded by the failure to restore or to improve their productivity or regeneration (Barbier, 1998). The environmental resources considered here correspond to the variable 'land' in Lutz and Scherbov (2000), which is the only dynamic ecological variable there. All other exogenous production-enhancing/-diminishing factors, such as water, investment, fertilizers, and food adjustments through import and storage losses are grouped together in the technology parameter in our framework.

Furthermore, we adjust the assumed functional forms to the PEDA model for comparison purposes. Such a comparison between the PEDA model and our reduced form derivative, applied to Mali, is also given in Lutz et al. (2002). All analyzed scenarios yield quantitatively similar results for both models. "This indicates that changes in the food distribution mainly affect the composition of population states as opposed to the age composition of the various population states" (Lutz et al., 2002). This result stimulates to provide a more detailed analysis of the reduced-form model, which is the purpose of this paper. For instance, we aim to relate the occurrence of the vicious circle to the degree of inequality of the food distribution. Furthermore, we analyze the model in dependence of the population growth rates of the food-insecure and food-secure population, respectively.

Allowing for negative population growth rates of the food-insecure population can help to dampen environmental degradation (as induced by the food-insecure population) and its repercussion on per capita food production. In particular, we investigate the role of the elasticity of the food-insecure population growth rate with respect to environmental degradation in mitigating or amplifying the negative effect of inequality in the distribution of food on environmental degradation. Moreover, we extend the model by allowing for mortality effects when resources and food production decline.

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population when distributing food according to the Lorenz curve and starting with the richest population segment. One can easily verify that, ceteris paribus, the method of Lutz and Scherbov yields a higher share of food-insecure population than our method. Clearly, the difference depends on the shape of the Lorenz curve. The higher is the degree of inequality in the distribution of food, the higher is the difference.

2 The Model

We consider the continuous-time version of the reduced-form PEDA model presented in Lutz et al. (2002). In what follows we will only shortly depict the dynamics. For a more detailed description refer to Lutz et al. (2002) or Dworak (2002).

The model consists of three differential equations which describe the dynamics of the food-insecure population P_I, the food-secure population P_S and the natural resource stock, R:[5]

$$\dot{P}_I = n_I P_I + M(P_I, P_S, R) \tag{1}$$

$$\dot{P}_S = n_S P_S - M(P_I, P_S, R) \tag{2}$$

$$\dot{R} = g(R) - D(P_I, P_S, R). \tag{3}$$

The population growth rates, n_I and n_S, determine the indigenous population growth rate of the food-insecure and food-secure population, respectively.

Additionally to the indigenous population growth, there will be a transition between these two sub-populations as described by the function $M(P_I, P_S, R)$. Obviously, the transition between the food security states is determined by the total amount of food and the food distribution.

In what follows, we postulate the following flexible and simple analytical form for the Lorenz curve, which was also used in Lutz et al. (2002), Prskawetz et al. (2000) and Dworak (2002)[6]:

$$L(F(z)) = (F(z))^{\alpha} \qquad \alpha > 1. \tag{4}$$

Lutz et al. (2002)and Prskawetz et al. (2000) demonstrate that the following functional relationship for $M(.,.,.)$ can be derived:

$$M(P_I, P_S, R) = \left[\min\left\{\left(\frac{y^*}{\alpha y}\right)^{1/(\alpha-1)}, 1\right\} - \frac{P_I}{P_I + P_S}\right](P_I + P_S), \tag{5}$$

where y denotes per capita income. The term in braces represents the food distribution function derived from the assumed Lorenz curve and evaluated at the minimum requirement. Hence, the term in braces gives the share of population, whose food entitlement is less or equal the subsistence level. If this share exceeds the current percentage of food-insecure population, the

[5]Time arguments of the variables, e.g. $P_I(t), P_S(t), R(t)$ are omitted in the following.

[6]See Chotikapanich (1993) for alternative functional forms of the Lorenz curve. For an extensive discussion about the implication of the assumption of the Lorenz curve see Prskawetz et al. (2000).

term in square brackets is positive and the number of food-insecure population rises. Vice versa, if the share of population which falls short of the minimum requirement is less than the share of food-insecure population, the term in square brackets is negative and the number of food-insecure people declines[7]

Total food production Y depends on the two sub-populations P_I and P_S, resources R and an exogenous technology T, i. e.

$$Y \quad = T(h_I P_I + h_S P_S)^{\beta_1} R^{\beta_2} \quad \beta_1, \beta_2 > 0, \tag{6}$$

where h_I and h_S denote constant values of the efficiency units of food-insecure and food-secure population with $h_S > h_I$, respectively.[8] The latter assumption relies on that "a person's consumption intake affects his productivity" (Dasgupta and Ray, 1987, p. 177)[9]. Furthermore, we postulate that the production function exhibits constant returns to scale w.r.t. resources and labor input, i.e. $\beta_1 + \beta_2 = 1$. Moreover, resources are essential for production implying that $Y \to 0$ if R becomes extinct.

Finally, the change of the resource stock is described by indigenous growth $a(\bar{R} - R)$ and reduced by environmental degradation $D(P_I, P_S, R)$. The coefficient $\bar{R}$ determines the saturation level of the resource stock (i.e. $\bar{R}$ is the stationary solution of R if the resources are not degraded) and the parameter a determines the speed at which the resource regenerates.

Degradation $D(P_I, P_S, R)$ of the resource stock depends on the stock of available resources and on population, where we assume that the impact of the food-insecure people is more severe, since poverty promotes environmental degradation as stated in the introduction. More specific, we assume that the scale of environmental degradation is a function of population density as measured by the function $f(P/\bar{R})$ with $f'(.) > 0$. Moreover, environmental degradation is positively linked to the stock of resources and the number of food-insecure people. We assume that the environmental degradation without scaling increases linearly in P_I but increases at a decreasing rate with the level of the resources. Moreover, we assume the scaling of environmental degradation f to be linear. Consequently, environmental degradation is

What are the reasons for the assumed functional forms?

[7]The term in the denominator of the brackets in formula (5) indicates the maximum food entitlement prevailing in the economy (Prskawetz et al., 2000, see). As it can be seen, the maximum food entitlement depends on the amount of food produced per capita and on the degree of inequality in the distribution of food and, hence, will vary over time. If the minimum requirement exceeds the maximum food entitlement, the share of population which falls short of the subsistence level equals one.

[8]We normalize efficiency units such that $h_I + h_S = 1$.

[9]An empirical confirmation of the nutrition-productivity hypothesis using household-level data from Sierra Leone is provided in Strauss (1986).

given by

$$D(P_I, P_S, R) = \gamma \frac{P}{\bar{R}} P_I \frac{R}{R + \eta} \tag{7}$$

where γ and η are fixed parameters. If resources are completely degraded, i.e. $R = 0$, environmental degradation is zero, since there is nothing to be depleted. Similarly, if the stock of food-insecure population is zero, environmental degradation will be zero.

3 Phase space analysis

To investigate the dynamics of the model we proceed in three steps. First, we assume zero natural population growth rates for both population groups. With this assumption we aim to elucidate the impact of inequality in the food distribution on resource degradation and its repercussion on food production, in isolation of natural population growth.

In a second step, we consider various combinations of positive and negative population growth rates for the food-insecure and food-secure population, respectively. For instance, a positive rate of natural population growth of the food-insecure population will reinforce the overall growth of the food-insecure population as it is already determined by the number of people who become food insecure each time period. In particular, we shall investigate, whether a sufficiently negative rate of population growth of the food-secure population could facilitate a sustainable steady state under these assumptions.

On the contrary, a negative rate of natural population growth of the food-insecure population will help to dampen environmental degradation and henceforth increase the likelihood of a sustainable steady state.

Note that these differing results are not caused by population growth itself. The driving force is environmental degradation as it is positively linked to the stock of food-insecure people and the inequality in food distribution.

This paragraph should eventually move to the conclusion.

Finally, we endogenize population growth, by allowing that mortality increases as food production declines. However, we leave birth rates constant but different for each sub-population.

Concerning the other system parameters, sub-Saharan African data about land dynamics, land degradation, food production, etc., is required to set them. Though, the quality of data from African countries, if they are available, is poor. Lutz and Scherbov (2000) admit that "since data sources for many African countries are limited, some of these data need to be based on estimations and assumptions". However, the scope of this paper is to analyze the qualitative dynamics of the reduced-form PEDA model. For this specific purpose, we assume ad hoc parameters in the first step. In following papers, the model is applied to real data from sub-Saharan African countries, as far as they are available.

3.1 Zero population growth – Isolating the impact of the inequality in the food distribution

The assumption of zero population growth implies that total population will be constant and we only need to concentrate on either equation (1) or equation (2). We are left with a system of two differential equations in the variables P_I and R:

$$\dot{P}_I = M(P_I, P - P_I, R) \tag{8}$$

$$\dot{R} = g(R) - D(P_I, P - P_I, R). \tag{9}$$

We can visualize the dynamics of the system by plotting the time evolution of both variables in a so called phase portrait (see Figure 1). The equilibria of the system are determined by the intersection of the two isoclines $\dot{P}_I = 0$ and $\dot{R} = 0$ which are given by the following implicit equations:

$$\dot{P}_I = 0: \quad P_I = \begin{cases} P\left(\frac{y^*}{\alpha y(P_I,R)}\right)^{1/(\alpha-1)} & \text{if } y^* \leq \alpha y \\ P & \text{otherwise.} \end{cases} \tag{10}$$

$$\dot{R} = 0: \quad P_I = \frac{a}{\gamma}\frac{\bar{R}}{P}(\bar{R} - R)\frac{(R+\eta)}{R}. \tag{11}$$

The line where the maximum per capita income αy coincides with the subsistence requirement $\alpha y(P_I, R) = y^*$ (as represented by the gray line in Figure 1) splits the plane into two regions. For values of P_I and R above this border, the first isocline is given by

$$P_I = P\left(\frac{y^*}{\alpha y(P_I, R)}\right)^{1/(\alpha-1)} \tag{12}$$

which can be solved for R. (Isoclines are indicated by dotted lines in Figure 1.) The first isocline might exhibit a hump shape pattern depending on the parameters of the production function β_1 (β_2), h_I (h_S) and on the degree of inequality α. (See Appendix A.1 for further calculations.) For values of P_I and R beneath the border, the first isocline is simply a vertical line from the

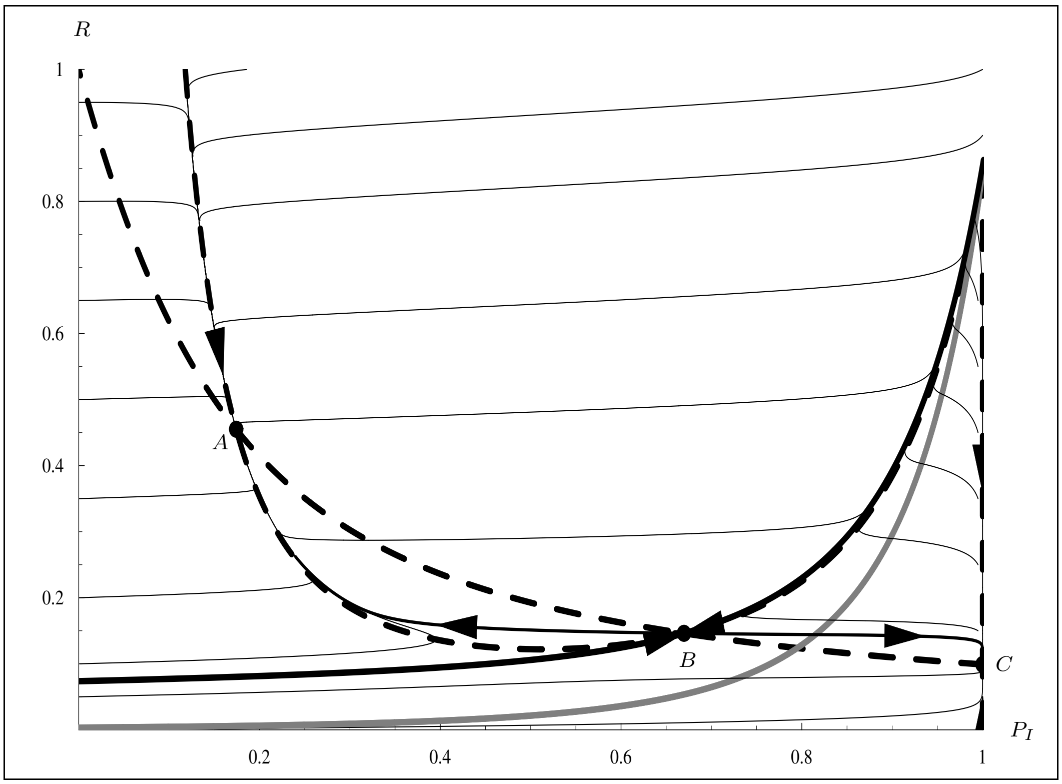

Figure 1: Phase portrait for the parameter values $a = 0.01, \alpha = 1.5, \beta_1 = 0.8, \beta_2 = 0.2, T = 1, \eta = 1, \gamma = 0.1, h_I = 0.2, h_S = 0.8, P = 1, \bar{R} = 1, y^* = 0.4$.

abscissa to the border at $P_I = P$. Along the second isocline the share of the food-insecure population is decreasing in the level of the resource stock.

As evident from Figure 1 the shapes of the isoclines might imply multiple equilibria. While the upper left (A) and lower right equilibrium (C) are stable nodes, the middle equilibrium exhibits saddle-point stability. The heavy bold arrows in Figure 1 indicate the stable manifolds of the saddle-point equilibrium which define the border of the basin of attraction to either stable equilibria. For initial values to the left of the stable manifold, the system will converge to a stable equilibrium characterized by a high stock of resources and a low level of the food-insecure population. But starting to the right of the stable manifold, resources become nearly extinct and the whole population becomes food insecure. The saddle point equilibrium is only reached for initial values on the stable manifolds. This equilibrium can be characterized by a modest stock of resources and a modest share of food-insecure population of the total population. Summing up, history, as represented by the initial values of the share of food-insecure population and resources, will determine the long run distribution of the population and the corresponding resource stock.

The shape of the stable manifold of the saddle point equilibrium, which delineates the region of attraction to either equilibrium (A) or (C), indicates that for low initial values of the share of food-insecure population the initial level of the resource stock will essentially determine whether the economy converges to a sustainable equilibrium or whether resources are almost depleted and all people become food insecure as represented by point (C). The higher the initial value of the share of food-insecure population, the more the long-term values of the share of food-insecure population and of the resource stock are determined by the initial value of the food-insecure population. In fact, these results highlight the role of degradation and its repercussion on production. The higher the initial value of the food-insecure population, the more severe the resource stock is degraded. By assumption the resources are necessary for production, i.e. $Y \to 0$ if $R \to 0$. Hence, the maximum income in the economy αy will fall below subsistence requirement y^* if the resource stock falls below the gray line.

Besides history, the degree of inequality α, will also influence the long-run distribution of population and the corresponding resource stock. A change towards a more unequal food distribution, as reflected by an increase in the degree of inequality, will, ceteris paribus, increase the share of the food insecure population as more food is distributed to the upper classes. But as more food is distributed to the upper classes the maximum food entitlement in the economy as given by αy will increase and henceforth the probability that all population will be food insecure will decline.

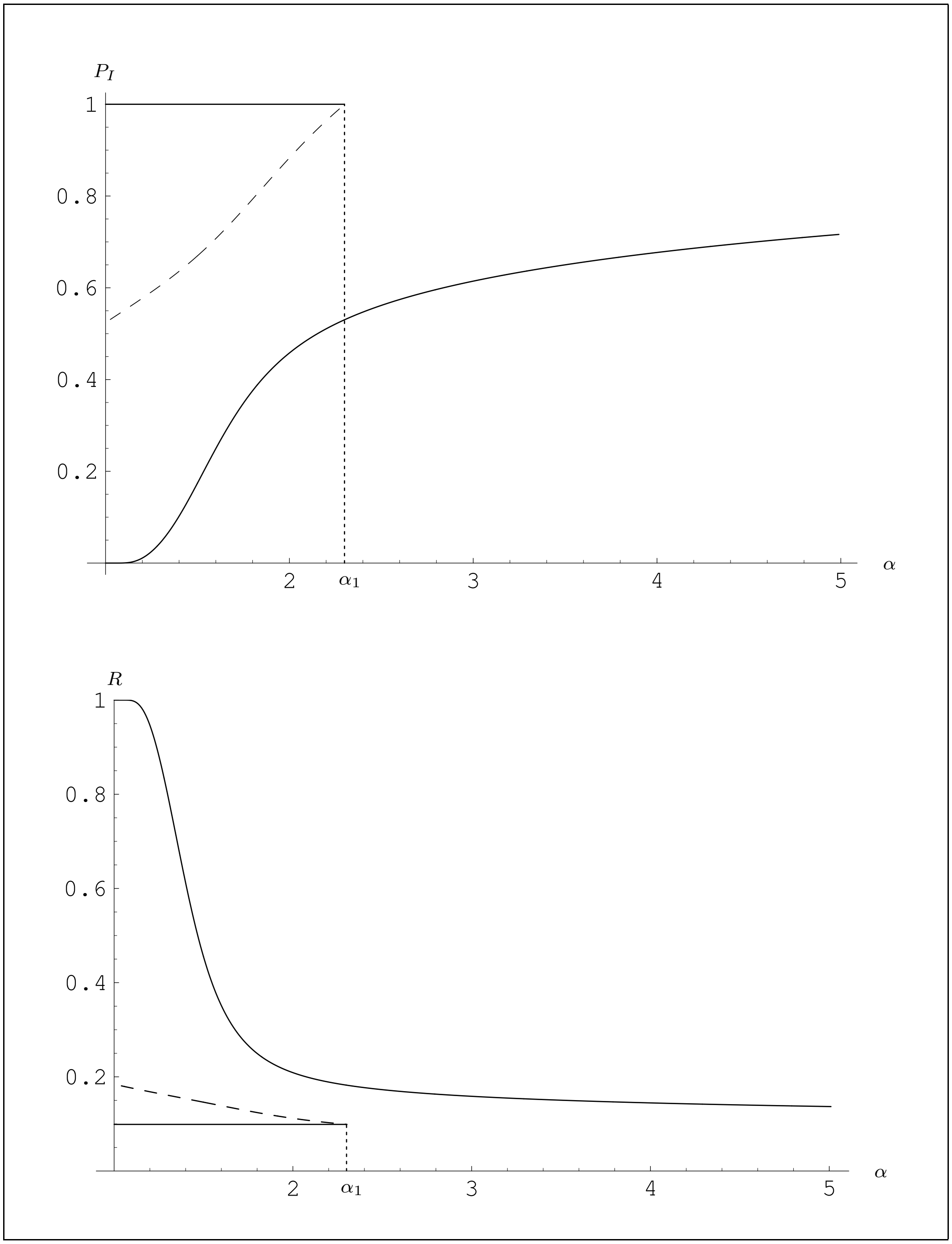

Figure 2: Bifurcation diagram of the steady state values of R and P_I with respect to the parameter α. All other parameters are set as in Figure 1.

Plotting the long-run value of the percentage of the food-insecure population against the degree of inequality in the food distribution α (Figure 2) confirms these results. For low values of α either all or only a small percentage of the population is food insecure where the dashed line separates the domain of attraction to either equilibrium. Clearly, if there does not persist any inequality in the distribution of income, i.e. $\alpha = 1$, all people will be either food secure ($P_I = 0$) if $y \geq y^*$ or food insecure ($P_I = 1$) if $y < y^*$. The specific value of per capita income y will be determined by the initial share of food-insecure population and the initial resource stock. This follows from the fact that the initial share of the food-insecure population will determine the value of the initial per capita output and hence the share of the food-insecure population. Since the labor efficiency of the food-insecure people is lower than the labor efficiency of the food-secure population, a higher stock of food-insecure people implies lower levels of per capita output. Similarly, the higher the initial resource is, the higher is per capita output.

As the degree of inequality increases, the maximum food entitlement in the economy αy increases so that at least part of the population will be food secure and the equilibrium where all the population is food insecure no longer exists. But at the same time an increase in the degree of inequality will increase the share of the food-insecure population (compare Figure 2).

The existence of multiple equilibria allows for a hysteresis effect. Suppose, the economy starts from an equilibrium configuration where the entire population is food insecure. Increasing the degree of inequality of the food distribution, the economy remains in the initial equilibrium configuration at first. If the degree of inequality rises beyond a certain threshold where the equilibrium vanishes (i.e. α_1 in 2), then the economy approaches a different steady state with a lower share of food-insecure population and a higher stock of resources. Reducing the degree of inequality, the economy stays at the new equilibrium branch, even if the threshold is passed. This lack of reversibility is called hysteresis (Strogatz, 1994, p. 60).

3.2 The role of population growth

In this section, we also allow indigenous population growth. Hence, we investigate the complete 3-dimensional model.

$$\dot{P}_I = n_I P_I + M(P_I, P_S, R) \tag{13}$$

$$\dot{P}_S = n_S P_S - M(P_I, P_S, R) \tag{14}$$

$$\dot{R} = g(R) - D(P_I, R). \tag{15}$$

A steady state is obtained by setting equations (13)-(15) equal to zero:

$$\dot{P}_I = n_I P_I + M(P_I, P_S, R) = 0 \tag{16}$$

$$\dot{P}_S = n_S P_S - M(P_I, P_S, R) = 0 \tag{17}$$

$$\dot{R} = a(\bar{R} - R) - \gamma \frac{P}{\bar{\bar{R}}} P_I \frac{R}{R+\eta} = 0. \tag{18}$$

Combining equations (16) and (17) implies $n_I P_I + n_S P_S = 0$. If we assume that n_S and n_I are both positive, then there does not exist an equilibrium. Furthermore, the system would always 'crash'. This is basically the result of exponential growth of population, while resource growth is constrained.[10]

Hence, for an equilibrium to exist we need that one of the population growth rates is less than zero. Solving equations (16)–(18) leads to a unique interior equilibrium (For details on the calculation of this equilibrium refer to Appendix A.2).

In order to determine the stability of this equilibrium, we consider two numerical examples, presented in the next paragraphs. As mentioned at the beginning of the section, we will first consider an example for a positive natural growth rate for the food-insecure population and a negative natural growth rate for the food-secure population. Within this numerical example, we want to ask, whether the negative indigenous population growth of the food-secure population is sufficient to ensure a sustainable steady state.

Finally, we consider a numerical example, where the natural population growth rate of the food-insecure population is negative, whereas the population growth rate of the food-secure population is positive. One might argue, that this assumption is unrealistic. But one could imagine, that the death rate of the food-insecure population is sufficiently high to exceed the birth rate, since the per capita food entitlement of the food-insecure population is lower than the per capita food entitlement of the food-secure population per definition.

As mentioned above, we consider first a numerical example, where the natural population growth rate is positive (negative) for the food-insecure (secure) population. Considering the parameter values given in Table 1 yields that the unique interior equilibrium is unstable with a two-dimensional stable manifold. Consequently, this equilibrium is approached only for initial values lying on the stable manifold. For all other initial values, the long-term behaviour can be described by either that population almost grows exponentially and resources become extinct or the resource stock grows up

[10] For the sake of completeness, the result with both negative population growth rates, although unrealistic, is mentioned. Since population is shrinking, the resources grow up to its carrying capacity and the population becomes extinct.

Parameters	Values
α	1.5
β_1	0.8
β_2	0.2
a	0.01
γ	0.01
η	1
T	1
y^*	0.4
$\bar{R}$	1
h_I	0.2
h_S	0.8

Table 1: Parameter values

to its carrying capacity, whereas population shrinks to extinction. The stable manifolds of the unstable equilibrium determine which of the long-term behaviours is approached like in the case of zero population growth.

The existence of a non-trivial (though unstable) equilibrium requires a continuous positive flow of former food-insecure to food-secure population in order to maintain strictly positive values of the food-secure population. This flow must be high enough to counteract the negative population growth rate of the food-secure population and to compensate the positive natural population growth of the food-insecure population. Therefore, for low values of α the resource stock must be low, otherwise the flow to the food-secure population would be too high and population would shrink. Similarly, for high values of α the resource stock must be high in equilibrium, otherwise the transition from food-insecure to food-secure population would be too low and we would end up with extinct resources and a growing population, where all are food insecure.

Plotting the equilibrium values as a function of the degree of inequality (see Figure 3) confirms these considerations. As Figure 3 shows, the corresponding equilibrium number of population is high if the degree of inequality is low. In the opposite, the steady-state population stock is low if the degree of inequality of the distribution of food is high. This result reflects the relationship of the environment and population in the concept of sustainable development. If the degree of inequality is low, the balanced resource stock is high and, hence, a high number of people (with constant share of food-insecure and food-secure population) can be sustained. However, the higher is the degree of inequality, the lower is the the equilibrium resource stock and the less people can be supported.

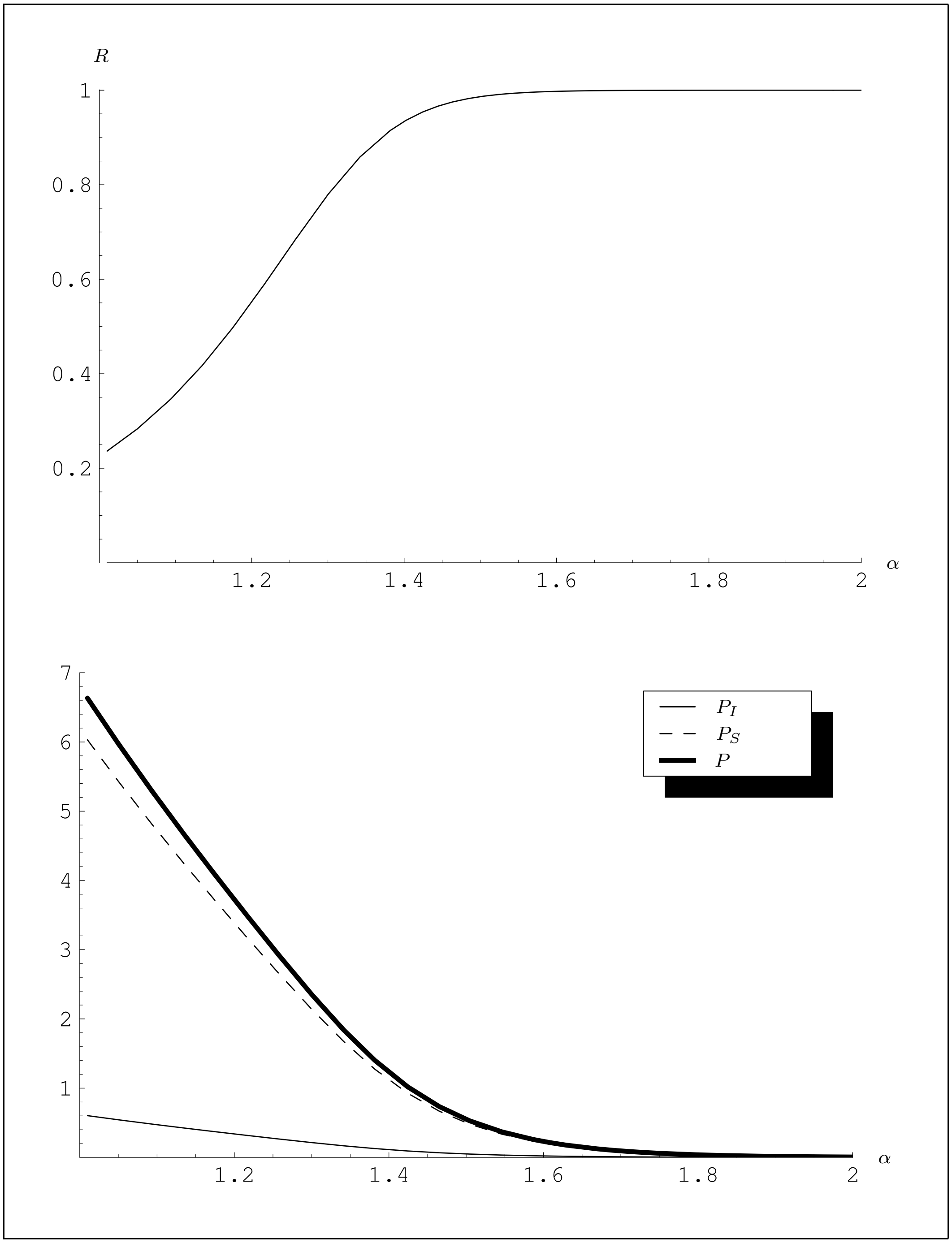

Figure 3: Bifurcation diagram of the unstable equilibrium with respect to the inequality parameter α. The natural population growth rates are assumed to be $n_S = -0.001$ and $n_I = 0.01$, all other parameters are set according to Table 1.

Finally, we consider positive natural population growth rates for the food-secure population and negative natural population growth rates for the food-insecure population. Therefore, we have assumed the natural population growth rates to be $n_S = 0.01$ and $n_I = -0.001$ and for all other parameters we stick to the set as given in Table 1.

In fact, this specific parameter constellation yields a limit cycle where the time series of resources and the food-secure and food-insecure population alternate over time. The limit cycle in the phase space is shown in Figure 4, while Figure 5 plots two dimensional projections of the cycle in the phase planes of total population versus the share of the food insecure population and total population versus resources. Figure 6 shows the time series of the share of food insecure population $\frac{P_I}{P}$, the total population P and resources R along such a cycle. The dynamics can be summarized as follows: if all population is food insecure, the total population will decline and resources can regenerate. Once a sustainable level of resources and population has been restored the share of food-insecure population declines. But as the share of food-insecure population declines, total population will grow again and resources will be jeopardized again. As resources decrease, the share of food-insecure people increases again until all population is food-insecure and the cycle starts over again.

Next, we want to answer the question if the cycle persists, if we vary the degree of inequality. Therefore, we provide bifurcation diagrams (see Figures 7, 8 and 9), where we plot the minimum and maximum values of the cycle and the equilibrium values of the share of food-insecure population, resources and total population as a function of the degree of inequality. As it can be seen from Figure 7 (and as derived in Appendix A.2) the equilibrium share of food-insecure population is independent of the degree of inequality. Not surprising, the equilibrium values of the resources fall and the equilibrium values of the total population increases as the degree of inequality rises.

Further, as the degree of inequality rises, the limit cycle shrinks, as indicated by a rising minimum value of the share of food-insecure population on pursuing the limit cycle.

Shouldn't be the period also be investigated?

As a certain threshold is reached, an unstable limit cycle emerges and the equilibrium becomes stable. In more mathematical terms, the equilibrium undergoes a sub-critical Hopf bifurcation. Now, there exist two different attractors – a stable equilibrium and a stable limit cycle, where the domain of attraction is separated by the unstable limit cycle. By the emergence of the unstable limit cycle, the basin of attraction of the stable limit cycle is reduced. If the degree of inequality rises further, the unstable limit cycle grows, further reducing the domain of attraction of the stable limit cycle. Hence, the probability, that all population is food insecure at least for a

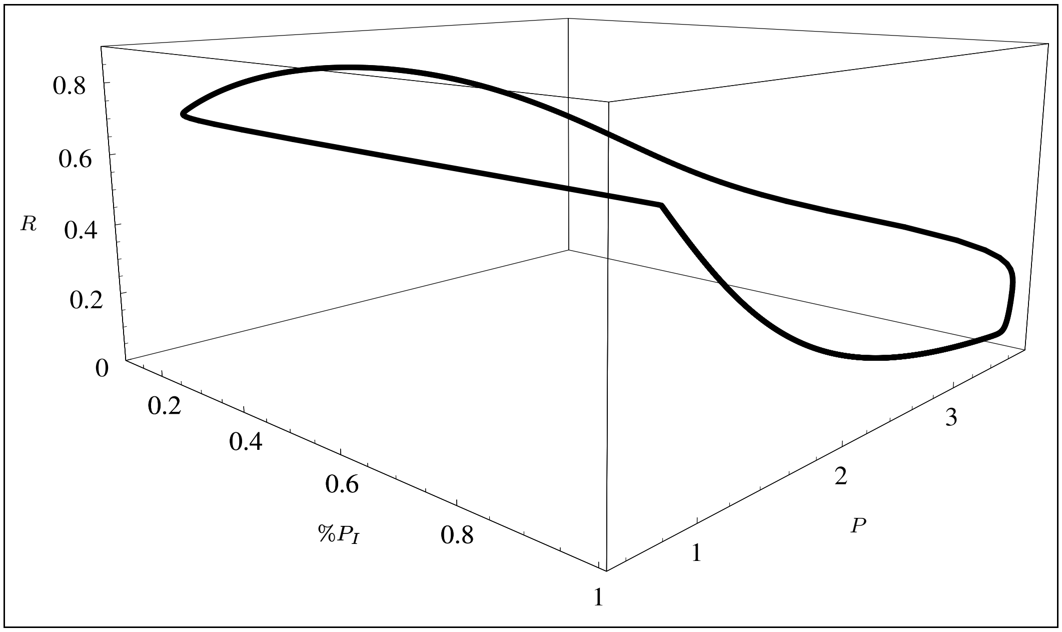

Figure 4: Limit cycle in the phase space of the share of the food-insecure population, s, versus total population, P, versus resource stock, R. The natural population growth rates are assumed to be $n_S = 0.01$ and $n_I = -0.001$, all other parameters are set according to Table 1.

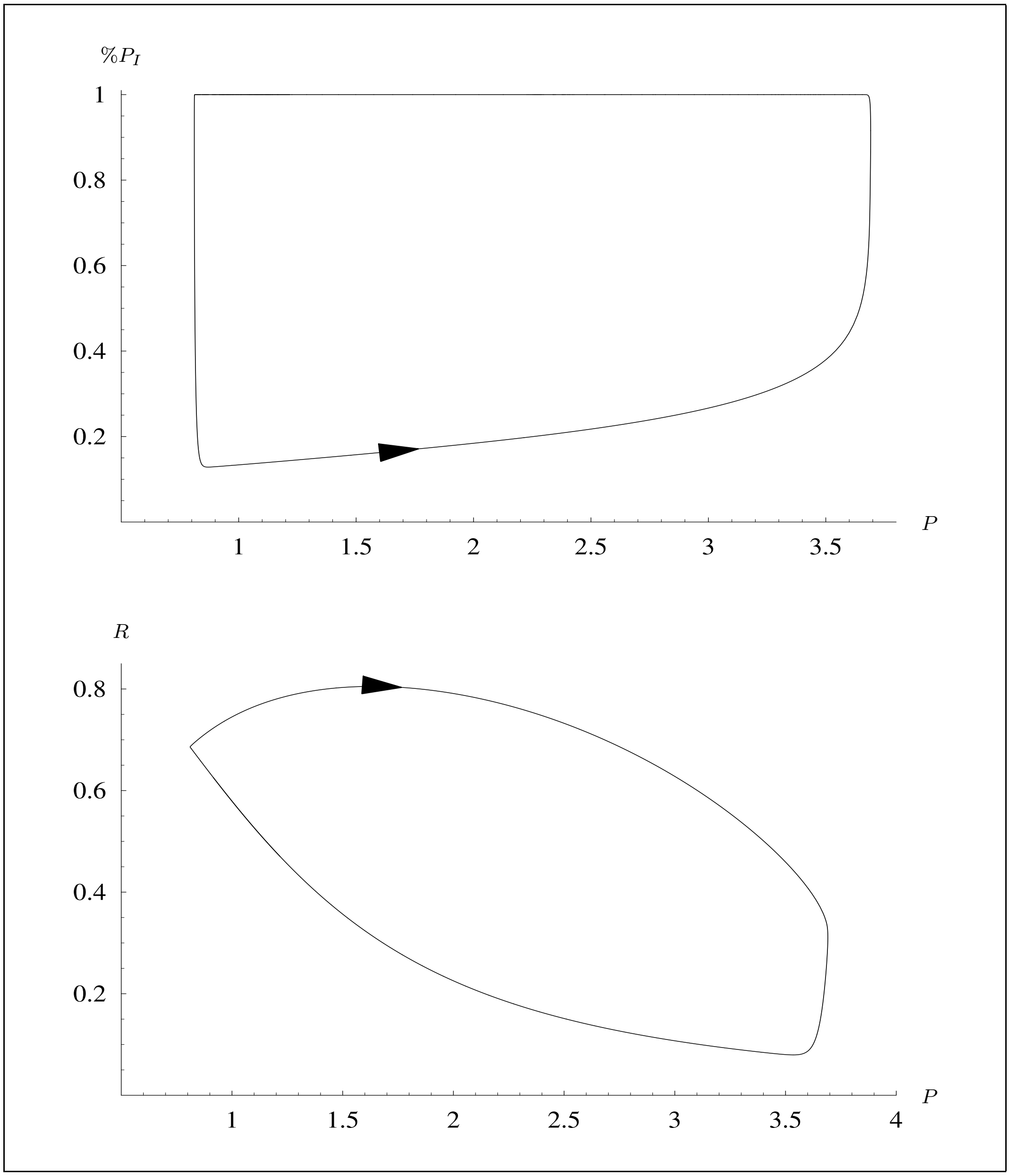

Figure 5: Limit cycle in the phase plane of total population versus the share of the food insecure population (above) and in the phase plane of total population versus resources (below). The natural population growth rates are assumed to be $n_S = 0.01$ and $n_I = -0.001$, all other parameters are set according to Table 1.

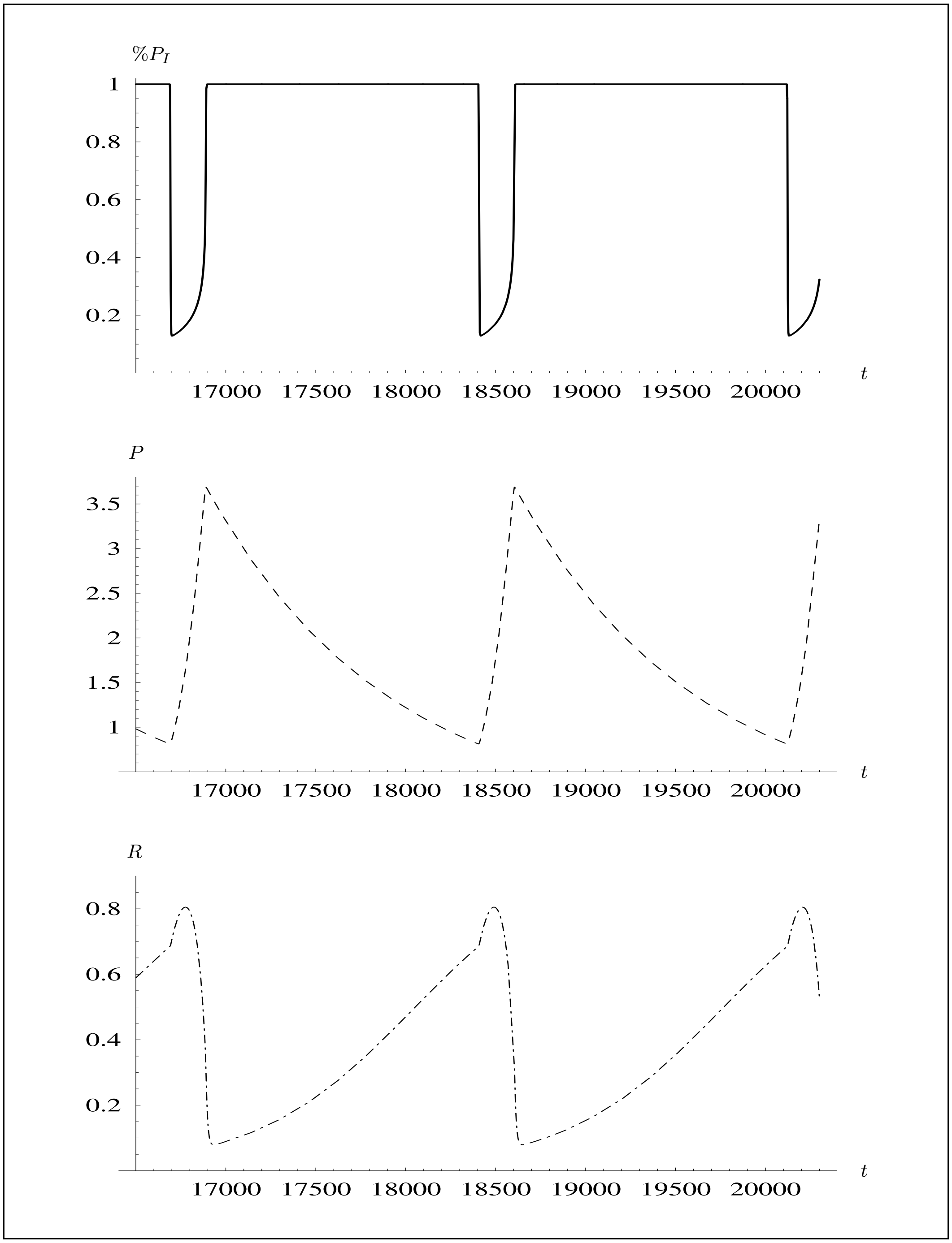

Figure 6: Time series of the share of food-insecure population, total population and resource stock. The natural population growth rates are assumed to be $n_S = 0.01$ and $n_I = -0.001$, all other parameters are set according to Table 1.

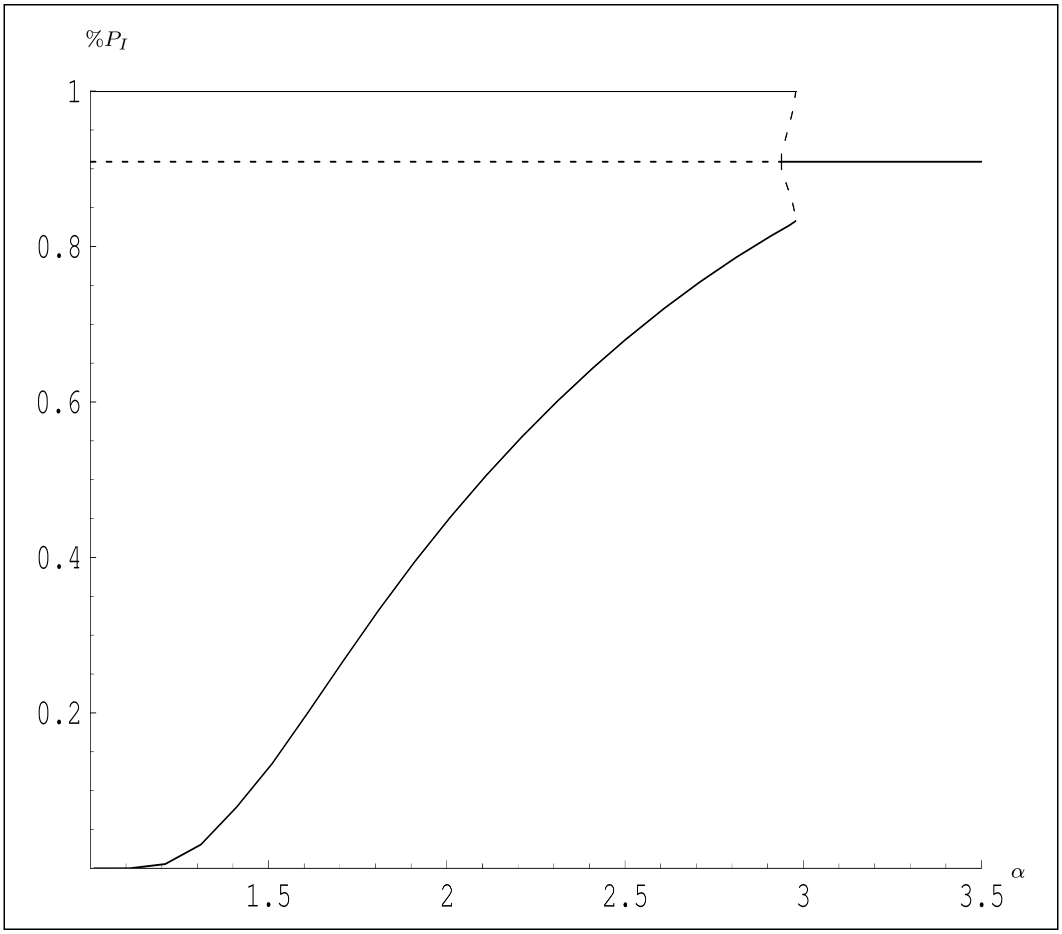

Figure 7: Bifurcation diagram of the share of food-insecure population, $\%P_I$, w. r. t. the inequality parameter α. The natural population growth rates are assumed to be $n_S = 0.01$ and $n_I = -0.001$, all other parameters are set according to Table 1.

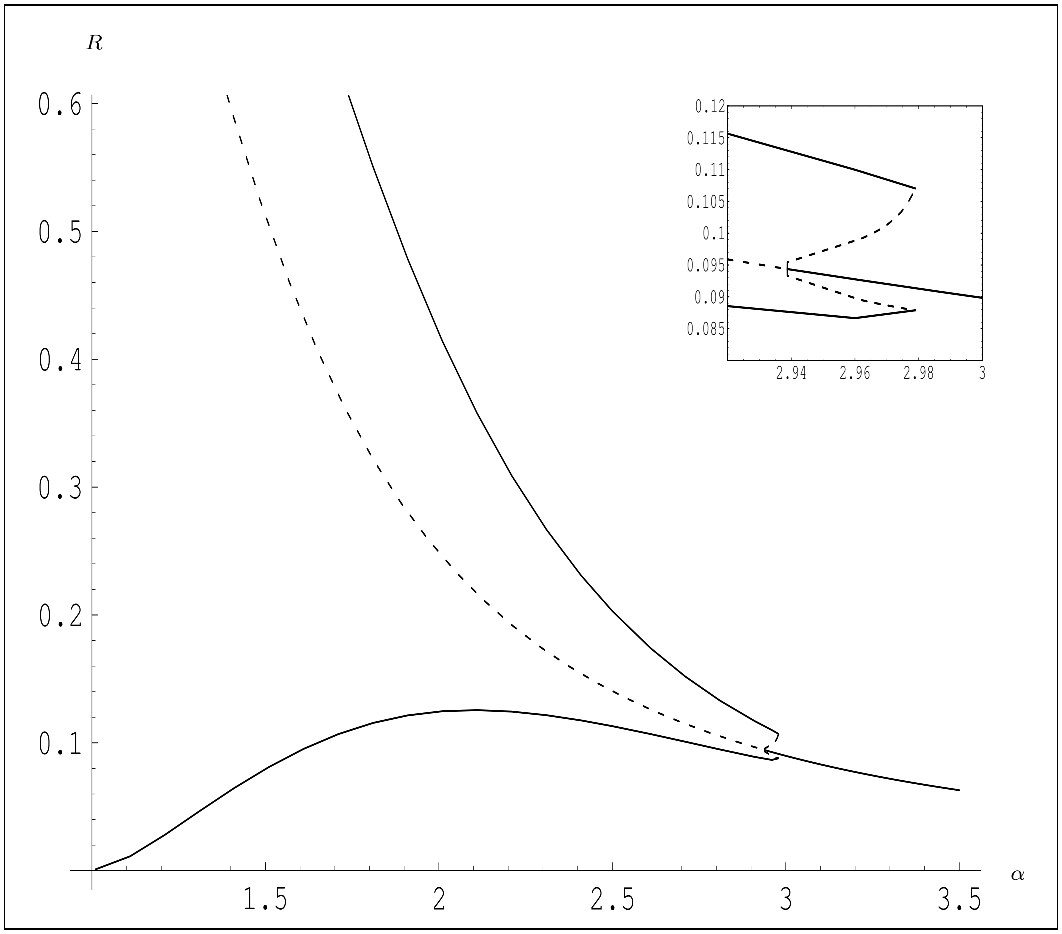

Figure 8: Bifurcation diagram of the resources, R, w. r. t. the inequality parameter α. The natural population growth rates are assumed to be $n_S = 0.01$ and $n_I = -0.001$, all other parameters are set according to Table 1.

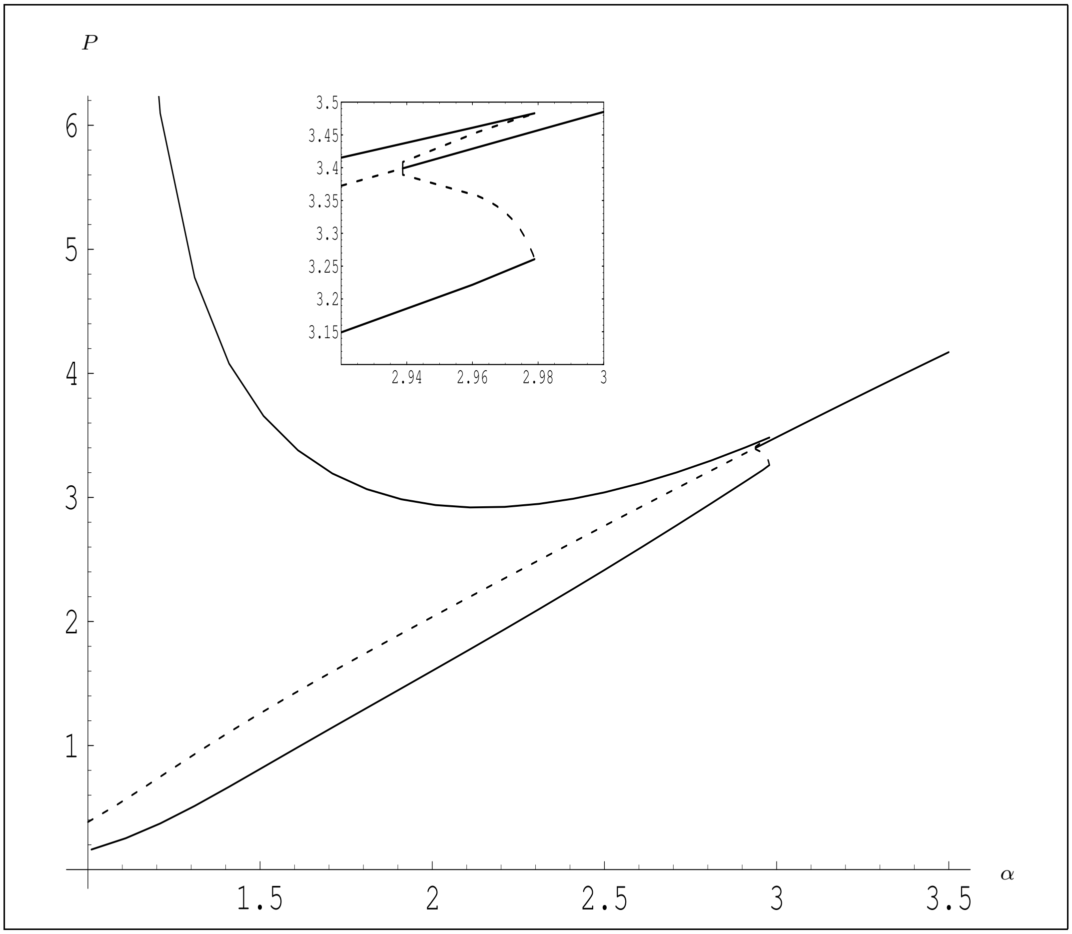

Figure 9: Bifurcation diagram of the total population, P, w. r. t. the inequality parameter α. The natural population growth rates are assumed to be $n_S = 0.01$ and $n_I = -0.001$, all other parameters are set according to Table 1.

short period (as happening on the stable limit cycle), shrinks as the degree of inequality increases. We conclude, that the reason for this result lies, similar to the case with zero population growth, in the fact, that the maximum food entitlement, prevailing in the economy, increasing with rising degree of inequality, thus reducing the probability that the entire population is food insecure.

As the inequality parameter α increases further, the unstable and stable limit cycle collide and vanish via a fold bifurcation, leaving the stable equilibrium.

Finally, we summarize our results in the following table 2, where the first column contains the assumptions about population growth and in the second column the long-run behaviour is described.

3.3 Endogenous Mortality

In the following paragraphs we want to ask whether the limit cycles persist if we allow mortality to increase as food production declines. Hence, we endogenously define death rates for the food-insecure and the food-secure population, i.e. $d_I = d(y_I)$ and $d_S = d(y_S)$, where y_I and y_S denote per capita food entitlement of the food-insecure and food-secure population, respectively. Recalling that the Lorenz curve plots the cumulative share of income versus the cumulative share of the population, if population is ranked in increasing order of income, then $L(P_I/P)$ denotes the share of total food entitlement the food-insecure population receives. Consequently, the per capita food entitlement for the food-insecure and the food-secure population are defined as following:

$$y_I = \frac{YL\left(\frac{P_I}{P_I+P_S}\right)}{P_I+\epsilon}, y_S = \frac{Y(1-L\left(\frac{P_I}{P_I+P_S}\right))}{P_S+\epsilon}, \tag{19}$$

where we add the small positive constant ϵ to guarantee non-zero denominators. In particular, we assume the following functional form for the death rate

$$d(y_j) = d_{max} - d_1\frac{y_j}{y_j+d_2}, \qquad \text{for } j = I, S. \tag{20}$$

Furthermore, the dynamical system (13)–(15) transforms to

$$\dot{P}_I = (b_I - d(y_I))\, P_I + M(P_I, P_S, R) \tag{21}$$

$$\dot{P}_S = (b_S - d(y_S))\, P_S - M(P_I, P_S, R) \tag{22}$$

$$\dot{R} = g(R) - D(P_I, R), \tag{23}$$

Population growth	**Long run behaviour**
$n_I = 0$, $n_S = 0$	At least one stable equilibrium, at most two stable equilibria and a saddle-point in between, where the stable manifolds define the border of the domain of attraction. The stable equilibria can be characterized either by a low share of food-insecure population and a modest stock of resources or total population is food insecure and resources are almost extinct.
$n_I > 0$, $n_S > 0$	Population grows nearly exponentially and resources become extinct.
$n_I < 0$, $n_S < 0$	Resources grow up to its carrying capacity and population shrinks.
$n_I > 0$, $n_S < 0$	There exist an unstable equilibrium, with a two-dimensional stable manifold, which defines the border of the domain of attraction like in the case of zero population growth. For all initial values not lying on the stable manifold, either resources become extinct and population grows nearly exponentially or resources grow up to the carrying capacity and population shrinks.
$n_I < 0$, $n_S > 0$	There exists either a stable equilibrium or an unstable equilibrium surrounded by a stable limit cycle or a stable equilibrium and a stable limit cycle with an unstable limit cycle in between.

Table 2: Summary of the results.

where b_I and b_S denote the crude birth rates of the food-insecure and food-secure population, respectively.

4 Conclusions and Extension

In our simple framework we were able to investigate the impact of the degree of inequality in the distribution of food on environmental degradation and its repercussion on per capita food production. By assuming zero population growth we were able to show that the inequality in the distribution of food is one of the driving factors of environmental degradation and persisting food insecurity in our framework. In particular, we showed that the economy may be locked by a poverty trap. For very low values of resources or for high initial shares of food-insecure population, the economy tends to a degraded environment where the entire population is food insecure in the long run. But, if the initial stock of resources is high enough, then the vicious circle may become virtuous. Then the economy may approach a situation with a low fraction of food-insecure population and a relatively high stock of resources. Summing up, these results highlight the role of poverty-driven environmental degradation and its repercussion on food production. Further, the results indicate, that the vicious circle can be broken if the resource stock is high enough.

Furthermore, we analyzed the dynamic behaviour of the food distribution mechanism. Prskawetz et al. (2000) showed with fixed resources, that the food distribution mechanism (employing a Lorenz curve) allows for the possibility of an hysteresis effect as the degree of inequality in the distribution of food is varied. As section 3.2 demonstrates, the hysteresis effect persists with dynamic resources. Hysteresis in this context means, that starting in an equilibrium configuration, where the entire population is food insecure and increasing the degree of inequality sufficiently may result in a long-run state, where a part of the population is food secure. Reducing the degree of inequality back to the initial value, the economy stays at the equilibrium branch, where a positive share of the population is food secure. The reason behind the hysteresis effect lies in the fact, that the degree of inequality also determines the maximum food entitlement prevailing in the economy. If the degree of inequality rises, then the maximum food entitlement increases and, hence, the probability, that the entire population is food insecure, shrinks eventually to zero.

Further, we investigated the role of population growth in amplifying environmental degradation and food insecurity. By assuming exogenous population growth rates we were able to analyze the impact of population growth on the vicious circle. Obviously, there does not exist an sustainable equilibrium with positive natural population growth rates for both sub-populations. Since the resources are constrained, the system will always ‘crash’. Consequently, opposite signs of the natural population growth rates for population is a necessary, but as our results demonstrate, not sufficient condition for sustainable development. For a negative natural population growth rate of the food-secure population, there exists a non-trivial steady state, but it is only approached for a non-generic set of initial conditions.

In the contrary, negative natural population growth for the food-insecure population may ensure sustainable development in terms of long-run stationary values of population size and resources or in terms of persistent oscillations. The latter is characterized by opposite developments of resources and population, i.e. when population grows, then the environment degenerates and when population shrinks, then the environment regenerates. Furthermore, when the population rises, then the share of food-insecure population increases and when population size decreases, then the share of food-insecure population stays at its maximum and decreases afterwards.

In a following step we extend the current work by also incorporating the feedback mechanism from per capita food production on natural population growth rates. That is, we endogenously define death rates of each subpopulation.

A Appendix

A.1 Discussion of the isocline in the case of zero population growth

The part of the first isocline given by (12) is monotonically decreasing/increasing in P_I for P_I smaller/greater than a threshold level $\tilde{P}_I$, which is defined as following

$$\tilde{P}_I = \frac{1}{1+\frac{\beta_1}{\alpha-1}} \frac{1}{2-\frac{1}{h_S}} P. \tag{24}$$

Summing up, this part of the first isocline exhibits a U-shaped pattern with the minimum at $P_I = \tilde{P}_I$. However, if $\tilde{P}_I > P$ holds, then this part of the first isocline is only a decreasing function of the number of food-insecure population.

A.2 Derivation of the unique equilibrium in the case of natural population growth rates with opposite signs

As already mentioned above, combining equation (13) and (14) yields

$$n_I P_I + n_S P_S = 0. \tag{25}$$

Equation (25) shows that the stock of food-insecure population is proportional to the stock of food-secure population in equilibrium and vice versa. Furthermore, the equilibrium share of food-insecure population is given by

$$\%PI = \frac{PI}{PI+PS} = \frac{1}{1-\frac{n_I}{n_S}}.$$

First, we have to consider the case that the maximum food entitlement is less than the required minimum level and therefore total population will become food insecure. In this case the transition is equal to the number of food-secure population and the dynamics of the food-secure population simplifies to

$$\dot{P}_S = (1-n_S)P_S.$$

Hence, the unique equilibrium of (A.2) is $P_S = 0$. This implies that also the number of food-insecure population is equal to zero. Therefore, we are left with the case

$$M = \left(\left(\frac{y^*}{\alpha y} \right)^{1/(\alpha-1)} - \frac{P_I}{P_I+P_S} \right) (P_I+P_S).$$

Substituting this expression in one of the equations (16) or (17) and taking equation (25) into account, we derive the following equilibrium condition

$$P_I = \underbrace{\left(\frac{\alpha T}{y^*} \frac{(h_I - h_S \frac{n_I}{n_S})^{\beta_1} (1 - n_I)^{\alpha - 1}}{(1 - \frac{n_I}{n_S})^{\alpha}} \right)^{1/\beta_2}}_{=:\Psi} R. \tag{26}$$

Taking equation (25) into account, equation (18) can be transformed to

$$P_I^2 = \frac{a\bar{R}(\bar{R} - R)(R + \eta)}{\gamma(1 - \frac{n_I}{n_S})R}. \tag{27}$$

Combining equations (26) and (27) yields an equation of third order, i.e.

$$\gamma\Psi^2(1 - \frac{n_I}{n_S})R^3 + a\bar{R}R^2 - a\bar{R}(\bar{R} - \eta)R - a\bar{R}^2\eta = 0. \tag{28}$$

According to the rule of Descartes, there exists one positive real solution for R. Hence, there exists a unique interior equilibrium.

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