A note on the compression of mortality

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Abstract

The rapid increase of human longevity has brought up important questions about what implications it may have for the variability of age at death. Earlier works reported evidence of a historical trend of mortality compression. However, the period life table model, which is widely used to address mortality compression, produces an artificially compressed picture of mortality as a built-in feature of the model. We base our study on examining the durations of exposure of birth cohorts (also as compared to period mortality schedules) to selected levels of mortality observed at old age. We also address the problem in a more conventional fashion, by examining the distribution of ages at death (in period tables and cohorts) above and below the mode. Overall, mortality has been significantly decompressing already since the 1970s. This finding contradicts with most previously reported results. The decompression of old-age mortality may indicate further optimistic prospects of ever-decreasing mortality. Mortality may well not be concentrated in the future within a narrow age interval but more dispersed along age groups, though at ever later ages on average.

INTRODUCTION

Human longevity continues to increase in developed countries. In currently low-mortality countries, the average duration of life has increased from the pre-modern level of about 35 years to about 75 years for the currently old cohorts. This increase is expected to continue. The United Nations (UN Statistics Division 2009) forecast period life expectancy at birth to increase to about 85 years by 2050 in low-mortality countries, and some even argue this is too pessimistic a scenario (Oeppen and Vaupel 2002, Christensen et al. 2009 and Vaupel 2010 suggest life expectancy at birth to continue to increase by about three months per year; White 2002 proposed a similar linear trend for life expectancy; Lutz et al. 1997 and Sanderson and Scherbov 2004 worked on similar scenarios; Ediev 2011 argues for a linear increase in life expectancy as a result of spreading health improvements along cohort lines).

The rapid increase of human longevity has brought up important questions about what implications it may have for the variability of age at death. Fries (1980), assuming a fixed upper limit to human lifespan, argued that the decline of mortality may lead to a rectangularisation of morbidity and mortality, i.e. compression of both to the oldest age. Such a view is supported by the age pattern of mortality decline, which is characterised by faster decline of mortality at young ages and nearly no change at oldest-old ages (with only a few exceptions, of which Japan is the most notable one).

Later works, while negating any strict link between mortality compression and existence of the upper limit to the lifespan (Carnes et al. 1996; Olshansky et al. 2002) and the very process of mortality decline (Wilmoth and Horiuchi 1999), brought up more evidence of a historical trend of mortality compression, which may have slowed down or even stopped in recent decades in developed countries (Wilmoth and Horiuchi 1999; Kannisto 2000; Canudas-Romo 2008; Thatcher et al. 2010; Engelman et al. 2010). Some authors (e.g. Myers and Manton 1984) argued against the mortality compression, but their evidence was based on methodology with built-in tendency to decompression (see further down).

Mortality compression was argued to be relevant for health-related behaviour (Wilmoth and Horiuchi 1999). Evidence for the compression, even for a stalled one, is important to better understand the prospects of ageing and senescence. Even though the direct link between mortality compression and the existence of an upper limit to the life span was discarded in the literature, the compression of mortality may still be associated with increasing efforts necessary to further extend life expectancy. The shifting mortality hypothesis, which is endorsed by the stalling mortality compression in developed countries during the recent decades (Canudas-Romo 2008), was associated with the inability, so far, to slow the pace of deterioration with age (Vaupel 2010). Understanding the developments of mortality compression is also important for modelling mortality and formulating assumptions about future mortality dynamics (Tuljapurkar and Edwards 2011).

Despite numerous advances, the methodology of studying the mortality compression remains an issue. Existing methods are affected by the in-built biases of the period life table model and age censoring (see critical review of methodological approaches in the next section). Building on earlier analysis of limitations to the conventional period life table model (Ediev 2011), we propose here a new approach to studying mortality compression. Using extensive mortality data and a new, cohort-oriented, methodology, we show that mortality compression at old ages was not a universal trend in the past in low-mortality countries. Furthermore, the whole period since the 1970s, i.e. the time of greatest reductions in old-age mortality, was characterised by strong mortality decompression for males and by contradictive trends concluded by a slight decompression for females in low-mortality countries. Meanwhile, period life tables in low-mortality countries were characterised by mortality compression which has only somewhat slowed in the most recent period. (As mentioned above, the very existence of the mortality compression was challenged already by Myers and

Manton (1984); their methodology, however, was biased towards showing mortality decompression).

In the following two sections, we briefly review existing methodological approaches, reiterate some concerns about the limitations to the conventional period life table model in case of dynamic mortality and show why the period life table model tends to artificially compress the mortality pattern when mortality is on the decline. This provides motivation for a new methodology based on cohort approach, which we present in Section 3. The empirical evidence is provided in Section 4. We present more traditional indicators of compression of the distribution of ages at death in relation to the modal age at death in Section 5 and follow with conclusions.

1 LITERATURE REVIEW ON THE METHODOLOGY OF STUDYING MORTALITY COMPRESSION

Fries (1980) as well as earlier researchers cited by him based their study on a visual inspection of period survival curves as well as on arguments in favour of the existence of fixed limits to the human lifespan. The latter arguments, while implying with necessity a mortality compression accompanying the approach to the lifespan limits, are to be discarded in view of recent and more comprehensive data (e.g., Oeppen and Vaupel 2002, Vaupel 2010).

Myers and Manton (1984) challenged the very existence of mortality compression examining a more complete life table for the US-American population and computing standard deviations of ages of death at ages 60 and over. Those indicators were criticised by Kannisto, however, as being prone to a built-in decompression of mortality (see further down).

Entropy, the Keyfitz's H (Keyfitz 1977), might also be used to objectively assess the degree of compression of mortality. Although it was used in some studies (including the study by Wilmoth and Horiuchi, see next), its usage was limited not least for a lack of clarity in the interpretation of its dynamics, as compared to other measures.

Wilmoth and Horiuchi (1999) stressed the importance of objective indices of compression and considered ten such indices. They chose as their favourite the Interquartile Range of the distribution of ages at death (*IQR*) for its convenience and good correlation with other measures. They reported a historical compression of mortality that was attributable to declining risks of death at young ages but also, to a lesser extent, at ages up to age 75. Finding Fries's prediction of a fixed limiting mortality pattern somewhat inconsistent, Wilmoth and Horiuchi also argued against 'derectangularisation' (Gavrilov and Gavrilova 1991) in favour of "a continuing pattern of stability in the variance of ages at death, as the entire distribution shifts upward."

Kannisto (2000, 2001), following earlier concepts, proposed distinguishing natural and premature deaths, of which the latter ones were responsible for most of the historical mortality compression. Since premature deaths are conceptually a phenomenon of transient effect, which must cease once they are reduced to a possible minimum, Kannisto advocated measuring mortality compression focusing on 'natural' deaths. He also pointed to biasedness of compression indicators, like those used by Myers and Manton (1984) (and, more recently, by Engelman et al. 2010), which are based on studying age intervals left-censored at a fixed old age. When life expectancy is growing and deaths are shifted upward, such indicators have a built-in tendency to show mortality decompression. Assuming the modal age at death, M, as the centre of the distribution of 'natural' deaths, Kannisto proposed using the mode instead of a fixed age limit to left-censor the distribution of deaths in order to eliminate the effects of the premature deaths and shifts of the deaths' distribution. Hence, his main indicator for mortality compression was the standard deviation from the mode of ages at death above the mode, $SD(M+)$. In accordance with Lexis's (1877) suggestion of normal distribution of age at 'normal' death, Kannisto demonstrated that the ratio of $SD(M+)$ to the life expectancy at mode $e(M)$ is close to theoretical value $\sqrt{\frac{\pi}{2}} \approx 1.25$ predicted by the normal distribution (Kannisto estimated this ratio to be about 1.24 in the data).

Notably, the standard deviation of age at death above the mode is also related to another measure, the Fastest Decline (FD), earlier proposed by Wilmoth and Horiuchi (1999) but discarded in favour of the *IQR*. FD is the tangent slope of the survival curve at its steepest decline or, equivalently, to the density function of the distribution of ages at death at the modal age. In this work, we will consider a modification of this index.

More recently, Thatcher et al. (2010) contributed to the research of mortality compression above the mode by examining the conditions for mortality compression in selected mortality models and presenting evidence for mortality compression above the mode in recent data in six countries. Another recent study by Engelman et al. (2010) was based on Myers and Manton's approach to studying mortality compression above selected fixed ages, a methodology, however, which has been criticised by Kannisto for its built-in tendency to show decompression.

Lynch and Brown (2001) developed a new approach to mortality compression by considering the slope of the mortality curve at the inflection point and suggested that mortality has been decompressing since the 1960s. Unfortunately, the inflection of the mortality curve (currently, around age 95 years) takes place at such an old age that the shape of the curve in its vicinity is not very informative about the distribution of most of the deaths; even above the mode (currently, around age 85 years), the overwhelming majority of deaths occur before the inflection point of the mortality curve.

Concluding this short (and incomplete) methodological review, we note the following two important aspects of contemporary approaches to studying the mortality compression, which provide the motivation for our study.

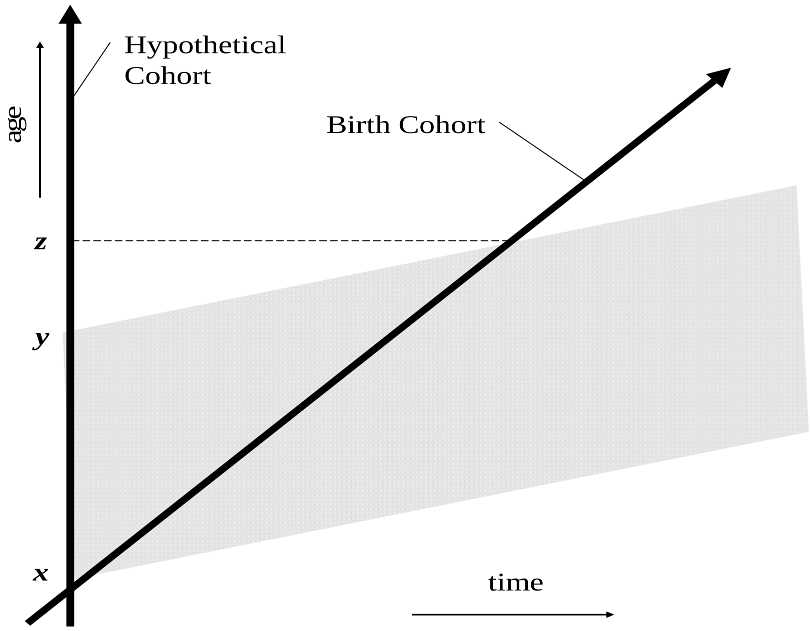
First, researches have been focusing recently on the compression of adult deaths, which is free from the effect of declining infant and child mortality. Both methods used so far to separate the effect of adult mortality, censoring at a fixed or the modal age, have their limitations, however. When censoring at a fixed old age, say, at 60, compression indicators are severely affected by the built-in tendency to show *de*-compression when the deaths' distribution is shifting upward (Kannisto 2001). Focusing on the deaths above the mode only has a disadvantage as well as it ignores a significant part of the mortality experience. Focusing on the distribution of deaths above the mode was motivated by the Lexis's assumption of normally distributed 'natural' deaths, which are topped up by premature deaths below the mode. Although this model is useful in many aspects and reproduces some characteristics of the above-the-mode distribution, it might be outperformed above the mode by the Kannisto's logistic model (Thatcher et al. 1998). Also, it is not quite consistent with mortality dynamics below the mode (see further down in this paper).

Second, most articles on mortality compression dealt with period life tables and reported a compression of the *life table* distribution of deaths. Although some authors (e.g. Wilmoth and Horiuchi 1999, Engelman et al. 2010) did study the compression of mortality from the cohort perspective, that line of research was limited because the results for cohorts who have completed their mortality history are already outdated. Meanwhile, these might be the most recent developments in mortality compression which are of greatest interest. As we have indicated in a different context (Ediev 2011; see also the next section), however, the compression is a built-in feature of the period life table model as such and might not indicate an actual compression of mortality for real people. The need to overcome this limitation of the period life table model is our main motivation for the current study.

2 WHEN THE PERIOD LIFE TABLE MODEL (ARTIFICIALLY) COMPRESSES THE LIFE SPAN

The period life table model describes the current mortality as a combination of currently observed age-specific mortality rates, each of which in fact describes the mortality of a different birth cohort. When there is no systematic temporal change of mortality, the period life table provides a relevant picture of the mortality pattern for each and every cohort observed. When mortality changes over time, however, the period life table induces age-specific compressions or decompressions of the mortality schedule.

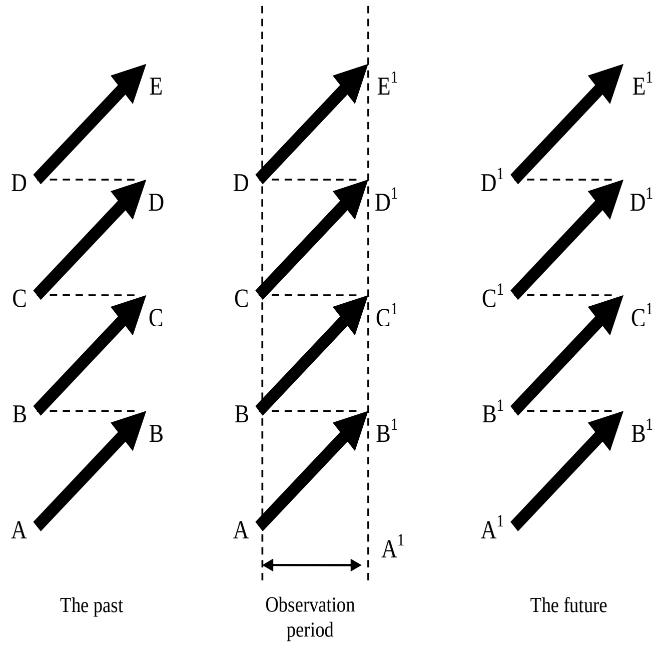
Figure 1 Illustration of the artificial mortality compression imposed by the period life table model



Source: Ediev (2011)

Consider the typical case of adult mortality declining with time and increasing with age. The gray strip in Figure 1 represents the area in the Lexis surface with a given level of mortality. The strip has a positive slope as the same level of mortality is observed at more and more advanced ages. Mortality is higher above the strip and lower below it. The hypothetical cohort of the conventional period life table (represented by the vertical line in the Lexis surface) is 'exposed' to a given mortality level during the period indicated by the age interval ***xy*** in the figure. But the birth cohorts (represented by the bisector) that experience what the period life table is supposed to be a combination of are exposed to the same mortality level over a longer period of time, as indicated by age interval ***xz***. Conventional life tables neglect the real exposures of birth cohorts, cutting off the part of cohorts' experience indicated by age interval ***yz*** in Figure 1. This leads to both overestimation and compression of mortality in the period life table by its very design.

Figure 2 Illustration of the artificial mortality compression assumed in the conventional 'constant mortality' scenario



Note: Arrows depict parts of the life span of birth cohorts in the Lexis surface (age goes along the vertical axis). Capital letters denote different levels of mortality. The panel to the left depicts the stagnant mortality prior to the observation period, the one in the middle depicts the declining mortality during the observation period and the third panel depicts the conventional 'constant mortality' scenario of the future.
Source: Ediev (2011)

Another way to note the artificiality of the mortality compression imposed by the period life table is to consider the conventional 'constant mortality' scenario. As the period combination of mortality rates (the period life table) is considered to provide a full account for the current mortality, the conventional 'constant mortality' scenario assumes time-constant, age-specific mortality rates.

An illustrative example is given in Figure 2 where the arrows correspond to parts of the life span of birth cohorts, and the three panels depict time-invariant mortality prior to the observation period, declining mortality in the observation period, and the future mortality assumed to remain constant at the most recently observed levels. Mortality levels are denoted by capital letters. So, prior to the observation period, mortality used to change from level **A** to level **B** in the first age group, from level **B** to level **C** in the second age group, and so on. Mortality is decreasing during the observation period, so that the youngest depicted cohort starts with mortality level **A** and ends up with mortality level $\mathbf{B}^{1}$, which is lower than the mortality level at the beginning of the observation period for the second youngest cohort (**B**). By a similar logic, every cohort followed in the observation period ends up with a mortality level lower than that of the older cohort at the same age in the beginning of the observation.

In spite of the intuitive soundness of the depicted conventional 'constant mortality' scenario, its closer evaluation reveals the artificial compression of mortality imposed by the period cross-section of mortality levels. Consider, for example, the second age group in Figure 2. In the future, the 'constant mortality' scenario assumes people to pass from mortality level $\mathbf{B}^{1}$ to the level $\mathbf{C}^{1}$ while passing through the age group under consideration. Already in the current year, however, the second youngest cohort in the illustration has moved from the higher mortality level **B** to the same eventual mortality level $\mathbf{C}^{1}$, all the while being in the same age group. Hence, against intuition, individuals' health will deteriorate faster in the future than it happens for those currently observed: although they start off with better health conditions (as indicated by lower mortality), they do not end up being healthier than the current population by the end of the age group. Paradoxically, time-invariant death rates in the future imply a sudden acceleration of health deterioration for individuals if the time-invariant phase is precluded by a period of mortality decline. The ‘ideal’ conventional period life table, skimming forces of mortality along a vertical (time) line in the Lexis surface, mistakes the difference between the age when an individual experiences a force of mortality $\mathbf{B^1}$ and the age when another individual experiences mortality level $\mathbf{C^1}$ for the duration of *time* over which an individual moves from level $\mathbf{B^1}$ to level $\mathbf{C^1}$. However, a difference between ages may indicate time intervals only within the same cohort. Replacing the actual time intervals over which individuals are exposed to different mortality conditions by cross-cohort differences in age compresses the age pattern of mortality and accelerates ageing in the conventional ‘constant mortality’ scenario.

Hence, distorting the age pattern of time-variant mortality is a built-in feature of the period life table (and of any other period schedules such as of morbidity, for example). When mortality is on the decline, the period life tables will tend to indicate mortality compression as a methodological artifact and not as the reflection of actual biomedical developments. By a similar logic, the period life tables may (artificially) indicate age decompression of time-increasing mortality (the mortality decompression reported for Russia in Tesárková et al. 2010 might be a candidate for such a situation). Therefore, studies of mortality compression (and exactly so of morbidity compression) should be conducted based on cohort mortality schedules. (Another possibility, which we do not consider here, would be to study period indicators after appropriate adjustments for exposure distortions: Ediev 2008, 2011. General theory shows that the period life table compresses the exposure to a given mortality/morbidity or any other life condition by $1/(1-r)$ times, where r is the annual change of age at which the condition is observed.)

3 NEW METHODOLOGY TO EXAMINE COMPRESSION

In view of the limitations of the period life table model, we base our study on contrasting the compression in cohort and period age schedules of mortality. We do recognise that the complete picture of cohort mortality is usually available only after it gets significantly outdated for studying contemporary developments in mortality. Therefore, we do not focus on the distributional characteristics of ages at death as it is usually done in the literature on mortality compression. Instead, our main method is based on examining the durations of exposure of birth cohorts (also as compared to period mortality schedules) to certain levels of mortality observed at old age. However, we will also consider, in a possible detail, distributional characteristics of ages at death beyond and below the modal age at death, which has attracted special attention in the literature (see Section 4).

To examine the compression of durations of exposure to various mortality levels, we have chosen six ranges of the annual death rate prevailing at old age: death rates 0.01 to 0.011, 0.02 to 0.021, 0.05 to 0.051, 0.1 to 0.101, 0.15 to 0.151, and 0.2 to 0.201. These levels of mortality are currently observed at ages around 61 to 90 years for males and 68 to 92 years for females in low-mortality countries and cover way beyond the modal ages. To estimate the exact durations of exposure to the selected ranges of the death rate, we use a linear approximation of the age schedule of the death rate's logarithm in the vicinity of the age when the given mortality level is observed:

$$Expos[M_1; M_2] = \frac{\ln(M_1/M_2)}{b} \qquad (1)$$

where $Expos[M_1; M_2]$ is the estimated duration of exposure to the death rate $M(x)$ ranging from M_1 to M_2; b is the slope parameter of the regression line $\ln(M(x)) = a + bx$ fit in the period of at least 9 years within which the given mortality rates were observed.

There is a considerable (partially, erratic) cross-country variation in mortality dynamics, which may have contributed to contradictive findings in the previous literature. Therefore we study mortality compression averaging results for all low-mortality countries with available mortality data. We use data from the Human Mortality Database (2011), HMD, excluding the countries of the former Eastern Bloc, which differ in social and mortality dynamics from the rest of the countries represented in the database and also Island and Liechtenstein for their small population size. Death rates from period life tables and from (partially incomplete) cohort mortality schedules of the HMD form the basis for our work. We study the dynamics of durations of exposure to the selected mortality levels over the long time period since 1900. For different years there were different numbers of countries with mortality data available in HMD but this does not affect our findings considerably.

In Figure 3, we present ages at which the selected mortality levels were first observed in period life tables (circles) and cohort life tables (dots) in the countries studied; we also present trends of the modal ages at death in period life tables (thin solid lines) and cohorts (thicker lines). As mortality was characterised by a declining trend in the past century in the low-mortality countries, the ages at which the selected mortality levels were observed used to move upwards with time (this process accelerated in the 1970s). In such a situation, our above discussion indicates the bias of the mortality compression picture revealed by the period life tables. The ages at which the selected mortality levels are observed have shifted by about 0.2 years per year on average in the recent decade. The general theory of tempo distortions (Ediev 2008) indicates that the period life table will compress the actual exposure durations to the selected mortality levels by about 1/(1-0.2)=1.25, i.e., by 25%. This would be an average bias of the period life table estimates for the mortality compression indicators. In the next section, we provide a more explicit and comprehensive account for these differential dynamics by considering cohort- and period-specific durations of exposure to selected mortality levels.

Trends presented in Figure 3 are also indicative about compression of ages at death above the mode. Standard deviation of ages at death above the mode is closely linked to the mortality level at the mode: the higher the mortality, the lower the standard deviation (Wilmoth and Horiuchi 1999). Hence, patterns in Figure 3 indicate rather different trends in the compression above the mode in period life tables and birth cohorts. The difference is more clearly pronounced for males: cohort-wise, mortality was decompressing above the mode, while in period life tables it shows compression in the short run and not much change in the long run. Given the interest in the literature to the compression in relation to the modal age, we also consider that aspect of the phenomenon in the section following the one below.

Figure 3a and b Age at which the selected mortality levels were first observed in period (circles) and cohort (dots) life tables as well as modal ages at death in period (thin solid lines) and cohort (thicker solid lines) life tables averaged over 22 low-mortality countries

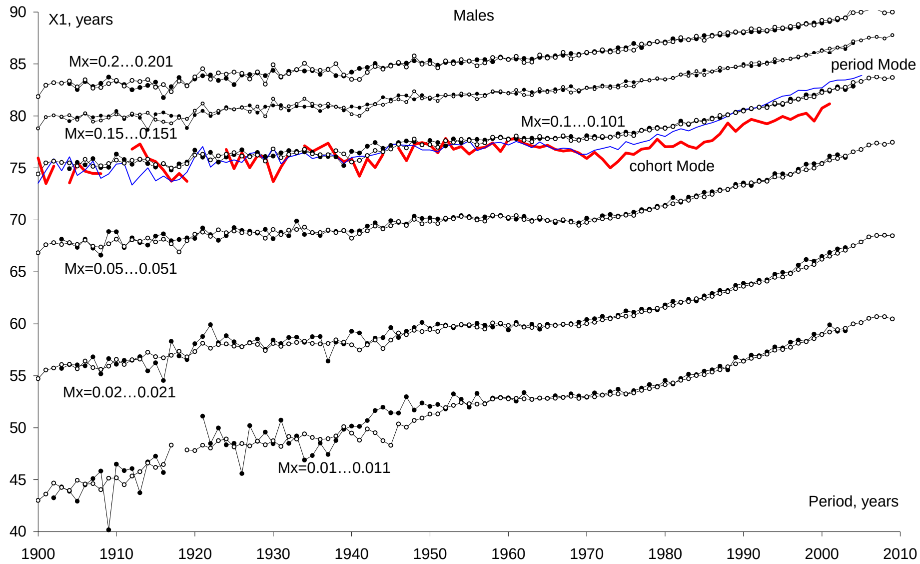


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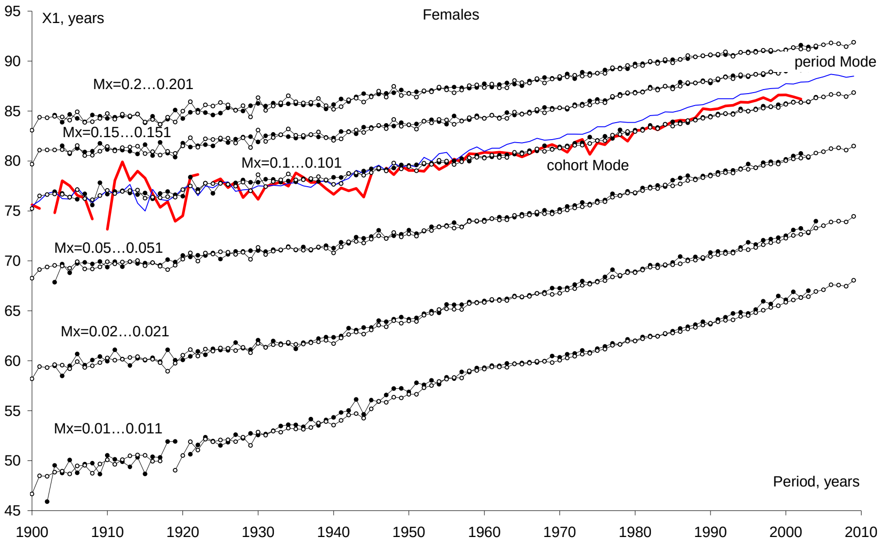
Figure 3 continued



Source: Own calculations based on data from HMD (2011)

4 EMPIRICAL EVIDENCE ON MORTALITY (DE)COMPRESSION

The development over time of durations of exposure to the selected mortality levels is presented in Figure 4 and Table 1. The overall trend after the 1960s was a decompression of mortality (and equally so even for an earlier period for males at higher mortality levels, i.e. at older ages). At the same time, period life tables would indicate a continuous mortality compression during the whole period, except for very recent developments in male mortality at lower mortality levels. This built-in bias of the period life table model has, apparently, widely affected the previous research. Males are exposed over longer (and still expanding) durations of time to the selected mortality levels. However, they experience those levels at younger ages than females, which is consistent with male's lower life expectancy.

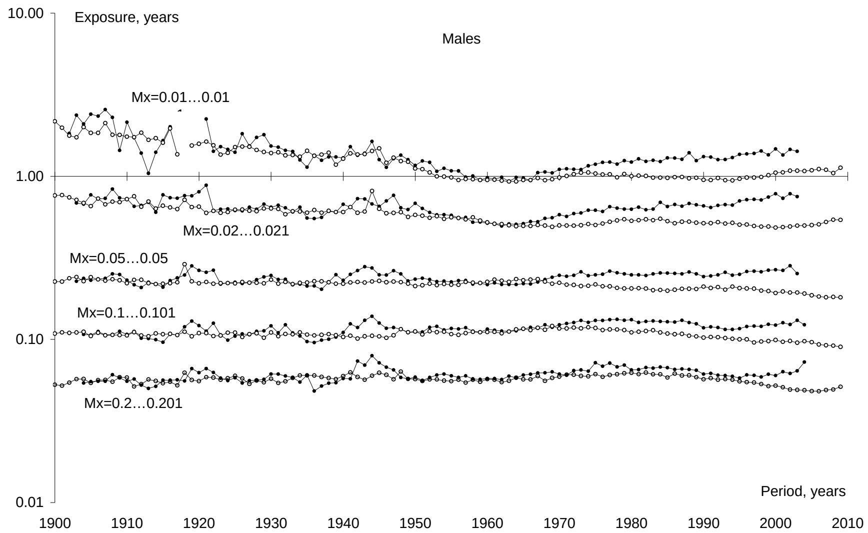
Over the last century, the age at which the selected mortality levels were experienced has shifted considerably. The death rate experienced by males at age 40 one hundred years ago is now experienced at age 60 in low-mortality countries. At age 65, females are exposed to the mortality level they would have been exposed to at age 45 a century ago. (Sanderson and Scherbov 2008 took account of such dramatic shifts by redefining the very notion of age.) Noting only limited success in reducing mortality at oldest-old ages and assuming no further success in those ages due to some biological constraints, one would naturally be led to expect mortality to be compressed while shifting along the age scale. Our results, however, do not support this anticipation. Exposures to the selected mortality levels arranged along the age scale are presented in Figure 5 and Table 2. Mortality is decompressing while moving up the age scale for males and only moderately compresses for females (except for the oldest-old ages, where female mortality does compress both in period life tables and cohorts). This may indicate a future mortality decline at oldest-old ages that has not been observed so far only because the cohorts to be involved in such decline are at younger ages yet. Period life tables would, nonetheless, indicate a pronounced mortality compression which, however, would only be a methodological bias due to the tempo distortion illustrated by the gap between the cohort and period estimates presented in the figure.

Table 1 Durations of exposure of birth cohorts (left panel) and period life tables (right panel) to selected mortality levels in 22 low-mortality countries averaged over selected periods of time

Mortality level (range of the death rate)	Birth cohorts' actual exposure to the selected mortality levels in periods:			Exposure estimates from period life tables		
	1900-1909	1960-1969	1997-2006	1900-1909	1960-1969	1997-2006
			males			
0.01-0.011	2.25	0.99	1.41	1.90	0.95	1.05
0.02-0.021	0.74	0.52	0.74	0.71	0.50	0.49
0.05-0.051	0.24	0.22	0.26	0.23	0.23	0.19
0.10-0.101	0.11	0.12	0.12	0.11	0.11	0.10
0.15-0.151	0.073	0.078	0.081	0.072	0.073	0.066
0.20-0.201	0.056	0.060	0.062	0.055	0.057	0.051
			females			
0.01-0.011	2.18	1.11	1.24	1.83	0.98	0.94
0.02-0.021	0.62	0.52	0.55	0.56	0.46	0.43
0.05-0.051	0.21	0.19	0.19	0.20	0.17	0.16
0.10-0.101	0.11	0.10	0.09	0.10	0.09	0.08
0.15-0.151	0.074	0.073	0.065	0.074	0.066	0.057
0.20-0.201	0.059	0.061	0.051	0.059	0.056	0.046

Source: Own calculations based on data from HMD (2011)

Figure 4a and b Dynamics over time of durations of exposure of birth cohorts (dots) and period life tables (circles) to selected mortality levels in 22 low-mortality countries



Source: Own calculations based on data from HMD (2011)

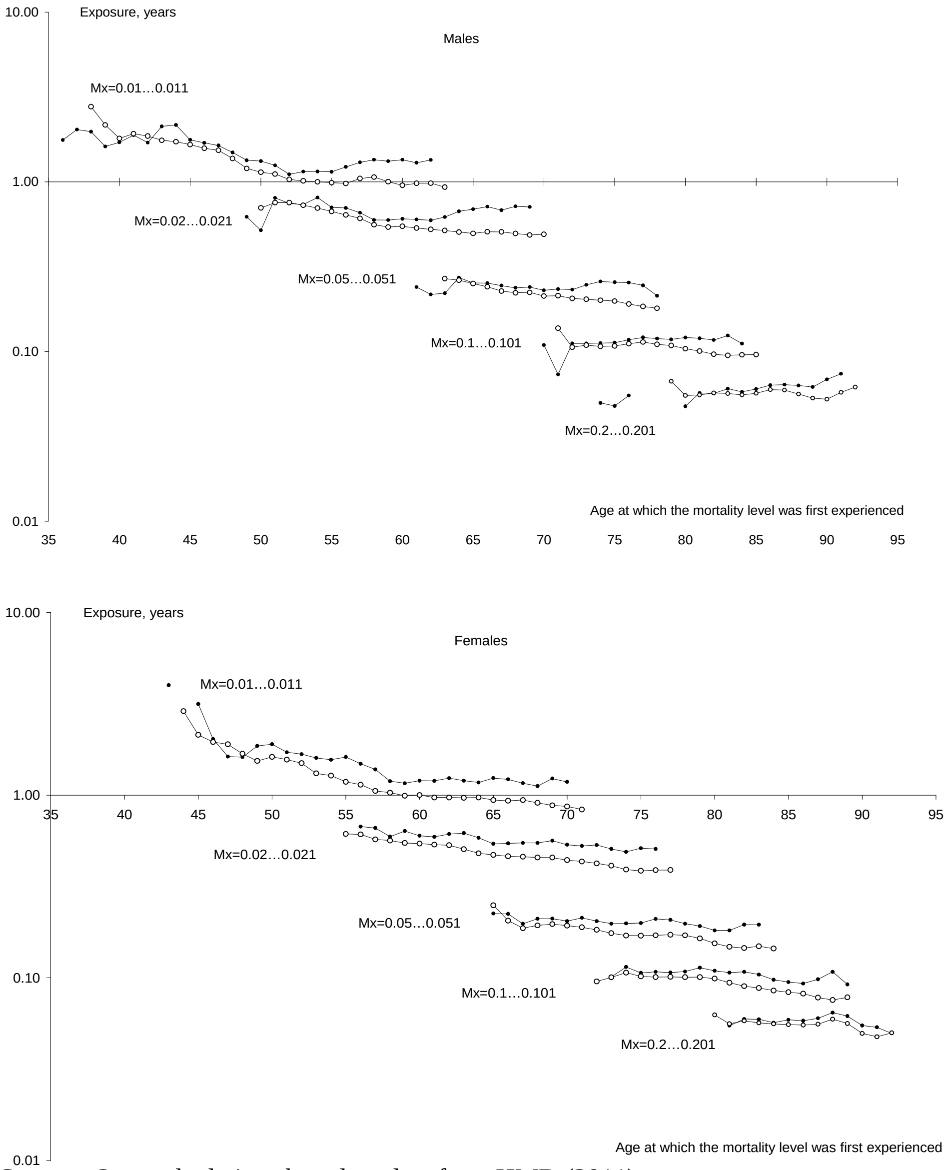
Table 2 Durations of exposure of birth cohorts and period life tables to selected mortality levels in 22 low-mortality countries averaged over selected ranges of age, when the mortality level is first experienced

Mortality level (range of the death rate)	35-40	40-45	45-50	50-55	55-60	60-65	65-70	70-75	75-80	80-85	85-90	90-95
Males, cohorts												
0.01-0.011	1.76	1.84	1.54	1.17	1.24	1.33						
0.02-0.021			0.62 [a]	0.75	0.62	0.61	0.70					
0.05-0.051						0.26	0.24	0.24	0.25			
0.10-0.101								0.11	0.12	0.12		
0.15-0.151								0.061	0.077	0.080	0.081	
0.20-0.201								0.050	0.051	0.058	0.063	0.070
Males, period life tables												
0.01-0.011	2.36	1.82	1.42	1.05	1.01	0.97						
0.02-0.021			2.12	0.73	0.58	0.53	0.50	0.49				
0.05-0.051					0.61	0.26	0.23	0.21	0.19			
0.10-0.101								0.11	0.11	0.10	0.10 [a]	
0.15-0.151									0.075	0.074	0.069	
0.20-0.201									0.067	0.056	0.057	0.055
Females, cohorts												
0.01-0.011		3.62 [a]	1.87	1.67	1.29	1.20	1.21	1.18 [a]				
0.02-0.021					0.63	0.60	0.55	0.53	0.51			
0.05-0.051							0.21	0.20	0.20	0.18		
0.10-0.101								0.11	0.11	0.11	0.094	
0.15-0.151									0.081	0.073	0.071	0.067
0.20-0.201										0.058	0.061	0.054
Females, period life tables												
0.01-0.011		2.83	1.75	1.45	1.06	0.98	0.93	0.86				
0.02-0.021				0.53 [a]	0.56	0.52	0.46	0.42	0.39			
0.05-0.051						0.19 [a]	0.20	0.18	0.17	0.15		
0.10-0.101								0.10	0.10	0.092	0.082	
0.15-0.151									0.076	0.069	0.062	0.059
0.20-0.201										0.057	0.056	0.049

[a] Values might be affected by small sample size (3 or less observations)

Source: Own calculations based on data from HMD (2011)

Figure 5a and b Dynamics over age of durations of exposure of birth cohorts (dots) and period life tables (circles) to selected mortality levels in 22 low-mortality countries



Source: Own calculations based on data from HMD (2011)

5 MORTALITY COMPRESSION IN RELATION TO THE MODE

Further down, we supplement our study by a more traditional analysis considering cohort and period distributions of ages at death above and below the modal age at death.

a. Mortality compression above the mode

Since it would not be possible to obtain the standard deviation of the ages at death above the mode for most of the cohorts alive, we use an indirect indicator of mortality compression above the mode: death rate at the modal age. Assuming, in line with Lexis and Kannisto, that 'natural' deaths due to senescence are normally distributed around the mode and there are no premature deaths above the mode, we obtain $d^*(M) = l^*(M)\mu(M) = 0.5\mu(M)$ as the height of the density function net of premature deaths ($l^*(M) = 0.5$ indicates that exactly 50% of 'natural' deaths happen before the mode). This indicator is an improvement to the *Fastest Decline* proposed by Wilmoth and Horiuchi (1999) in that it is cleaned from the effect of supposedly premature deaths. In the normal distribution, the root mean square deviation from the mode is $\frac{1}{\sqrt{2\pi}} \approx 0.40$ times $\frac{1}{d^*(M)}$. This ratio is close to the empirical ratio 0.37 (with sample standard deviation 0.01) that we obtain from distributions of deaths above the modal age in the Human Mortality Database (2011). The upward standard deviation from the mode of age at death above the mode may then be approximated as

$$SD(M+) = \sqrt{\frac{\int_M^\infty d(x)(x-M)^2 dx}{\int_M^\infty d(x)dx}} \approx \frac{0.74}{\mu(M)} \text{ (years).} \qquad (2)$$

This simple relation allows to considerably widen the set of (incomplete) cohort mortality schedules for which the mortality compression above the mode can be studied. To refine and stabilise the estimates, we smooth the rates by a quadratic parabola applied to the age interval of ten years around the mode; the mode itself is estimated using the formal relation equating the square and the derivative of the death rate at the mode:

$$\tfrac{d}{dx}\mu(M) = \mu^2(M) \qquad (3)$$

(an equivalent to the relation was derived, in the context of the Gompertz law, by Pollard 1991; it was rediscovered later as a general relation by Canudas-Romo 2008, Thatcher et al. 2010, and Tuljapurkar and Edwards 2011).

The dynamics of the modal ages at death and death rates at the modal age are presented in Figure 6. For males, there has been mortality *decompression* above the mode since the 1900s followed by stagnation in the period of fastest increase at the modal age (since the 1970s). The period life table model, on the contrary, was misleading as it indicated compression above the mode since the 1900s and even faster compression since the late 1960s. In the case of females, period life tables suggest steady mortality compression above the mode since the 1930s, while cohorts have been showing compression since the 1970s only, indicating stagnation in 1900-1960s and, possibly, since the 1990s.

b. Mortality compression below the mode

Deaths above the mode constitute only about one-third of deaths above age 30 and may not be representative of all adult deaths. Therefore, it is also necessary to consider the distribution of ages at death below the mode, which is presented next. To eliminate effects due to younger-age mortality (presumably, the premature deaths, in the Lexis-Kannisto approach) and widening the observation window (in case of left-censoring at a fixed age), we consider the distributions of ages at death in an age interval of fixed width (30 years) below the mode:

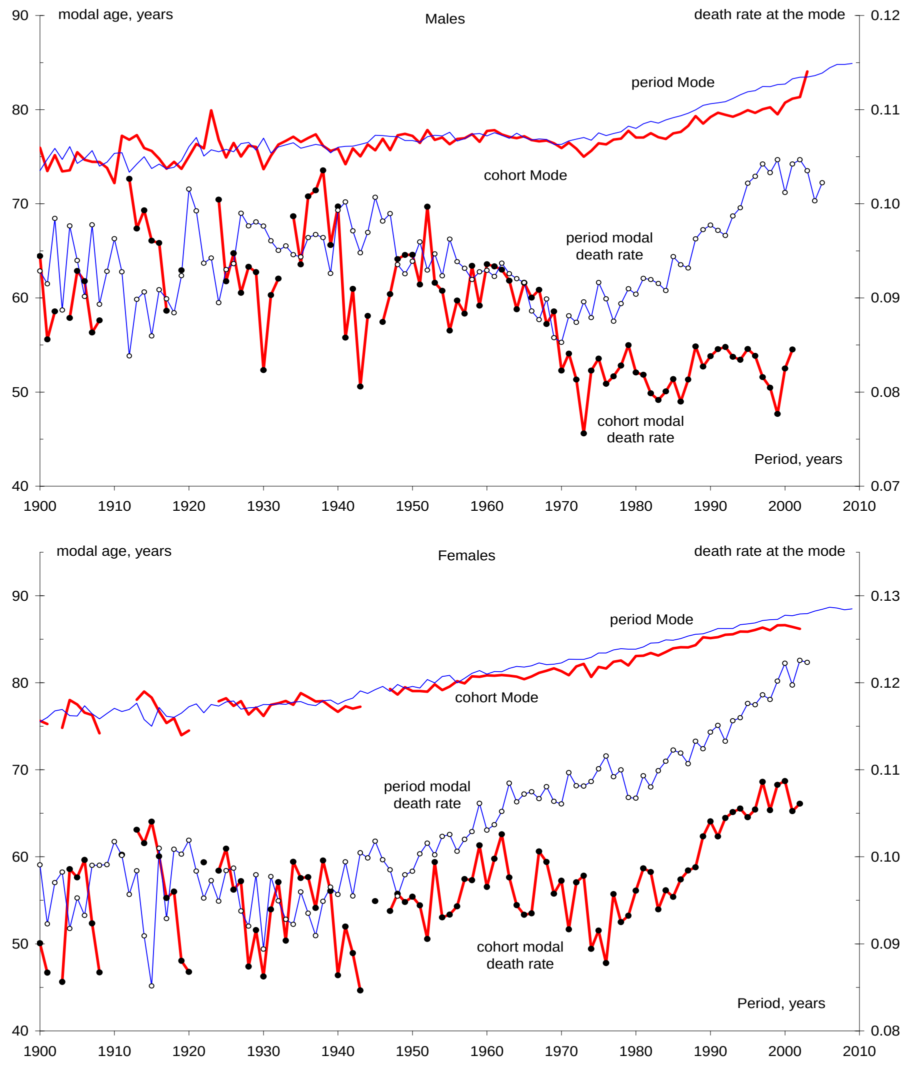
$$SD({}_{30}M) = \sqrt{\frac{\int_{M-30}^{M} d(x)(x-M)^2\,dx}{\int_{M-30}^{M} d(x)\,dx}} \text{ (years).} \qquad (4)$$

The dynamics of this indicator are presented in Figure 7. Just like in the cases considered above, the mortality compression below the mode is read differently from period and cohort records. The period life table model suggests compression below the mode for both males and females. This differs from the experience of individuals as seen from cohort schedules: mortality below the mode has been decompressing for males and largely stagnating for females since the 1970s.

A gradual long-term trend of compression since the 1900s (more obviously so for females), could have been taken as support for Lexis's assumption of a mixture of natural and pre-mature deaths below the mode. If premature deaths were to decline to a small fraction, the distribution below the mode might become more symmetrical to the one above the mode. Given the results reported above about the death rate at the mode, Lexis's assumption would imply the standard deviation of natural deaths below the mode to be currently about $\frac{0.74}{0.08} \approx 9.3$ years for males and $\frac{0.74}{0.105} \approx 7.0$ years for females (cohort-wise). The huge discrepancy between these estimates and the data presented in Figure 7 indicates that deaths below the mode do not follow the normal pattern and presumable unnatural deaths responsible for the deviation are not going to free the way anytime soon. A better assumption would be that the very 'natural' deaths below the mode show a left-skewed distribution rather than the normal one. This distribution is currently expanding for males and shifting up for females.

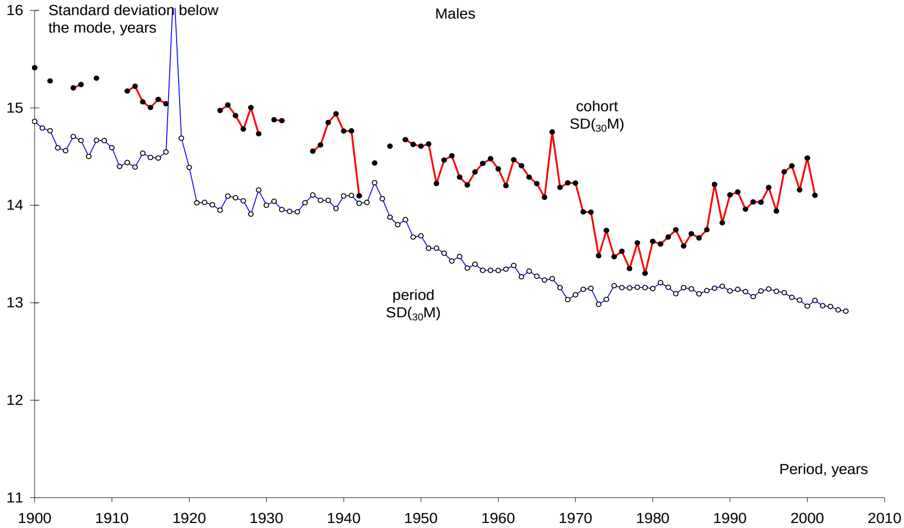
Such interpretation is also supported by the fact that the probability to survive from age 30 to the modal age at death (which we do not present in detail here) has been about constant for females and risen only slightly for males, constituting about 35% for both sexes currently. Hence, about two-thirds of all deaths above age 30 happen below the mode, and only one-third above it; this proportion does not seem to be moving toward a balanced 50% level in the foreseeable future.

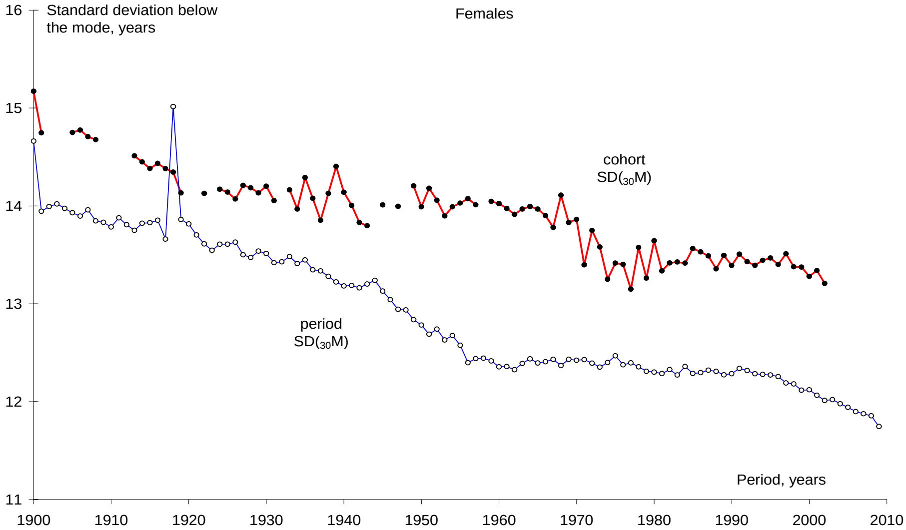
Figure 6 Modal ages at death in period (thin solid lines) and cohort (thicker solid lines) life tables and death rates at the modal age (filled circles for cohorts and circles for period life tables) averaged over 22 low-mortality countries as a function of the calendar period when the modal age was observed (data for periods with less than 5 observations are excluded)



Source: Own calculations based on data from HMD (2011)

Figure 7 Cohort (dots) and period (circles) root mean square deviations form the modal age at death of ages at death within 30-year age interval below the mode averaged over 22 low-mortality countries as a function of the calendar period when the modal age was observed (data for periods with less than 5 observations are excluded)





Source: Own calculations based on data from HMD (2011)

6 Conclusions

We showed in this paper that mortality compression may not be studied through the dynamics of period life tables which by their very design tend to provide a compressed picture of the human life span when mortality is on the decline. The conventional way to study mortality (and also morbidity) compression based on period cross-sections must be discarded or, at least, supplemented by adjustment procedures to overcome methodological biases. The conventional reliance on period data as well as limitations of age-censoring methods and results based on selected country cases have contributed to contradictory and questionable findings in the literature.

A direct estimation of the durations of exposure of males and females to six selected old-age mortality levels in 22 low-mortality countries shows mortality decompression—not compression—to characterise its dynamics since the 1970s, i.e. in the period of biggest success in reducing old-age mortality.

Even though the selected mortality levels used to be observed at increasingly older ages, this did not result in a significant compression of the duration of exposure to them; for males, mortality is decompressing while shifting to older age. This indicates the possibility of a future mortality decline at oldest-old ages which may not yet have been observed in many countries just because the cohorts involved are still younger.

Although deaths above the modal age at death form only a relatively small fraction of the distribution of deaths, they tell important stories about mortality at ages approaching the end of the human lifespan. Compression above the mode is indicative of an important aspect of mortality compression which attracted special attention in the literature—the rectangularisation of the survival curve. However, cohort-wise, mortality compression is not supported by data even above the mode. For males, there was a long period of decompression above the mode which has been succeeded by stagnation since the 1970s. For females, a long-term compression above the mode has also been succeeded by stagnation. Currently, mortality above the mode is largely shifting upwards.

Below the mode, mortality was compressing until the 1970s. Since then, cohort-wise, mortality has been decompressing for males and shifting upwards for females. The period life table model, on the contrary, due to built-in biases, suggests a universal pattern of mortality compression above and below the mode.

Unlike most of the previous works on mortality compression, we do not come to the conclusion about a future concentration of mortality (nor, seemingly so, of morbidity either) in narrow age intervals at some old age. Rather, deaths and illness seem to stay dispersed over age, while shifting to ever more advanced ages.

Vaupel (2010) made a case for an optimistic scenario of the future with ever-increasing longevity stating that people may live longer simply by reaching advanced ages in better health despite “[s]lowing the pace of deterioration has proved intractable, at least until now”. Here, we support this view and even show that the pace of deterioration is in fact slowing down already, which adds an even more optimistic tone to the scenarios of future mortality.

Our findings support the vision (Ediev 2011) that the durations of exposure of cohorts to currently observed mortality levels form a genuine part of mortality conditions and should be taken into account when formulating future mortality scenarios.

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