The twin hypothesis of education and retirement

Master's thesis

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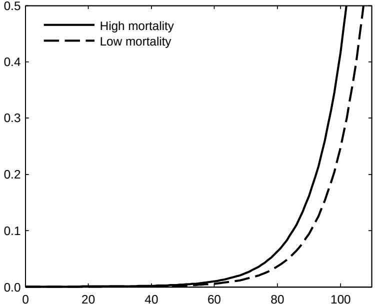
Overview

- Starting point:
  - Ben Heijdra and Ward Romp. Human capital formation and macroeconomic performance in an ageing small open economy. *Journal of Economic Dynamics and Control*, 2009
  - Ben Heijdra and Ward Romp. Retirement, pensions, and ageing. *Journal of Public Economics*, 2009
- Goals:
  - combine models to allow for endogenous schooling **and** retirement
  - How does individual behavior differ from the original papers?
  - How are the dynamics of aggregate variables affected?

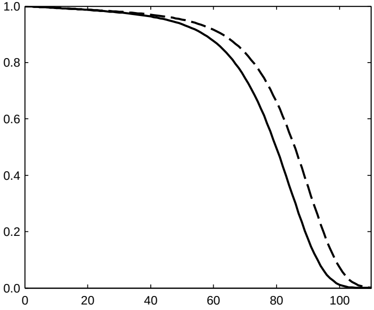
Outline

Model demography

- overlapping generations model in continuous time
- age-dependent mortality rate $m(u, \psi_m)$
- cumulative mortality rate $M(u, \psi_m) := \int_0^u m(\alpha, \psi_m) d\alpha$



Mortality rate, $m(u) = \mu_0 + \mu_1 e^{\mu_2 u}$



Survival function, $e^{-M(u)}$

Individual maximization problem

At time t, a generation v individual ($v \leq t$) maximizes the expected value of remaining lifetime utility:

$$\int_t^\infty \Big[\underbrace{U(\bar{c}(v,\tau))}_{\substack{\text{utility from} \\ \text{consumption}}} - \underbrace{I(\tau - v, R(v)) D(\tau - v)}_{\text{disutility of work}} \Big] \underbrace{e^{-\theta(\tau - t)}}_{\substack{\text{discount} \\ \text{factor}}} \underbrace{e^{M(t-v,\psi_m) - M(\tau - v,\psi_m)}}_{P[T \geq \tau | T \geq t]} d\tau$$

with respect to

- consumption, $\{\bar{c}(v,\tau)\}_{\tau=t}^{\infty}$,
- length of schooling, $e(v)$,
- retirement age, $R(v)$.

Individual maximization problem

Asset accumulation equation:

$$\frac{\partial \bar{a}(v,t)}{\partial t} = [r+m(u)]\bar{a}(v,t) + I(u,e(v))\bar{s}_E(v) + [I(u,R(v)) - I(u,e(v))](1-t_L)\bar{w}(v) + [1-I(u,R(v))]\bar{p}(v) - \bar{c}(v,t) - \bar{z}(v,t)$$

It incorporates:

- an education subsidy $\bar{s}_E(v)$ while in school, $u < e(v)$,
- net wage income $(1-t_L)\bar{w}(v)$ while at work, $e(v) \leq u < R(v)$,
- a pension benefit $\bar{p}(v)$ after retirement, $u \geq R(v)$.
- lump sum taxes $\bar{z}(v,t)$ and consumption expenditures $\bar{c}(v,t)$ at any time

Individual maximization problem

Individual human capital:

$$\bar{h}(v) := \begin{cases} 0 & \text{for } 0 \leq u < e(v), \\ A_H h(v)^\phi e(v) & \text{for } u \geq e(v), \end{cases} \qquad 0 \leq \phi \leq 1$$

Wages: $\bar{w}(v) := w \cdot \bar{h}(v)$

Pensions: $\bar{p}(v) := \vartheta \int_{e(v)}^{R(v)} \bar{w}(v) ds$

Subsidies: $\bar{s}_E(v) := s_E w A_H h(v)^\phi$

Taxes: $\bar{z}(v,t) := z(t) w A_H h(v)^\phi$

Aggregate economy

Small open economy $\Rightarrow$ output proportional to human capital.

Per capita human capital:

$$h(t) := \int_{-\infty}^{t} \left[I(t-v, R(v)) - I(t-v, e(v)) \right] \bar{h}(v)\, l(v,t)\, dv$$

where $\bar{h}(v) = A_H h(v)^{\phi} e(v)$.

Steady state:

$$\hat{h}^{1-\phi} = A_H \cdot e^* \cdot \hat{x}, \qquad \hat{x} = \int_{e^*}^{R^*} l(u)\, du$$

Comparative static analysis

How do individuals react to *demographic shocks* and *policy reforms*?
How is per capita human capital affected (in the long run)?

The effects of population aging

Reduced adult mortality: $\partial m / \partial \psi_m \leq 0$

$$\frac{\partial e^*}{\partial \psi_m} > 0, \qquad \frac{\partial R^*}{\partial \psi_m} > 0$$

$$\frac{\partial \hat{h}^{1-\phi}}{\partial \psi_m} = \underbrace{\frac{\partial \hat{h}^{1-\phi}}{\partial e^*} \frac{\partial e^*}{\partial \psi_m}}_{>0} + \underbrace{\frac{\partial \hat{h}^{1-\phi}}{\partial R^*} \frac{\partial R^*}{\partial \psi_m}}_{>0} + A_H e^* \underbrace{\frac{\partial \hat{x}}{\partial \psi_m}}_{<0} \gtreqless 0$$

Transitional dynamics

Initial steady state: $e_0 = 22.3$, $R_0 = 62.5$.

Scenario: Life expectancy at birth increases from 76.6 to 82.3 years.

Three model specifications:

1. schooling time adjusts, retirement stays fixed at R_0
2. retirement adjusts, schooling time is fixed at e_0
3. education and retirement adjust at the same time

Long-run effects: Comparison

Mortality shock

Long-run impact of a reduction in old-age mortality:

Scenario	$\hat{x}$	$\hat{h}$	%-change in $\hat{h}$
initial SS	46.5%	28.2	—
uncontrolled	44.8%	26.8	–5.0%
e endogenous	44.6%	26.9	–4.8%
R endogenous	45.4%	27.3	–3.3%
full model	45.2%	27.5	–2.5%

$\Rightarrow$ The negative impact of aging on human capital (and thus output) is overestimated by 32% if we do not control for e and R at the same time.

Long-run effects: Comparison

Education reform

Long-run impact of a 20% increase in the schooling subsidy:

Scenario	$\hat{x}$	$\hat{h}$	%-change in $\hat{h}$
initial SS	46.5%	28.2	—
e endogenous	44.9%	28.7	+1.6%
R endogenous	46.2%	28.0	−0.7%
full model	45.2%	29.1	+3.1%

$\Rightarrow$ The macroeconomic impact of the education reform is underestimated by almost 50% if we do not control for both e **and** R.

Conclusion

- Neglecting the interaction in education and retirement decisions may result in wrong expectations about the quantitative effects of demographic shocks and policy reforms.
- BUT: Most of the economic literature only considers one of these decisions!

$\Rightarrow$ **Twin hypothesis** of education and retirement

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Optimal individual behavior

First order condition for retirement:

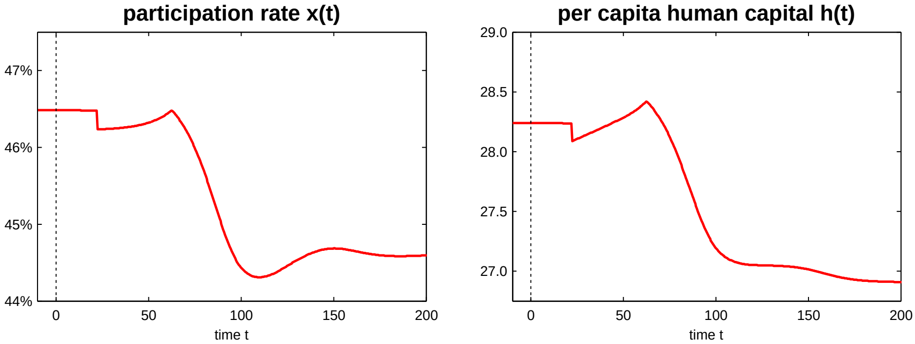
$$U'(\bar{c}^*(v,t))\left[(1-t_L)\bar{w}(v)-\bar{p}(v)+\frac{\partial\bar{p}(v)}{\partial R}\Delta(R,r)\right]=D(R)e^{(r-\theta)(R-u)}$$

First order condition for education:

$$(1-t_L)\frac{\partial\bar{w}(v)}{\partial e}\Delta_1(e,r,R)+\frac{\partial\bar{p}(v)}{\partial e}\Delta_2(e,r,R)=(1-t_L)\bar{w}(v)-\bar{s}_E(v)$$

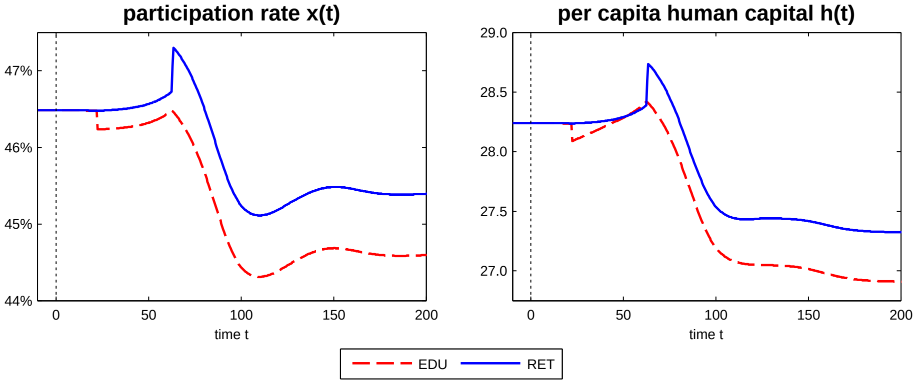
Aggregate behavior: Transitional dynamics

Mortality shock under $R_0 = 62.5$



Aggregate behavior: Transitional dynamics

Mortality shock under $e_0 = 22.3$



Aggregate behavior: Transitional dynamics
Mortality shock, e and R endogenous