Lifespan inequality and the first demographic dividend

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Abstract

This paper investigates how inequalities in life expectancy affect the timing and extent of the first demographic dividend. Beyond the expected direct influence of the age structure on the support ratio, lifespan inequality imposes various unexpected influences on the dynamics of the support ratio.

Inequalities in life expectancy are often caused by socio–economic inequalities. Socio-economic status has a significant influence on life expectancy. People with higher income and education tend to have better health literacy and better access to healthcare and resources that promote healthy living. We apply a formal model that allows for an analytical investigation of the combined influence of inequalities in life expectancy and declining fertility on the dynamics of the age structure. This framework is capable to project the characteristics of the first demographic dividend under different scenarios with respect to lifespan inequality and the life table in general, but also with respect to other relevant parameters such as the speed of fertility decline, generation length, or age at entering and leaving the labour market. This investigation provides insights into the influence of inequalities within a population on the demographic dividend.

We use continuous analytical survival functions with the capability to incorporate changes in lifespan inequality. We then examine how these inequalities affect the age structure of populations with declining fertility. The analysis shows that higher inequality in life expectancy leads to an earlier peak, but at the cost of a shorter beneficial period, lower intensity and lower overall gains. Therefore, in addition to the overall benefits of reducing inequalities in life expectancy, the gains from the first demographic dividend increase as inequalities become smaller. This means a benefit for the economy and for the population as a whole.

1 Introduction

If a population, starting from an originally high level of fertility, experiences a sustained decline in its fertility, this temporarily leads to an economically advantageous age structure in which the share of persons of working age is particularly high. This results in a "demographic window of opportunity" that holds the potential for higher economic growth. This frees up resources for investments in physical capital but also in education and family support. Consequently, income per capita grows and this is often addressed as the first demographic dividend. The latter term is widely used but with varying meanings. In general there are two concepts of the first demographic dividend. The first one addresses changes in the age structure, in particular the growth in the support ratio which boosts economic growth (Mason, 2005); the second one addresses the age structure itself with a focus on the level of the support ratio (United Nations Population Fund, 2012). In this paper we apply the second concept for two reasons. Firstly, the current state — age structure, support ratio — of the population under investigation is more relevant for the current level of economic performance than its changes. A high support ratio that has already started to shrink is better for the economy than a low support ratio that is on the rise but only reaches a high level later. Secondly, as we will show later, this concept allows for an exact and intuitively understandable quantification of the first demographic dividend within our modelling framework.

Fig. 1 shows the support ratio over time for India, Indonesia, Malaysia and the Philippines. These four countries are chosen because they are all in a stage of the demographic transition where support ratios are still expected to continue to rise substantially. Therefore, these countries still have a substantial part of the first demographic dividend ahead of them. The actual estimates are shown in bright colours and cover the period from 1950 to 2021, the projections (plotted in light colours) cover the period from 2022 to 2100. It is of great importance for the economic prosperity of these countries to know how high the peak will be, when the peak will be reached and how long the support ratio will remain at a high level. These future dynamics of the support ratio do not only depend on fertility and mortality per se (and of course migration) but also on lifespan inequality as we will show in this paper. We apply a formal model that allows for an analytical

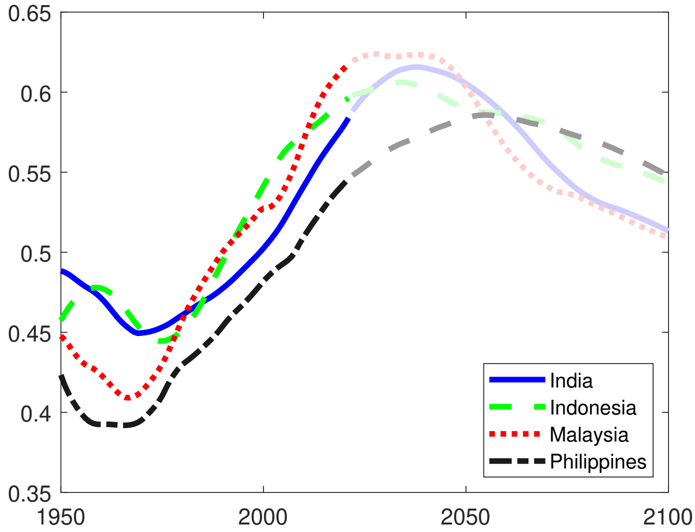

Figure 1: Support ratio $S(t)$, data source: United Nations (2022).

investigation of the combined influence of inequalities in life expectancy and declining fertility on the dynamics of the age structure. This investigation provides insight into the influence of inequalities within a population on the demographic dividend.

Inequalities in life expectancy are often caused by socio–economic inequalities. Socio–economic status (SES) has a significant influence on life expectancy. People with higher income and education tend to have better health literacy and better access to healthcare and resources that promote healthy living. On the other hand, people with lower SES may face economic, social and environmental challenges that can affect their health and reduce their life expectancy. Studies have consistently shown that people with higher SES tend to have longer life expectancies compared to those with lower SES (Luy et al., 2011). People who live in poverty or have lower SES may experience a range of health disparities. They are less likely to have health insurance and may have limited access to medical care. This can result in delayed diagnosis and inadequate treatment, which can lead to poorer health outcomes. People living in poverty may have a higher exposure to pollution and other environmental hazards that can harm their health. People with lower SES may have aggravated access to healthy nutrition, which can contribute to a higher risk of chronic diseases such as obesity and diabetes. Finally, they may experience chronic stress caused by financial hardship, job insecurity, and other social and economic challenges. This can negatively impact their health and reduce their life expectancy.

2 The model

For a mathematical investigation of the dynamics of the support ratio during a decline in fertility we revisit a model proposed by (Coale, 1972, chap. 4). The basic assumption of this model is a population where fertility is fixed in age structure but declines at a constant rate $k < 0$. The mortality schedule is fixed and the survival function $l(a)$ denotes the probability of surviving to age a. Moreover, we denote the generation length μ, which is close to the mean age at childbearing, and assume that this value is constant over time.

To analyse the model, we normalise the time scale such that $t = 0$ when the net reproduction rate $NRR = 1$. Under these assumptions fertility approaches infinity for $t \to -\infty$ and zero for $t \to \infty$. Although these extreme values are unrealistic, the model is appropriate for describing the development of the age structure during the transition phase from high to low levels of fertility. For the limiting case $k = 0$, we assume that the NRR is constantly equal to 1. This special case actually represents a stationary population but it is relevant for our analysis as it marks the boundaries for the dynamics in the case of a slow fertility decline.

To investigate the age structure, we denote $N(a, t)$ the number of females aged a at time t, which is equal to the number of births at time $t - a$ times the survival probability $l(a)$, thus $N(a, t) = B(t - a)l(a)$.

The support ratio is defined as the share of the working age population in the total population, i.e

$$S(t) = \frac{\int_W N(a,t)da}{\int_0^\omega N(a,t)da}, \tag{1}$$

where ω denotes the maximum age of the life table and W denotes the age range of the working age population. Throughout this paper we compare the actual support ratio $S(t)$ vs. a benchmark level that can be sustained over the long term. As long as the support ratio exceeds this benchmark, we consider the respective age structure as advantageous. For this benchmark we choose the support ratio of a stationary population S_0 with the same mortality schedule $l(a)$, which we express as

$$S_0 = \frac{\int_W l(a)da}{\int_0^\omega l(a)da}. \tag{2}$$

We choose this level as a benchmark as it is a neutral level that can be sustained with a constant age structure and consider support ratios above S_0 advantageous.

3 Lifespan inequality

We denote $\mu(a)$ the force of mortaliy (hazard rate or risk of death) at age a. Then the probabiliy of a newborn individual to survive to age a, i.e. the life table survival function, is given as

$$l(a) = \exp\left(- \int_0^a \mu(y)dy \right). \tag{3}$$

Vice versa we get the force of mortality

$$\mu(a) = -\frac{d}{da} \ln l(a) = -\frac{l'(a)}{l(a)}.$$

Following Keyfitz (1977) we use the survival function $l(a)$ to compute life table entropy

$$H[l(a)] = -\frac{\int_0^\infty l(a) \ln(l(a))da}{\int_0^\infty l(a)da}. \tag{4}$$

This entropy H is a dimensionless indicator of the relative variation in the length of life, i.e. lifespan inequality (Aburto et al., 2019). It can be interpreted as a weighted average of $\ln(l(a))$, weigthed by $l(a)$ but also as a weighted average of life expectancy at age a relative to life expectancy at birth e_0 (Goldman and Lord, 1986). If mortality is concentrated at one fixed age, H becomes zero, if the force of mortality μ is constant over all ages, H is equal to one (Keyfitz and Caswell, 2005) and if remaining life expectancy $l(a)$ increases with age, $H > 1$ holds (Goldman and Lord, 1986). Moreover, entropy H measures the elasticity of life expectancy to changes in mortality (Goldman and Lord, 1986; Keyfitz and Caswell, 2005).

Fig. 2 shows life table entropy H vs. life expectancy at birth e_0 for all life tables provided by United Nations (2022). The graph on the left highlights the data points for 1950, 1985 and 2021, the graph on the right shows a three–dimensional view of the data. The graphs show a clear negative nonlinear relationship between H and e_0, which means that an increase in life expectancy is associated with a decrease in lifespan inequality. Moreover, the graphs show that in the period from 1950 to 2021, life expectancy at birth increased and lifespan inequality decreased. The gain in life expectancy is therefore also due to the fact that more people are reaching a higher age, and not just to an increase in the highest age that people reach. Cross–country differences in life expectancy and in inequality in life expectancy both decreased over this period. The variance of life expectancy at birth among all countries decreased from 133.56 in 1950 to 56.53 in 2021 and the variance of lifespan inequality decreased from 0.0414 (1950) to 0.0039 (2021).

Fig. 3 illustrates the relationship between life table entropy H and life expectancy at birth e_0 for selected countries for the period 1950 to 2021. The selected countries are those that had the highest (South Sudan, Cambodia, Dem. People's

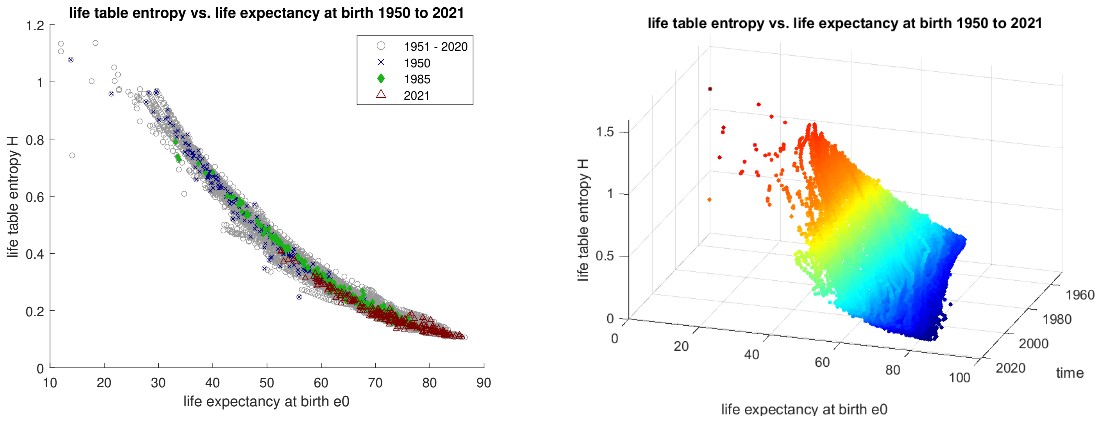

Figure 2: Life table entropy vs. life expectancy at birth, all countries, 1950 to 2021. data source: United Nations (2022).

Republic of Korea, Timor-Leste) and lowest (Monaco) life table entropy values, those covering the largest range in H (South Sudan, Cambodia, Dem. People's Republic of Korea, Republic of Korea and Timor-Leste) and those covering the smallest range in H (Latvia, Sweden, Netherlands, United States of America and Denmark). The charts again confirm that in the period from 1950 to 2021, overall life expectancy increased and lifespan inequality, measured in terms of life table entropy, decreased.

4 Analysis

For a decrease in fertility, i.e. $k < 0$, the support ratio initially increases and then decreases. In fig. 4 the solid blue line illustrates the dynamics of the support ratio of a population with fertility declining at a rate $k = -0.02$ and the dashed red line shows the corresponding benchmark S_0. We define the intersections of the support ratio with the benchmark as start time t_1 and end time t_2 and the time difference $t_2 - t_1$ as the duration or length l of the first demographic dividend. Moreover, we define the maximum difference between the support ratio and the benchmark as the height h and the area between the support ratio $S(t)$ and the benchmark S_0 from time t_1 to time t_2 as the total amount A of the demographic dividend. The size of this area informs us about the total amount of the surplus a population gains from a declining fertility.

Since the support ratio is the share of the working age population, i.e. the number of working-age persons divided by the total population, and time is measured in years, the total amount A can be interpreted as the potential additional working years per person. Goldstein et al. (2023) show how the speed of fertility decline and

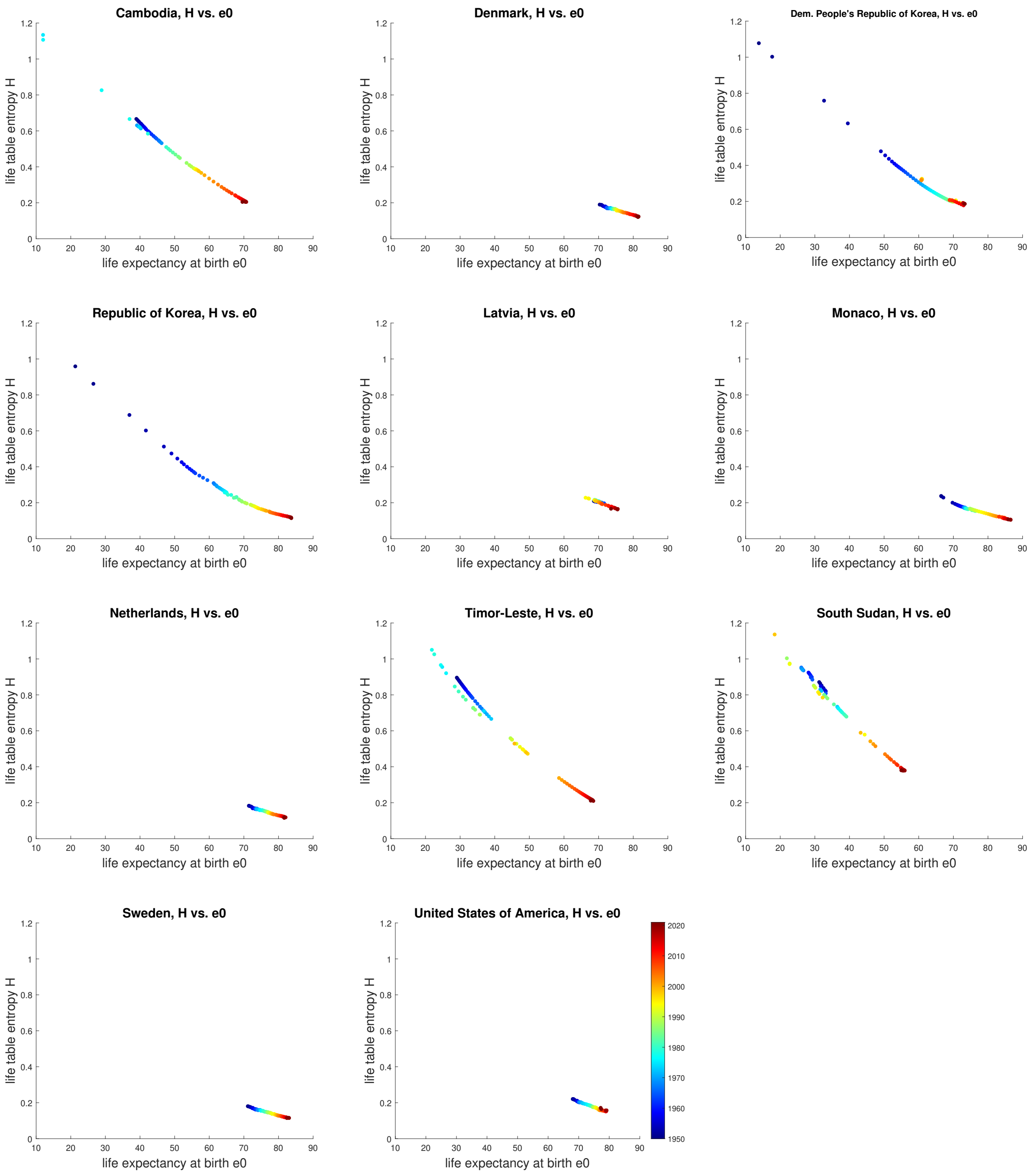

Figure 3: Life table entropy vs. life expectancy at birth for selected countries for the years 1950 to 2021, data source: United Nations (2022).

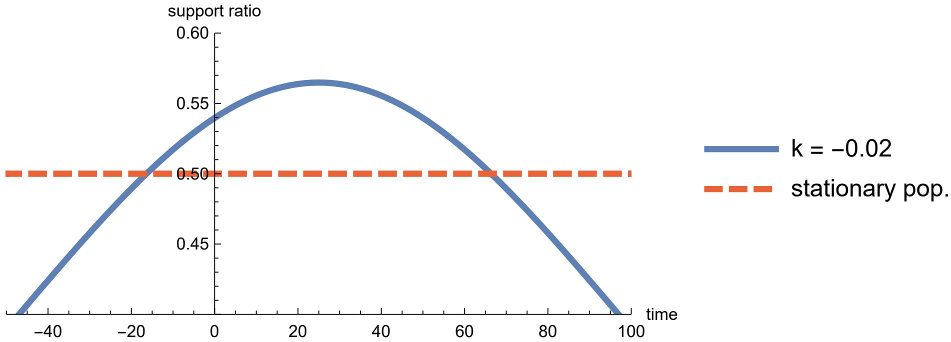

Figure 4: Support ratio $S(t)$ and benchmark S_0.

the generation length influence the timing (start, end and peak), length, height and total amount; in this paper we investigate the influence of lifespan inequality on those characteristics.

Goldstein et al. (2023) showed that the peak of the suppurt ratio occurs at time

$$t^* = \frac{(A_0 - A_{W,0})\frac{\mu}{k} + t_0\sigma_0^2 - t_{W,0}\sigma_{W,0}^2}{\sigma_0^2 - \sigma_{W,0}^2} \tag{5}$$

where t_0 ($t_{W,0}$) denotes the time when the mean age of the total population $A(t)$ (working age population $A_W(t)$) is equal to the mean age of the stationary population A_0 (stationary working age population $A_{W,0}$) with the same survival function $l(a)$. The variables σ_0^2 and $\sigma_{W,0}^2$ denote the variance in age of the corresponding stationary total population and stationary working age population, respectively. These variances depend on the lifespan inequality induced by the survival function $l(a)$.

Using (3), we introduce the ith moments L_i of the stationary population implied by the life table $l(a)$, (Keyfitz and Caswell, 2005, p. 104),

$$L_i = \int_0^\omega a^i l(a)da. \tag{6}$$

With (6) we can express life expectancy at birth $e_0 = L_0$ and the mean age $A_0 = {}^{L_1}/_{L_0}$ and variance $\sigma_0^2 = {}^{L_2}/_{L_0} - ({}^{L_1}/_{L_0})^2$ of the corresponding stationary population. Goldstein (2021) and Feichtinger and Vogelsang (1978, eq. (8.12)) introduced the parameter

$$\theta = \frac{\frac{L_3}{L_1} - \frac{L_2}{L_0}}{2\sigma_0^2}. \tag{7}$$

to approximate the time t_0 when the mean age of a pseudo–stable population equals the mean age of the stationary population with the same life table function,

$$t_0 \approx A_0\theta - \frac{\mu}{2},$$

where μ denotes the generation length. Since this parameter θ is correlated with lifespan inequality H (see appendix A) this points to another influence mechanism of lifespan inequality on the first demographic dividend.

To obtain a deeper understanding of the relationship between specific survival schedules, life table entropy and the dynamics of the support ratio we apply the mortality law of Siler (1983),

$$\begin{aligned}\mu(a) &= \alpha\, e^{\beta a} + \gamma + \delta\, e^{-\zeta a} \\ l(a) &= \exp\left[\frac{\alpha}{\beta}\left(1 - e^{\beta a}\right) - \gamma a - \frac{\delta}{\zeta}\left(1 - e^{-\zeta a}\right)\right].\end{aligned} \tag{8}$$

Using (8), we get life expectancy at birth e_0

$$e_0 = \int_0^\infty l(a)da = \int_0^\infty \exp\left[\frac{\alpha}{\beta}\left(1 - e^{\beta a}\right) - \gamma a - \frac{\delta}{\zeta}\left(1 - e^{-\zeta a}\right)\right] da \tag{9}$$

and life table entropy

$$H[l(a)] = -\frac{\int_0^\infty \exp\left[\frac{\alpha}{\beta}\left(1 - e^{\beta a}\right) - \gamma a - \frac{\delta}{\zeta}\left(1 - e^{-\zeta a}\right)\right]\left[\frac{\alpha}{\beta}\left(1 - e^{\beta a}\right) - \gamma a - \frac{\delta}{\zeta}\left(1 - e^{-\zeta a}\right)\right] da}{\int_0^\infty \exp\left[\frac{\alpha}{\beta}\left(1 - e^{\beta a}\right) - \gamma a - \frac{\delta}{\zeta}\left(1 - e^{-\zeta a}\right)\right] da}.$$

We estimate the parameters α, β, γ, δ and ζ in (8) for the four countries India, Indonesia, Malaysia and the Philippines in the year 2021. Since we are interested in the influence of lifespan inequality on the dynamics of the age structure and in particular on the gains in the support ratio during a decline in fertility, we modify the estimated parameters α and β in opposite directions. This allows us to construct a set of life table functions, that are close to the original data but lifespan inequality varies while life expectancy at birth remains constant.

Fig. (5) shows the life table survival functions for the four countries under consideration. The circles indicate actual life table data (United Nations, 2022), the solid lines represent the estimated life table functions using the mortality law of Siler, the dashed lines show modified life tables with lower lifespan inequality and the dotted lines show modified life tables with higher lifespan inequality. Life expectancy at birth remains constant across all mortality scenarios. The graphs show that a reduction in lifespan inequality is achieved by lower mortality at younger and middle ages and higher mortality at older ages and vice versa.

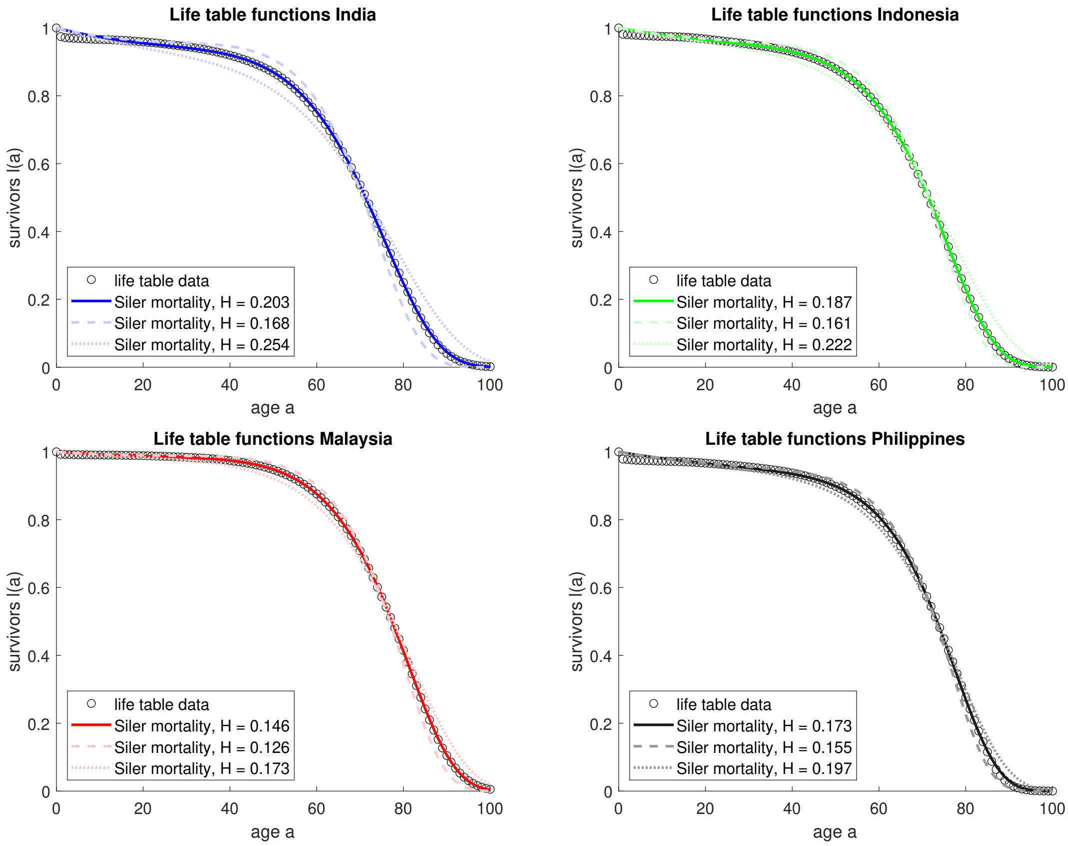

Figure 5: Life table survivors, Siler estimates and mortality scenarios, 2021. data source: United Nations (2022).

5 Results

Since we are interested in the influence of lifespan inequalities on the first demographic dividend, we first have a look on the dynamics of the support ratio under different mortality scenarios. Fig. 6 depicts the support ratio over time under the assumption that fertility declines at a constant rate. Again we focus on the four countries India, Indonesia, Malaysia and Philippines. The speed of fertility decline

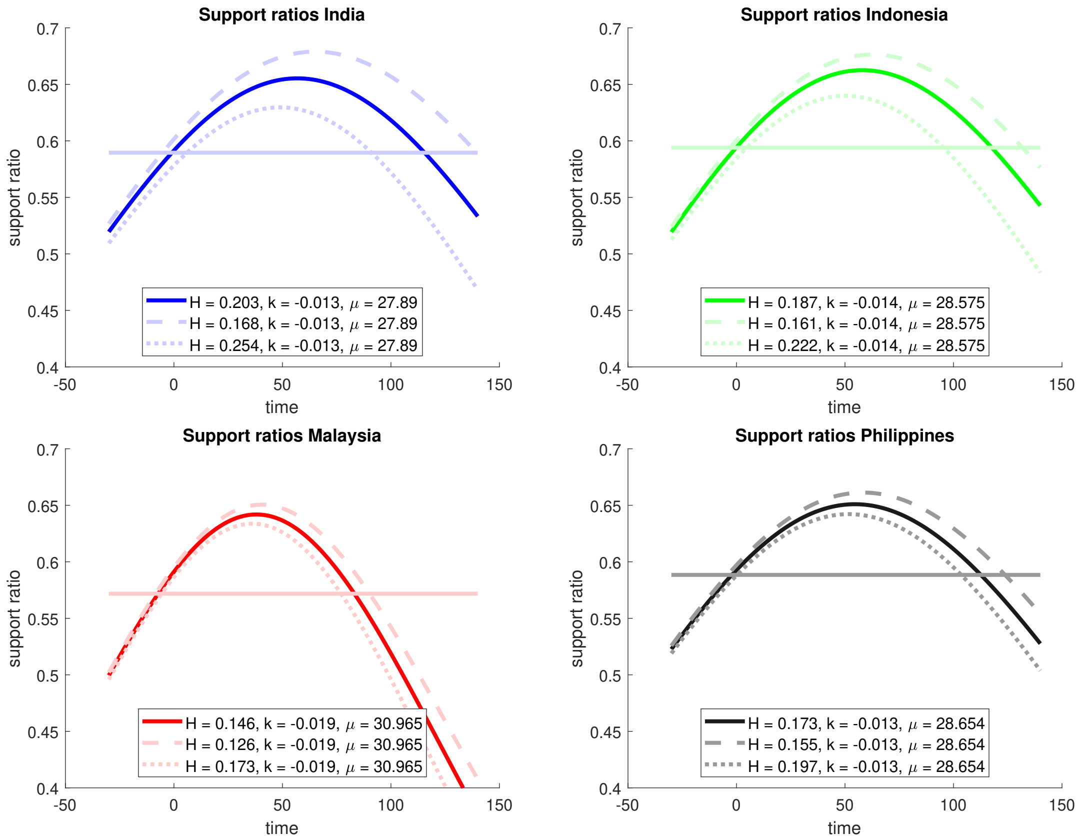

Figure 6: Support ratios over time for different mortality scenarios.

k is the average rate of decline experienced since the peak in the net reproduction rate (see table 1). The generation length μ is approximated by the mean age at birth observed in 2021. Analogous to fig. 5, the solid lines represent the support ratio trajectories using the 2021 life tables, the dashed lines show the support ratios if the life tables are adjusted to obtain a lower lifespan inequality, and the dotted lines show the support ratios if the life tables are adjusted to obtain a higher lifespan inequality. The horizontal line indicates the benchmark support ratio S_0 that would be obtained by a stationary population with the actual life table.

The results are very similar for all four countries. In the case of a reduction in

country	NRR_{max}	year
India	1.945	1964
Indonesia	2.063	1969
Malaysia	2.67	1961
Philippines	2.905	1959

Table 1: Maximum net reproduction rate and peak time.

lifespan inequality — i.e. a lower value of lifetable entropy H — the benchmark level S_0 is reached earlier, thus the population arrives at an advantageous age structure more quickly. Moreover, the peak value of the support ratio is higher, thus the intensity of the advantage from the favourable age structure increases. Finally, the second intersection with the benchmark, indicating the end of the advantageous age structure, occurs later. Thus, the gains from the advantageous age structure can be accumulated over a longer period of time.

Fig.7 provides a systematic investigation of the sensitivity of the results with respect to changes in lifespan inequality. These results are generated with various modified life tables. The markers indicate the results obtained with the actual life tables of the respective countries. For the other results again the parameters α and β are modified in opposite direction in order to vary life table entropy H but keep life expectancy at birth e_0 constant. The curves confirm the conclusions we drew from the support ratio dynamics discussed previously. The peak of the support ratio occurs later if lifespan inequality decreases. The maximum support ratio, the time interval during which the advantageous age structure can be sustained, and also the total amount of the surplus in the support ratio increase as lifespan inequality decreases.

The life table scenarios for these four countries are generated with the same variation in the parameter α. We multiplied the original parameter α by a factor between 0.4 and 2.5. The appropriate multipliers for the parameter β vary from country to country. The range of the multipliers for β varies between 0.855 to 1.1494 in the case of the Philippines to 0.78 to 1.22 in the case of India. Consequently, the variation of H is smallest in the case of the Philippines — from 0.1546 to 0.1968 — and largest in the case of India — from 0.1679 to 0.2537. This results in different variations in the peak time, the maximum support ratio, the duration and the total amount. The shift of the peak time takes values between 4.4 years (Malaysia) and 16.15 years (India). The variation of the maximum support ratio ranges from 0.0169 (Malaysia) to 0.0493 (India). The variation of the duration ranges from 98 years (Malaysia) to 144 years (India). Finally, the variation of the total amount ranges from 4.4 working years (Malaysia) to 7.8 working years (India).

We interpret the total amount as the potential additional working years per person in the population. The graph in the lower right panel of fig. 7 shows that this potential amounts to around 4 to 5 additional working years per person based on the current life tables. The sensitivity analysis indicates that this potential can be decreased or increased between 2 to 4 years if lifespan inequality increases or decreases while life expectancy remains constant. This analysis covers a wide range of parameter variations in order to make the influencing mechanisms clearly visible. Nevertheless, even small and realistic variations in lifespan inequality have a significant influence, allowing for substantial changes in the labour force potential provided by the first demographic dividend. We conclude that reducing lifespan inequality is a value in itself, but also offers major economic benefits.

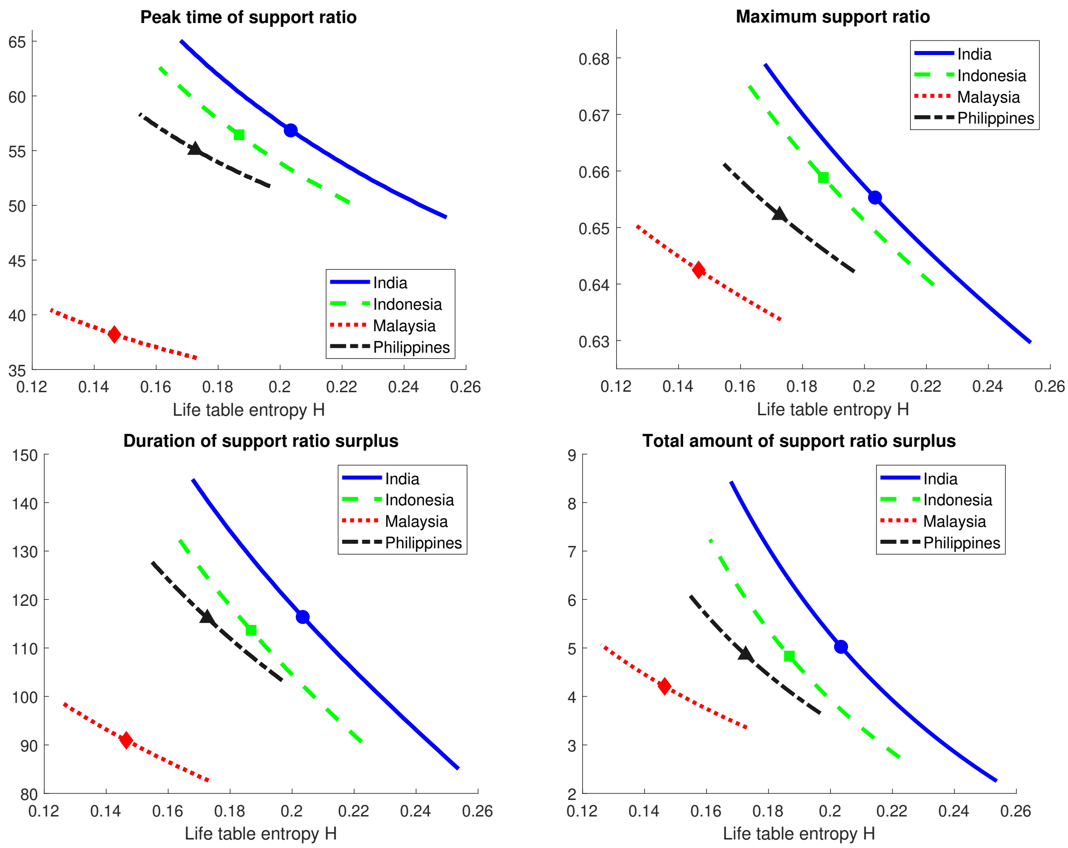

Figure 7: Sensitivity of timing, height, length and total amount of surplus support ratio with respect to changes in life table entropy H.

6 Summary and Conclusions

Throughout this paper we apply a concept of the first demographic dividend that focuses on the level of the support ratio rather than its growth as it is used among others for instance by United Nations Population Fund (2012). The support ratio is seen as favourable as long as it exceeds a given benchmark level. This interpretation allows for an exact quantification of the first demographic dividend within the applied modelling framework.

The mathematical and numerical derivations presented in this paper allow for a quantitative assessment of the influence of lifespan inequality on the dynamics of the age structure of a population and, in turn, on the characteristics of the first demographic dividend. Moreover, this framework is capable to project the characteristics of the first demographic dividend under different scenarios with respect to lifespan inequality and the life table in general, but also with respect to other relevant parameters such as the speed of fertility decline, generation length, or age at entering and leaving the labour market. In particular, we investigate four characteristics of the first demographic dividend.

- The *peak time* is the point in time when the support ratio reaches its maximum level and thus indicates the point in time when the benefits generated by the age structure are highest.
- The duration or *length* of the first demographic dividend is given by the time interval over which the support ratio exceeds the benchmark.
- The *height* indicates the maximum surplus the support ratio reaches compared to the benchmark.
- The *total amount* is defined as the area between the support ratio and the benchmark over the respective time interval.

The *peak time* indicates how long it takes to arrive at the maximum level. The results from the analysis of the formal model reveal that this peak is reached earlier in the case of higher lifespan inequality. From that perspective, a higher level of lifespan inequality seems favourable because the economic advantage gained from a decline in fertility can be savoured earlier. The *length* reveals how long the advantageous period lasts. Within the given framework, higher lifespan inequality shortens the duration of the advantageous age structure. The *height* indicates the intensity of the advantage gained from fertility decline. Higher lifespan inequality reduces this intensity. Finally, the *total amount* indicates the potential gain in terms of working years per person over the whole time interval when the support ratio is beyond the benchmark. According to the investigated model, higher lifespan inequality reduces the gain from the demographic dividend.

To sum up, higher inequality in life expectancy leads to an earlier peak, but this comes at the price of a shorter beneficial period, lower intensity and lower overall gains. Therefore, in addition to all the other benefits of reducing inequalities in life expectancy, the benefits from the first demographic dividend increase as inequalities become smaller, which means a benefit for the economy and for the population as a whole.

A Lifetable entropy

In this section we investigate the relationship between life span inequality and the paramter θ introduced in (7). To obtain flexibility in terms of lifespan inequality, we generalise the mortality law of de Moivre (1725) by adding an exponent j to the fraction a/ω. Then the life table function $l(a)$ becomes a polynomial of a/ω with non–integer power. With this we obtain a survival function that depends on maximum age ω and the exponent j,

$$\mu_j(a) = \frac{j(\frac{a}{\omega})^j}{a - a(\frac{a}{\omega})^j} = \frac{j}{\frac{\omega^j}{a^{j-1}} - 1} \quad \text{and} \quad l_j(a) = 1 - \left(\frac{a}{\omega}\right)^j. \tag{10}$$

The additional parameter j allows the creation of life tables with any degree of lifespan inequality.

Fig. 8 illustrates the shape of the force of mortality $\mu_j(a)$ and the survival function $l_j(a)$ for exponents j varying from 0.1 to 10. In the special case $j = 1$ (solid red lines) this function is equivalent to the mortality law of de Moivre, in the limiting case $j \to \infty$ all mortality is concentrated at age ω. The graphs show that lower exponents j equate to higher lifespan inequality.

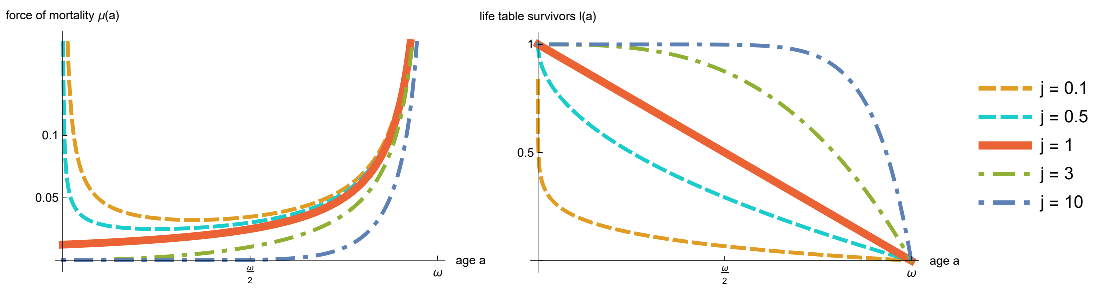

Figure 8: Force of mortality $\mu(a)$ and survival function $l_j(a)$ of the generalised survival function.

From (10) we get life table entropy,

$$H = \gamma + j - \frac{1}{1+j} + \psi\left(\frac{1}{j}\right),$$

where γ denotes the Euler–Mascheroni constant and $\psi(.)$ is the digamma function. The moments (6) become

$$L_i = \frac{j\omega^{1+i}}{(1+i)(1+i+j)}.$$

This results in

$$e_0 = \frac{j\omega}{1+j}, \quad A_0 = \frac{(1+j)\omega}{4+2j}, \quad \sigma_0^2 = \frac{(1+j)(7+j(4+j))\omega^2}{12(2+j)^2(3+j)}$$

and

$$\theta = \frac{(2+j)^2(10+j(5+j))}{(1+j)(4+j)(7+(4+j))}.$$

Assigning $j = 1$ (de Moivre) we get $\theta = 6/5$, the limit value for $j \to \infty$ (concentrated mortality) is $\theta = 1$ and for $j = 0$ we get $\theta = 10/7$, which serves as an upper bound for θ.

Fig. 9 shows parameter θ vs. life table entropy H. We see that this survival function allows an entropy H greater than one for very small values of j. The graph shows a clear monotonic relationship between the parameter θ and life table entropy H.

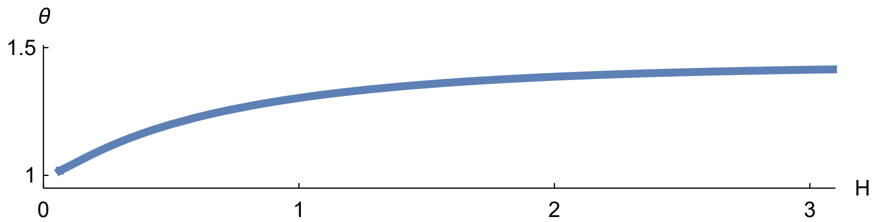

Figure 9: Parameter θ vs. life table entropy H.

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