The Formal Demography of Peak Population

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Talk for "The Causes and Consequences of Depopulation"

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Thanks

- Organizing committee and Wittgenstein Centre for the invitation
- Tom Cassidy, my co-author, for many hours of useful discussions
- Carl Schmertmann (and rest of twitter) for helpful comments on the app.

An imagined dialogue

Depulation "denier":

> *This depopulation thing is a myth. Lots of countries – even those with few migrants – have already had below-replacement fertility for a long time, but they just keep growing.*

Demographer's answer:

> *In age-structured populations, there's a lag. Just you wait!*

In this talk we'll learn how long the wait is likely to be.

Today's agenda

1. Coale's answer using his scenario of 'forever" fertility decline
2. Extending Coale's model
   (Longevity and flattening)
3. The Real World
   (China, India, Brazil, and the planet)
4. Conclusion and open questions

(Sorry, I'm going to skip migration. But happy to discuss)
(We'll have a couple of interactive survey Qs; and also use an interactive on-line app)

Where do babies come from?

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Demographer's answer:
"Today's babies come from past babies"

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A useful approximation:
"Today's babies come from mothers born about 30 years ago"

$$\hat{B}(t) \approx \hat{B}(t-\mu_0)NRR(t), \tag{2}$$

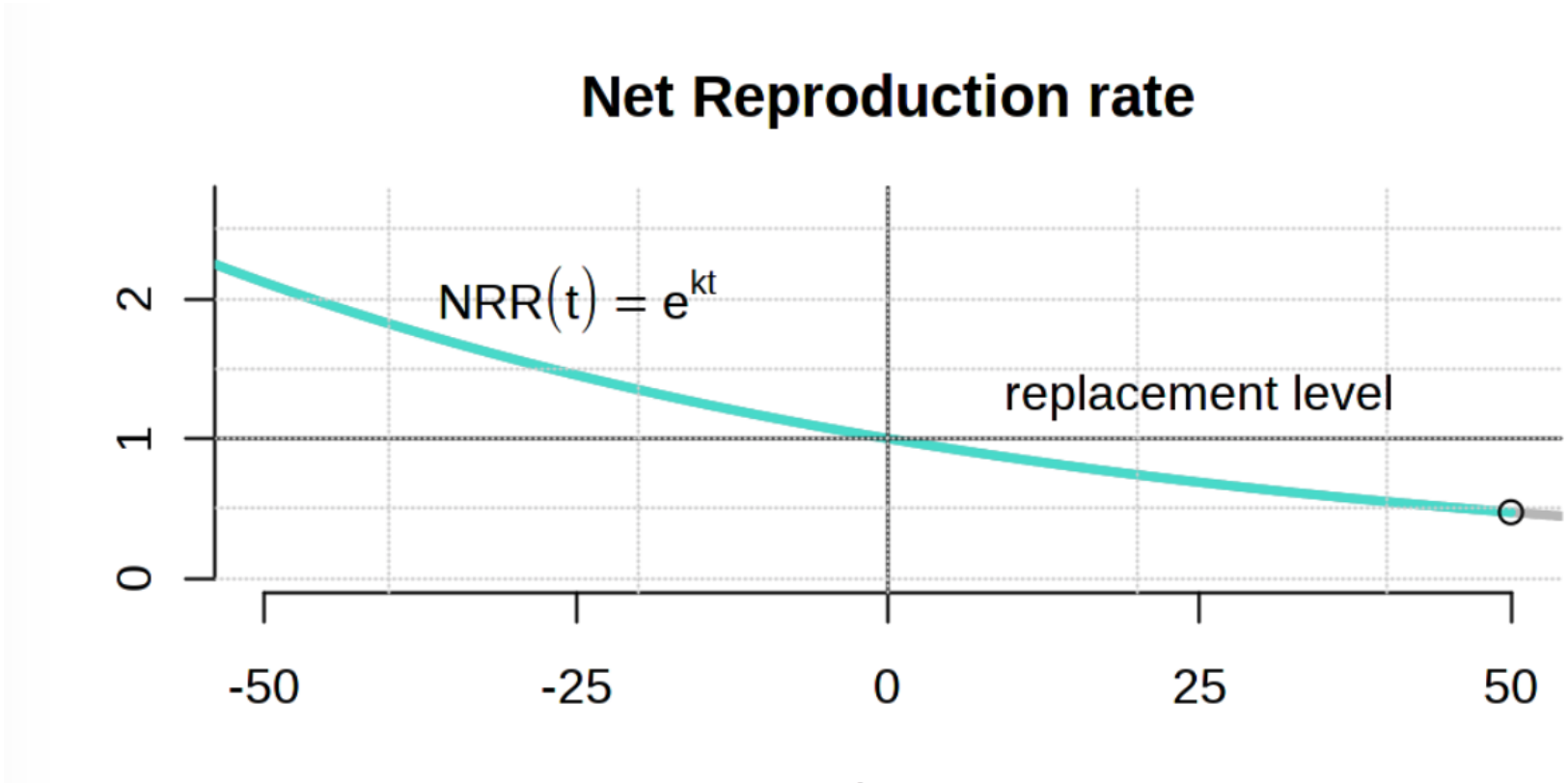
(Not exact. Because for time-varying renewal, $\mu_t \neq \mu_0$. But quite accurate.)

Coale's Scenario

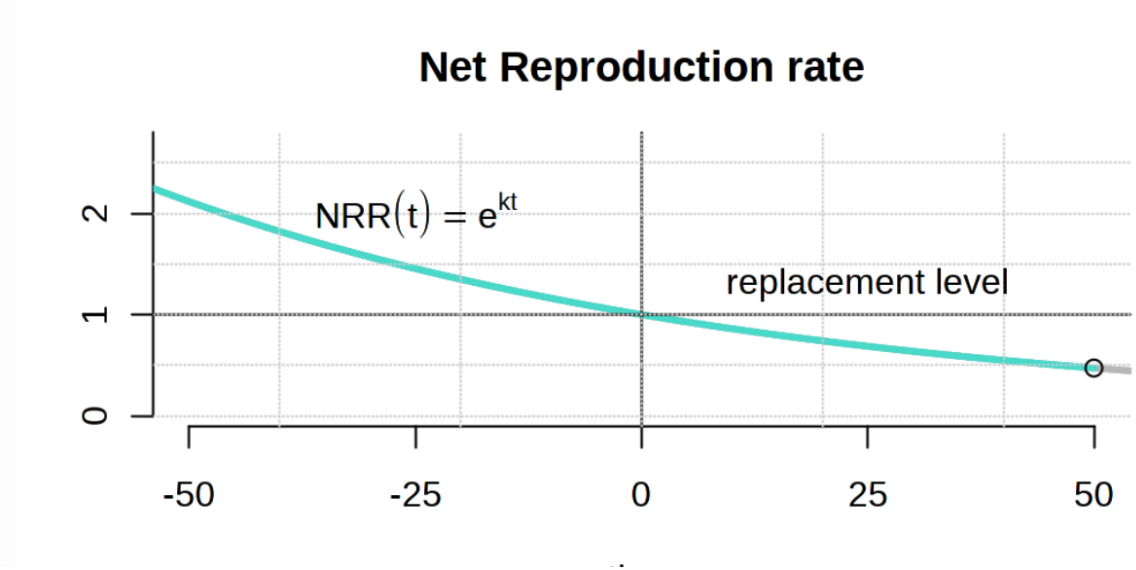
Coale curious about age-structure of populations undergoing fertility decline. Supposes,

$$NRR(t) = e^{kt},$$

where k is negative.



1st quiz: *When fertility declines, when do births peak?*



(Battle between increasing moms and decreasing babies-per-mom).

A. Before NRR reaches replacement?
B. At replacement ($t = t_0$)?
C. After replacement?

Type A, B, or C into chat. Moderators, which is most popular answer?

Our “shiny” simulation

- See when it happens
- See if speed of NRR decline makes a differenc
- Some intuition based on generational slope

What we learned about peak births

1. Births peak before t_0
2. In Coale scenario, exactly half a generation before:

$$t_B = t_0 - \mu_0/2$$

3. Invariant to speed of NRR decline (k). So should hold quite generally.

2nd Quiz: When does *population* peak?

A. Same time as births
   (When biggest generation arrives)
B. About 40 years (A_0) later
   (When biggest generation reaches average age of population)
C. About 80 years (e_0) later
   (When biggest generation "exits")

(Try chat again)

Answer is "about 40 years"

Population comes from births that have survived

$$N(t) = \int B(t-x)\ell(x)dx \tag{3}$$

Another useful approximation, concentrating population at average age of survival,

$$N(t) \approx B(t-A_0)\int \ell(x)dx \tag{4}$$

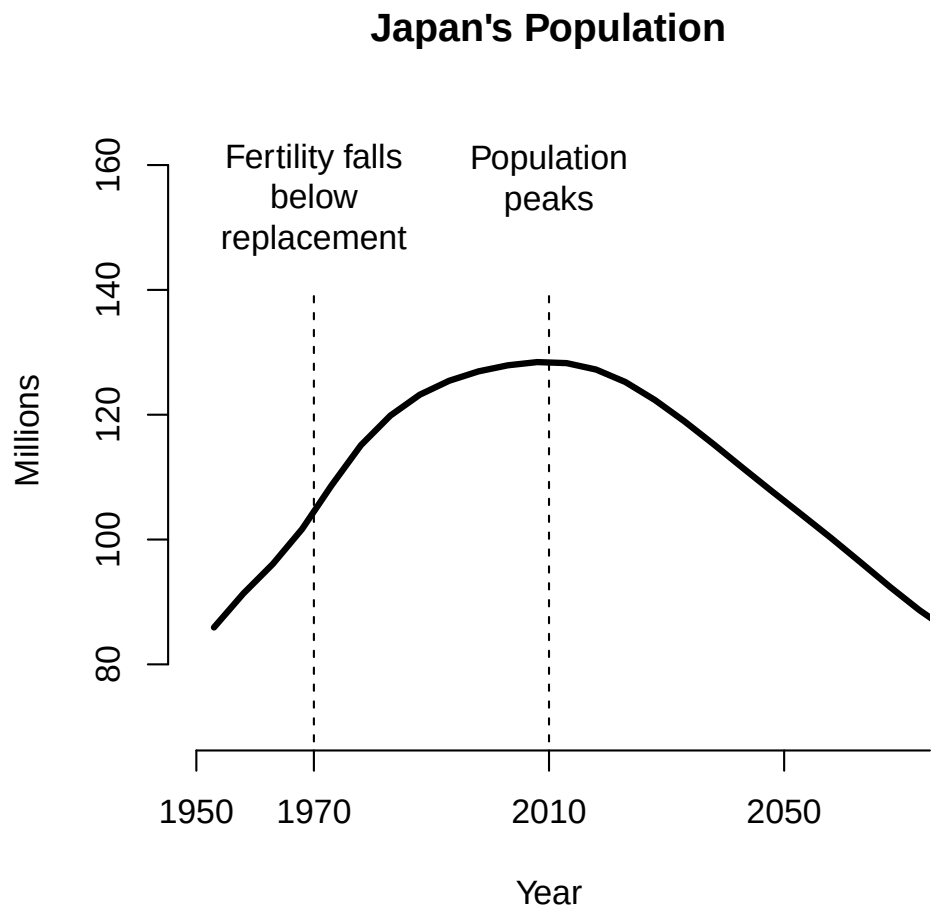
So, pop will peak about $A_0 = 40$ years after peak births.

Our “shiny” simulation

- When it happens
- If changing k matters

Example: Japan

Coale scenario tells us it should take 25 years (0 - 15 + 40 = 25) from t_0 to decline.



But observed lag is 40 years. What is going wrong?

1. Longevity

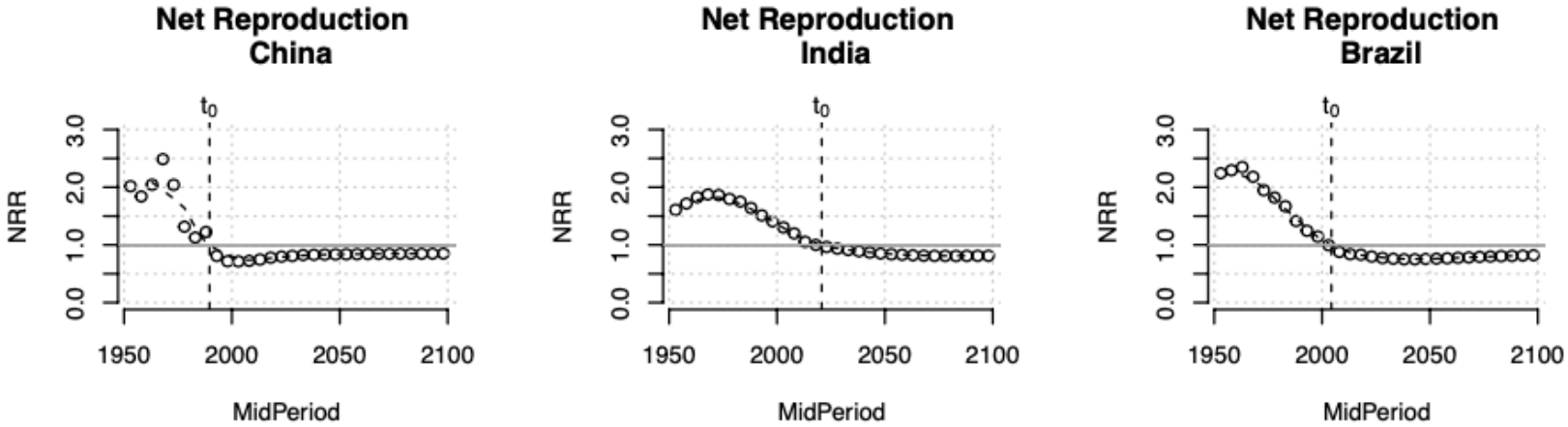
As people live longer, the population will – all other things equal – grow.

This tilts the right-side of population curve upward, and delays peak.

Result, we find, is a delay of 5-10 years.

2. "Real world" fertility differs from Coale's model

According to the UN



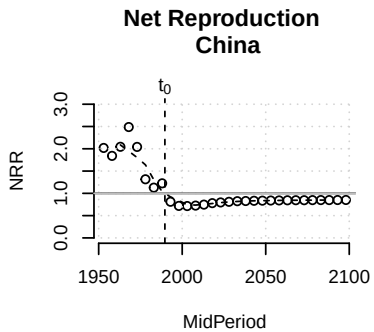
Exponential fall until replacement, and then some flattening.

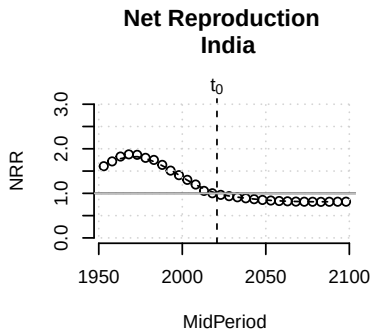
Result, we find, is a delay of another 5-10 years.

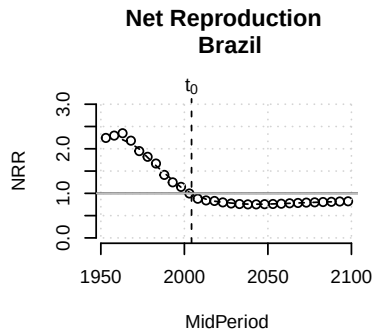
One final formula

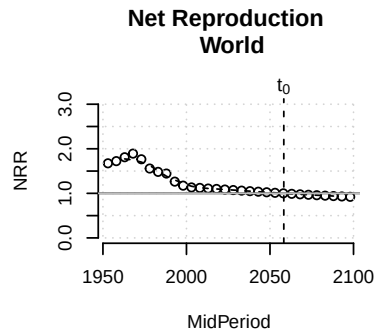
$$
\begin{aligned}
t_N &\approx \textit{birthpeak} + \textit{growingup} + \text{longevity} + \text{curvature} \\
&\approx (0-15) + (40) + (5 \text{ or } 10) + (5 \text{ or } 10) \\
&\approx \text{about 40 years}
\end{aligned}
$$

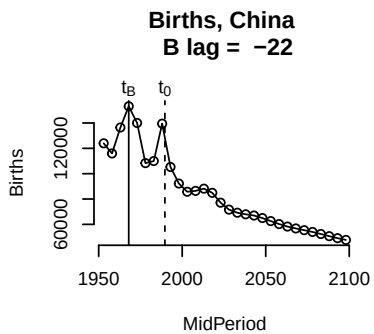
Taken together, how do we do?

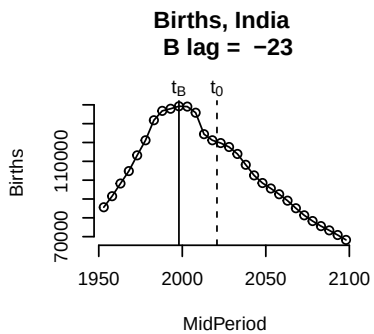


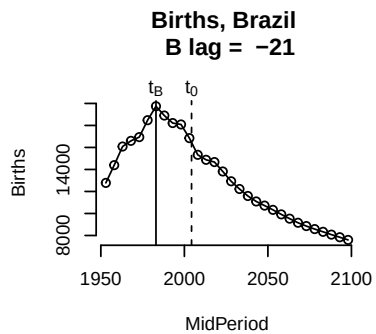


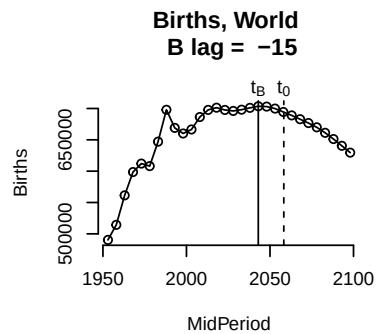


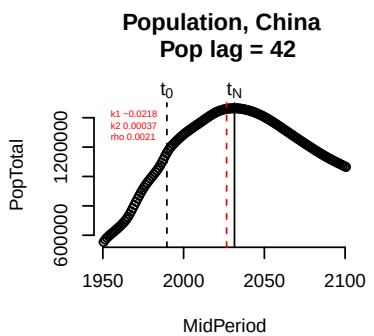


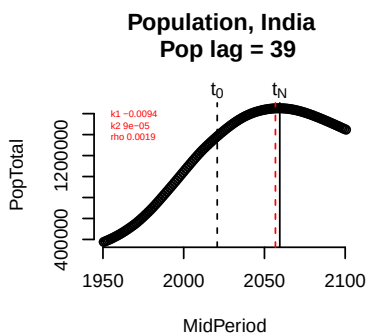


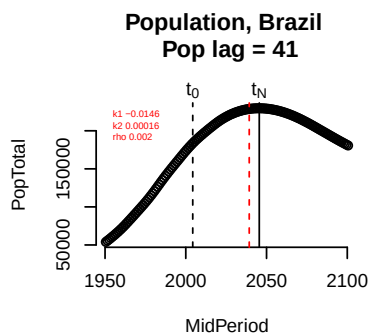


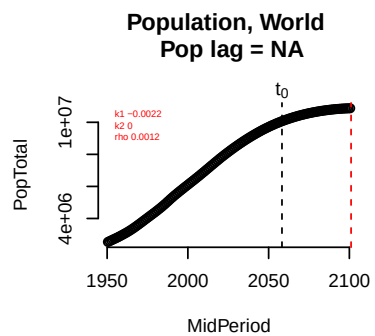












A comment on migration

Can model net migration (of demographically identical people) as a shift (up or down) of NRR curve, advancing or delaying t_0.

Lessons

- Be patient: pop will start falling about 40 years after replacement fertility
- There is demography beyond stable population theory
- Re-read the classics – Coale (1972) – and add to them.

Coda: Some open research questions

1. *Demographic dividend*
   And other functions of age structure
2. *Historical complications*
   Effects of rising NRR before decline – or "starting" age structure.
3. *Aggregation*
   How sub-pops with different peaks add up to peak for whole?
4. *Simple stochastic forecasts?*
   Distribution of when replacement fertility occurs to get uncertainty about population peak

Thank you.

Extra slides

t_0 explains differences in forecasts

Scenario	t_0	t_N	Delay
IIASA "scenario name"	$\sim$ 2015	$\sim$ 2055	40 years
IIASA "other scenario name"	$\sim$ 2035	$\sim$ 2070	35 years
IHME "Reference scenario"	$\sim$ 2030	$\sim$ 2063	33 years
UN "Medium"	2065	$\sim$ 2105?	40? years

Our extensions

1. Increasing longevity
   If e_0 increasing at rate ρ (e.g., .01 per year), we find

$$t_N^+ \approx t_N + \frac{\rho}{-k}\mu_0 \approx \text{5-10 years later}$$

2. "Flattening-out" of NRR decline below replacement.
   Our approach is to allow NRR decline to have curvature

$$NRR(t) = \exp(k_1 t + k_2 t^2), \qquad \text{for } k_1 < 0, k_2 > 0$$

   In this case, births fall a bit slower than they rise. So, peak is later. We find

$$t_{N2} \approx t_N + \frac{-k_2}{k_1}\frac{\sigma_\ell^2}{2} \approx \text{5-10 years later}$$

(See simulation)

A poor man's stochastic forecast

Tell me when you think t_0 will be and what your uncertainty is about that date, and I'll give you back a range of when population will peak.

Caveats

- Exact date of pop peak very sensitive since several years of near zero growth.
- Exact date of sub-replacement often not so clear
- Early rises in NRR seem to make t_B earlier, but we don't understand yet.