The Formal Demography of Peak Population

Extended Abstract for PAA 2022

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Abstract

When will population peak? In this paper, we build on classical results by Ansley Coale. We show that births are at their maximum before fertility has reached replacement, and that population reaches its maximum several decades after replacement levels. We extend Coale's results, modeling increases in longevity, net immigration, and log quadratic fertility decline that resembles more closely the projected slowdown in fertility decline. Our models predict a typical lag between replacement and fertility and population decline of about 40 years, consistent with projections by the United Nations projections.

1 Introduction

Even though a much of the world has had sub-replacement fertility for decades, few countries are actually declining in size. Japan is a good example. In Japan, fertility fell below replacement in about 1974, but the total population did not begin to decline until about 2009, some 35 years later. The world is forecast by the United Nations to reach replacement fertility later this century. Should we expect that same delayed onset of decline in other below-replacement fertility countries and for the future of the world population?

In this paper, we use mathematical models to shed light on the determinants of when populations peak in size and begin shrinking. The basic framework we use was introduced by Coale in his 1972 book *Growth and Structure of Human Populations*. His framework was useful for studying the age-structure of populations undergoing steady fertility decline. We extend it to consider increases in longevity, changing rates of fertility decline, and net migration.

We find that with these extensions, the stylized mathematical results match well with age-structured projections made by the United Nations. Populations will tend to decline 3 to 4 decades after replacement fertility is reached.

2 Previous Results

2.1 Background

Coale analyzed age-structured population dynamics for populations undergoing exponentially declining fertility at rate k "forever". The model is unrealistic for the distant past and the distant future but is plausible for the decades surrounding a transition from population growth to population decline. The population trajectory of the model considered by Coale "bell" shaped, reaching a peak several decades after fertility reaches replacement levels.

Coale's early work has been built on by Schoen and Kim's "hyperstable" population models in which fertility decline occurs exponentially but with a particular age pattern that makes the results more tractable.

We briefly describe Coale's approach and then summarize the additional results we have found.

3 Coale's approach

Beginning with the renewal equation

$$B(t) = \int B(t-a)\phi(a,t)\,da,$$

Coale assumes exponential decline in net maternity, such that $\phi(a,t) = \phi_0(a)e^{kt}$, for $k < 0$ and $\int \phi_0(a)\,da = 1$.

Generally, there is some time varying value μ_t satisfying

$$B(t) = B(t - \mu_t) \int \phi(a, t)\, da.$$

Coale's approach is to approximate μ_t as a constant μ_0, the mean age of childbearing in the stationary population with net maternity $\phi_0(a)$. This gives

$$B(t) \approx B(t - \mu_0)) \int \phi(a, t)\, da.$$

The approximate birth stream $\hat{B}$ then obeys

$$\hat{B}(t) \approx \hat{B}(t - \mu_0))e^{kt},$$

and has as its solution

$$\hat{B}(t) = \exp\left(\frac{k}{2}t + \frac{k}{2\mu_0}t^2\right).$$

Accepting this approximation yields two important and interesting results:

- Peak births t_B occur half a generation before t_0, when fertility:

  $$t_B = t_0 - \mu_0/2. \tag{1}$$

  This is typically about 15 years before $NRR(t) = 1$.

- Population will peak about when the largest cohort reaches the mean age of the stationary population,

  $$t_N = t_0 - \mu_0/2 + A_0. \tag{2}$$

  Typically, A_0 is about 40, so in Coale's scenario, population peaks about 25 years after fertility falls beneath replacement.

4 New Results

4.1 Longevity

At a global level life expectancy is increasing at a rate of about .. per year. The logic of comparative statics of stationary populations would suggest that this is adding roughly the same amount to population growth. A dynamic calculation involves using the "cohort average life expectancy" CAL. CAL increases slightly slower than period life expectancy.

We consider improving longevity at older ages, which allows us to leave the NRR tragectories unchanged. Population change from longevity at older ages can be modeled as the change in "cohort average life expectancy" CAL (Broullard, Guillot) . Assume – somewhat unrealistically – that life expectancy is increasing exponentially at rate ρ. In this case, we obtain the results that peak population will occur at time

$$t_N^+ \approx t_N + \frac{\rho\mu}{-k} \tag{3}$$

In order to obtain some intuition behind this result, think of longevity as tilting upwards slightly the trajectory of the log-population without mortality change. The original trajectory is roughly quadratic, and so when it is tilted, it takes longer to peak. The original peak is at year t_N. The tilt (of magnitude ρ) is then offset when the original growth rate would have reached $-\rho$. Our result tells us that this takes about $-\mu/k$ years.

Typical values might be $k = -.015$ and $\rho = 0.0025$ (corresponding to a rate of increase of about 2 years per decade for life expectancy of 80 years). In this case the population decline will be about 5 years *later* than it would have been without longevity change. (Note: here we consider only the Coale scenario).

4.2 Migration

Migration, like longevity, is also complicated to model because it depends on age-structure and time trends. However, a simple approach is to assume, following Alho, that immigrants have the same fertility and mortality as natives and arrive as a constant proportion γ of native births.

This formulation allows us to roll immigration into a somewhat expanded notion of the NRR, which reaches replacement at time

$$t_0^{(M)} = t_0 + \frac{\log(1+\gamma)}{-k} \approx \frac{\gamma}{-k}. \tag{4}$$

A population with considerable immigration might have $\gamma = .15$ and $k = -.015$, so $-\gamma/k \approx 10$ years. In this case, the critical timing of reaching replacement rates of fertility and immigration would be delayed by about 10 years and the timing of peak population by about the same amount.

4.3 Slowing fertility decline

In line with empirical observations, UN population projections assume a slowing rate of fertility decline over time. They use a double exponential, but a stylized alternative (that is locally accurate) is to assume the log(NRR) is quadratic over time, or

$$\phi(a,t) = \phi_0(a) \exp\left(k_1 t + k_2 t^2\right),$$

where typically $k_1 < 0$ and $k_2 > 0$.

In this scenario, the birth trajectory is approximately cubic in logarithms. Coale's conclusions can be represented in modified form:

- Births peak just a small bit earlier, with

  $$t_{B2} \approx t_0 \frac{-\mu}{2} + \frac{k_2}{2k_1}\mu, \tag{5}$$

  With k_2/k_1 typically being negative and $k_2 << k_1$.

- Population peaks somewhat longer after peak births, with

  $$t_{N2} \approx t_{B2} + A_0 + \left(-\frac{k_2}{2k_1}\sigma_\ell^2\right) \tag{6}$$

For example, in Brazil, $k_1 \approx -0.013$ and $k_2 \approx .0002$. This means that factor involved in the more complicated birth trajectory $\mu k_2/(2k_1)$ is on the order of 1/100. The change in the timing of peak births is very small, $<< 1$ year. But the delay the timing of peak population is more substantial $\approx$ 5 to10 years.

5 Empirical examples

The United Nations projects that many of the world's countries will experience population decline before centuries end. We can compare these projections, which combine historical experience with relatively complex projected trajectories of vital rates, with the approximations that we have formally derived.

[see last page for Figure]

Figure 1 shows the Net Reproduction Rate (NRR), births (per 5 year period), and annual population totals, according the 2019 United Nations "medium" scenario. We have chosen 3 large countries that are forecast to experience population decline, as well as the world as a whole, which is expected to still be growing very slightly at century's end.

The first row of the figure shows the historical and projected net reproductive rates. Three features emerge: a slight increase prior to 1970 due not to fertility increase but to declines in infant mortality, steady decrease until replacement, and finally a flattening out of fertility (or, in China, a slight rise) once below-replacement fertility is reached. The dashed vertical line labeled t_0 marks the year when the 5-period moving average of the NRR crosses replacement level of 1.0.

The next row shows the observed and projected births in each country and in the world. We see that in all cases, as our modeling predicts, births peak a decade or two before replacement NRR is reached. Our model does not allow for rises or ups and downs prior to replacement – such as seen in China, India, and Brazil, and in these countries the temporal gap between peak births and replacement fertility is a bit longer (on the order of 21 years) than predicted. For the world, however, where the projected decline is constant for many decades, peak births occur at exactly the expected time, 15 years, or one-half-a-generation, before t_0.

Finally, the bottom row shows the observed and projected populations. The timing of the "observed" peak population is marked with t_N and a solid black line. The red dashed line shows the approximation of $\hat{t}_N$. (These estimates use location-specific estimates of k, k_1, k_2, and ρ, and letting $A = 42$ and $\sigma_\ell^2 = 630$, the values of the stationary population of Sweden in 2015.)

The lag observed between replacement fertility and peak population is about 40 years in China, India, and Brazil. This is considerably longer than roughly 25 years predicted by the Coale scenario (with log-linear NRR decline and no longevity increase), but is just slighly later than the magnitude of effects we estimate with our extensions, as shown by the red dashed line.

For the world, replacement fertility is projected by the United Nations to occur in about 2060. The lag is until population decline is not yet observed by 2100. Applying our results to the world case, we would expect the peak to be reached in 2100. If it follows the pattern of the other countries examined here, we would expect the actual peak to be only a few years later.

6 Future plans

For our paper for PAA, we would like to

- Better understand the role of ups and downs in net fertility prior to the replacement.
- Develop expressions for the trajectory and size of peak population, not just the timing.
- Provide some examples of how migration affects the timing and magnitude of population decline

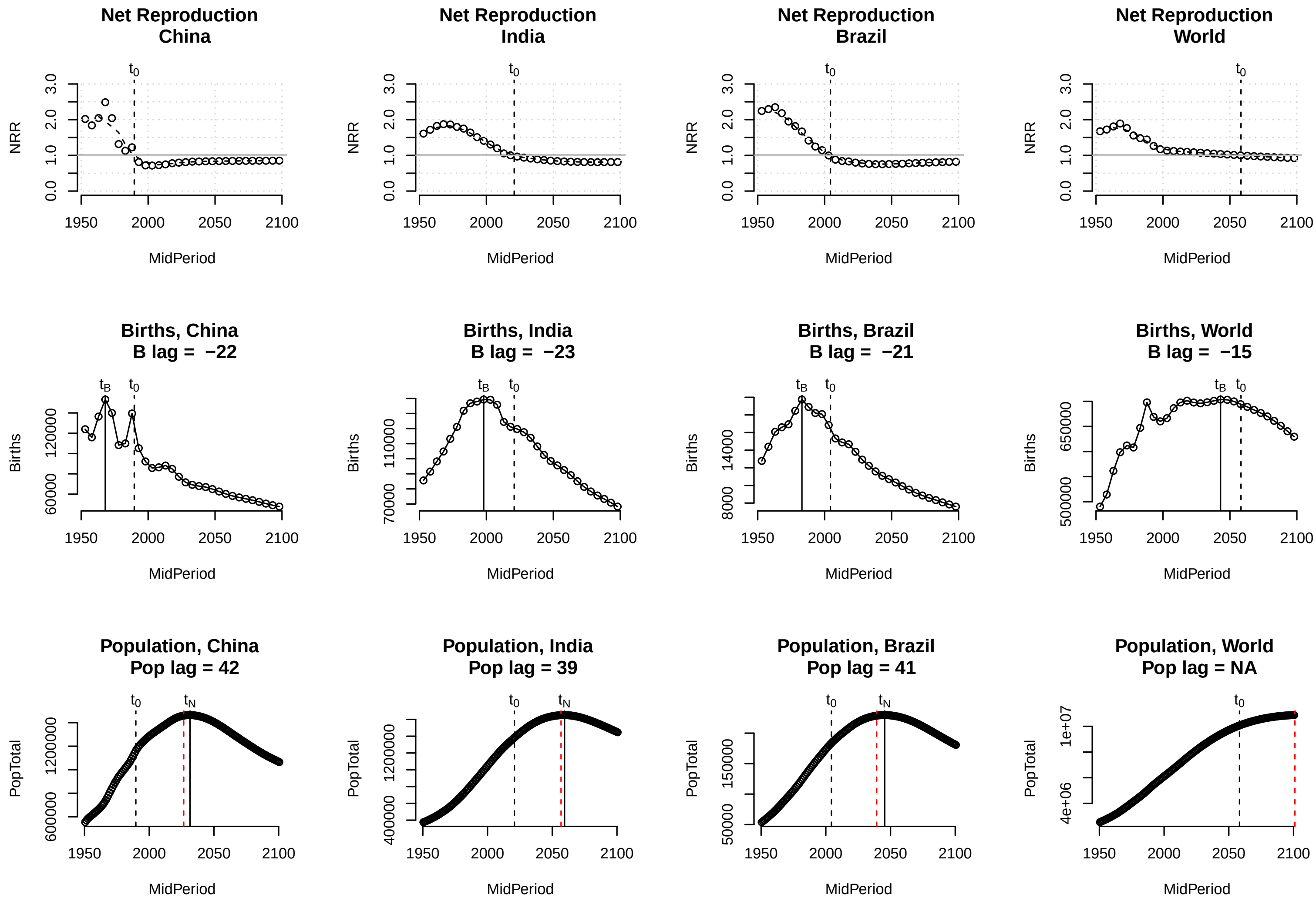

Figure 1: Empirical examples of Peak Population from United Nations Projections. The top row shows projected NRR trajectories according to the 2018 "Medium" projection, with the timing t_0 when replacement fertility is reached. Note the leveling after replacement. The middle row shows the 5-year totals of births, together with the same t_0 value and the observed peak. Note that births peak before t_0. The bottom row shows annual population totals together with the observed peak (the solid black line) and the predicted peak from our approximations (dashed red line). Note that the lags are close to that predicted by our modeling.