Disparities in careers of scientists

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Roadmap

“Science of Sciences”

Questions:

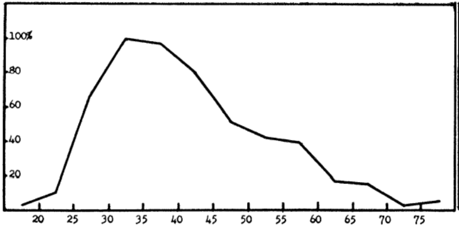
- How can a young scientist assess her/his chances for a scientific career?
- How should an evaluation committee take into consideration disparities in scientific production of candidates for a position?
- How to deal with a slump in scientific production (“midlife crisis”)

Incredible flood of scientific production (research papers, book etc.)

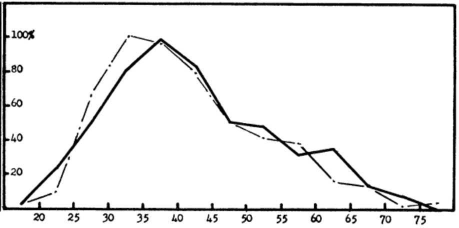
Essential inequalities and disparities in scientific production

Quetelet(1835!)

Lotka (1926): concentration: frequency of authors with n publications proportional to n^{-2}



(a)



(b)

Figure 1: Distribution of scientific publications over time (a) Age versus creative production rate for Russians only, in science and mathematics (b) Solid line: age versus creative production rate for Englishmen only, in science and mathematics. Broken line, same as in panel (a)

Steep increase and gradual decline of output over age

Way et al (2017), Feichtinger (2019)

Descriptive and normative models

Tipping Behavior

Life cycle models of human capital accumulation Becker (1962),

Ben-Porath (1967), McDowell (1982), Levine & Stepan (1991), Stepan (1996)

Individual productivity patterns

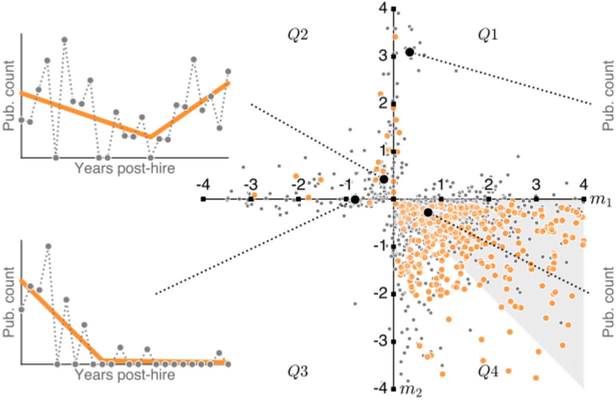


Figure 2: Distribution of individuals' productivity trajectory parameters. Plots show example publication trajectories to illustrate general characteristics of each quadrant. (Source: Way et al., 2017).

- Q1: busy scientists
- Q2: slump pattern (Schwandt, 2016)
- Q3: fading behavior
- Q4: one-peaked career (Feichtinger et al., 2018)

Rinaldi et al. (2000)

Simonton: (2014): Scientific and artistic work, Wiley Handbook of Genius

"Once a nobel laureate, always a nobel laureate"

Merton (1968): MATTHEW EFFECT
The winner lakes all, the loser (almost) nothing.

- Symonds al (2006): Gender differences "Academicians live longer"
- Feichtinger et al. (2006): Life expectancy of a 50 year old academician is 3 years higher than those of an Austrian with university education and 6 years longer than those of a "normal" Austrian male.
- Gould (2012): Human capital and socio-economic status of families.
- Disparitives between various desciplines (humanities vs natural sciences)
- Clauset et al. (2015): "faculty hiring network" of American ivy league universities

Grimm's Märchen (fairy tales)

Die Bremer Stadtmusikanten (The musicians of the city of Bremen)

Age problems of scientific production:
"Als alter Esel ist man auf den Hund gekommen, nach dem kein Hahn mehr kräht, und wo alles für die Katz' ist."

Retirement because shrinking power versus experiences.

Model description

Decision variables:

$K(t)$ ... reputation of the scientist

$I(t)$ ... investments into improving reputation (e.g. by research, networking, etc.)

Parameters:

r ... discount rate

c ... investment costs (individual characteristics/talent of scientist)

δ ... depreciation rate of reputation

a ... self-enforcement rate of reputation

d ... discrimination parameter

τ ... small parameter (for numerical calculations)

Matthew function:

$$M(K) := \frac{aK - d}{K - \tau}$$

The model

$$V(K_0, I(\cdot), T) := \int_0^T \mathrm{e}^{-rt} \left(\ln(K(t)+1) - cI(t)^2\right) \mathrm{d}t \tag{1a}$$

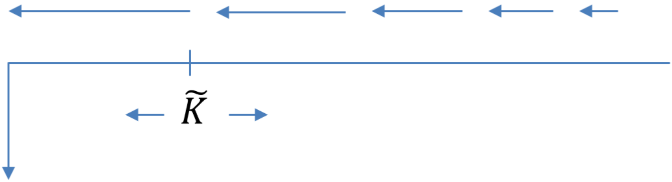
$$\max_{I(\cdot),T} V(K_0, I(\cdot), T) \tag{1b}$$

$$\text{s.t.} \quad \dot{K}(t) = \begin{cases} I(t) - \delta K(t) + \dfrac{aK(t) - d}{K(t) + \tau} & K(t) > 0 \\ 0 & K(t) = 0 \end{cases} \tag{1c}$$

$$0 \leq I(t) \leq I_{\max} \tag{1d}$$

$$K(0) = K_0 \geq 0. \tag{1e}$$

River Paradigm: an oar moves against a current in a river A; rowing up current



$\tilde{K}$ ... stalling equilibrium
(maximum investment I_{max} required to stay in $\tilde{K}$)

$I < I_{max} \ldots\ K(t)$ decreasing

Parameter

r	δ	c	τ	d	a	$I_{\max}$
0.03	0.1	$*$	10^{-5}	1	$*$	1

Table 1: Base case parameter values.

Necessary optimality conditions

$\lambda(t)$ ... current-value costate variable

λ_0 ... factor for the objective value in the Hamiltonian

$\mu(t), \nu(t)$ ... Lagrange multiplier

Hamiltonian

$$\mathcal{H}(K, I, \lambda) = \lambda_0 \left(\ln(K+1) - cI^2\right) + \lambda \left(I - \delta K + \frac{aK - d}{K + \tau}\right),$$

Lagrangian

$$\mathcal{L}(K, I, \lambda, \mu, \nu) = \mathcal{H}(K, I, \lambda) + \mu I + \nu(I_{\max} - I).$$

The derivative of the Hamiltonian and Lagrangian with respect to the control I yields

$$\partial_I \mathcal{H} = -\lambda_0 2Ic + \lambda$$
$$\partial_I \mathcal{L} = \partial_I \mathcal{H} + \mu - \nu.$$

Thus, from the maximum principle

$$I^*(t) = \underset{0\leq I\leq I_{\max}}{\operatorname{argmax}}\, \mathcal{H}(K^*(t), I, \lambda(t)) \tag{2}$$

the following expressions for the control and Lagrangian multipliers can be derived

$$I^* = \begin{cases} 0 \\ \dfrac{\lambda}{2c\lambda_0} \\ I_{\max} \end{cases}, \quad \mu = \begin{cases} -\lambda \\ 0 \\ 0 \end{cases}, \quad \nu = \begin{cases} 0 \\ 0 \\ -\lambda_0 2Ic + \lambda \end{cases}. \tag{3}$$

The canonical system writes as:

$$\dot{K}(t) = I^*(t) - \delta K(t) + \frac{aK(t) - d}{K(t) + \tau} \tag{4a}$$

$$\dot{\lambda}(t) = r\lambda(t) - \partial_K \mathcal{H}(t). \tag{4b}$$

with

$$\partial_K \mathcal{H} = \frac{\lambda_0}{K+1} + \lambda \left(\frac{a\tau + d}{(K+\tau)^2} - \delta \right).$$

An optimal solution $(K^*(\cdot), I^*(\cdot), T^*)$ has to satisfy the following (limiting) transversality conditions

$$K^*(T^*) = 0 \quad \text{and} \quad \mathcal{H}(K^*(T^*), I^*(T^*), \lambda(T^*)) = 0, \quad \text{for} \quad T^* < \infty \tag{5a}$$

or

$$\lim_{t \to \infty} \mathrm{e}^{-rt} \lambda(t) = 0, \quad \text{for} \quad T^* = \infty. \tag{5b}$$

Minimal admissible control value $\underline{I}$ such that zeros of the state dynamics (1c) exist for $I \in [\underline{I}, I_{\max}]$:

$$\underline{I} := \begin{cases} 0 & 2\sqrt{d\delta} - a \leq 0 \\ 2\sqrt{d\delta} - a & 0 \leq 2\sqrt{d\delta} - a \leq I_{\max} \\ \infty & \text{otherwise.} \end{cases} \tag{6}$$

Proposition 1

For $\tau = 0$ the following cases can be distinguished

$\underline{I} < I_{\max}$: *for every $I \in [\underline{I}, I_{\max}]$ there exist two branches of equilibria $K_i(I), i = 1, 2$ with*

$$K_{1,2}(I) = \frac{I + a \mp \sqrt{(I+a)^2 - 4d\delta}}{2\delta} \tag{7a}$$

and

$$\tilde{K}_i := K_i(I_{\max}), \quad i = 1, 2. \tag{7b}$$

For $I \in (\underline{I}, I_{\max})$ and $\underline{I} < \infty$ the equilibria in (7a) satisfy

$$\tilde{K}_1 < K_1(I) < K_2(I) < \tilde{K}_2. \tag{7c}$$

$\underline{I} > I_{\max}$: *there exists no equilibrium of (4a) for $0 \leq I \leq I_{\max}$.*

$\underline{I} = I_{\max}$: *the only equilibrium is $\tilde{K}_1 = \tilde{K}_2$.*

Stalling equilibria

There exist two solutions $\tilde{K}_1$ and $\tilde{K}_2$ of (4a) for $I = I_{\max}$

$$\tilde{K}_{1,2} = \frac{I_{\max} + a - \delta\tau \mp \sqrt{(\delta\tau - I_{\max} - a)^2 - 4(d - I_{\max}\tau)\delta}}{2\delta} \tag{8}$$

and the solutions are real valued iff

$$D := (\delta\tau - I_{\max} - a)^2 - 4(d - I_{\max}\tau)\delta \geq 0.$$

Optimal solution patterns

Proposition 2

The optimal control problem (1) exhibits an optimal solution. For $T^* = \infty$ *and* $\lambda_0 = 1$ *the Arrow sufficiency conditions are satisfied.*

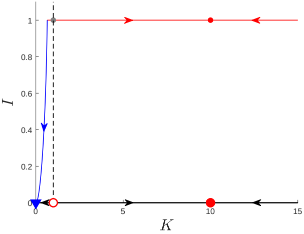
Abnormal case

Normal case: $\lambda_0 \neq 0$
Abnormal case: $\lambda_0 = 0$

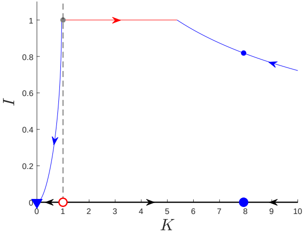
Halkin (1974), Aseev and Veliov (2015)

Proposition 3

Let the problem (1) be abnormal for $K(0) = \tilde{K}_1$ and let $\underline{I} \leq I_{\max}$. Then if $I_{\max} \neq I_{\text{crit}}$ the equilibrium $(\tilde{K}_1, 0)$ with $\tilde{I} = I_{\max}$ is an admissible equilibrium of the canonical system (4).

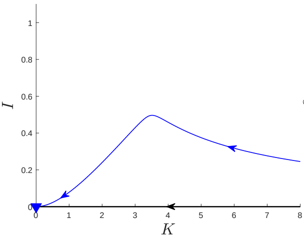


(a) $c = 0.25$ (I)

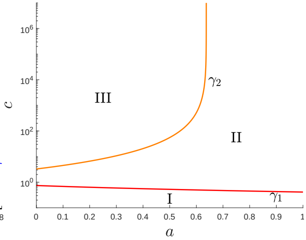


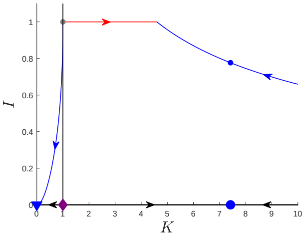
(b) $c = 0.6$ (I)

Figure 3: Bifurcation diagram for parameter values a and c is at the center. The examples are calculated for the parameter values taken from 1 with $a = 0.1$ and increasing c.

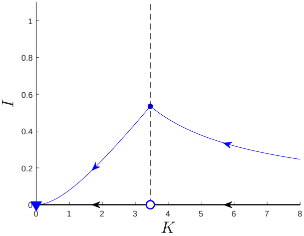


(g) $c = 4.55$ (III)

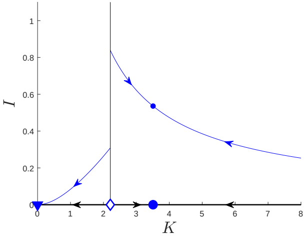




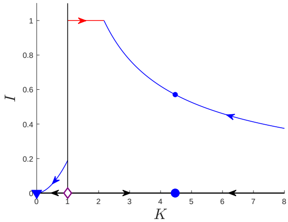
(c) $c \approx 0.684,\ \gamma_1$



(f) $c \approx 4.515,\ \gamma_2$



(e) $c = 4.25$ (II)



(d) $c = 2$ (II)

REGION I: a locally stable and an unstable equilibrium exist ($\tilde{K}_2$ and $\tilde{K}_1$)

low investment costs: large domain of $K(0)$ exists for convergence to $\tilde{K}_2$

BIFURCATION CURVE γ_1: stalling equilibrium becomes Skiba equilibrium;

For $K(0) = \tilde{K}_1$ decision maker indifferent either to stay in $\tilde{K}_1$ with $I = I_{max}$ or choose $I^*(0) < I_{max}$ moving to 0

REGION II: Skiba equilibrium instead of stalling equilibrium;
c increasing ... $I^*(0) < I_{max}$;
domain leaving academia becomes larger;
optimally discontinous investments

BIFURCATION CURVE γ_2: Skiba point coincides with stable equilibrium and becomes semi-stable

REGION III: Always optimal to move against 0 in finite time for large values of c and low values of a.

Conclusions

- Merton (1968): better-known scientists tend to receive more academic recognition than lesser-known scientists for similar work.

- Matthew effect also in other socio-economic fields as education, health status, income and pension dynamics.

- History-dependence of the solution
  - → Matthew effect can be crucial for success of scientist
  - → Supportive environment important
  - → Making mistakes in early career stages more devastating than in later stages

- Threshold in state space: stalled equilibrium, investment at its upper boundary without any growth: Putting maximum efforts into work is not always rewarded, but necessary to stay in academia

Conclusions

- Paradigm of oarsman in a rowing boat
  Other examples: collapse of empires; capital accumulation under increasing returns and a self-financing constraint (Skiba on the boundary)
- Multi-stage framework
- Abnormal optimal problems
- Economist (2019) / Wang et al. (2019): "What doesn't kill me makes me stronger"
- Stroebe (2010): The graying of academia: Will it reduce scientific productivity.