International Workshop on Heterogeneous Dynamic Models of Economic and Population Systems

1 December – 2 December 2016

Venue: Vienna University of Economics and Business – Campus WU,, Welthandelsplatz 1, 1020 Vienna

Building AD, Conference room 1

Abstracts

Regional control in optimal harvesting

Author: Anita, S.
Friday, Dec. 2, session 5, 9:00-9:35.

Abstract:
Here we investigate the regional control for some optimal harvesting problems related to population dynamics; namely we consider the problem of maximizing the profit for spatially structured harvesting problems with respect to both the harvesting effort and the selected subregion ω (of the whole domain Ω) where the effort is localized. For a fixed subregion ω we state necessary optimallity conditions and use them to get the structure of the optimal effort and to reformulate the maximization problem with respect to the subregion ω, where the harvesting effort is localized, in a more convenient way. We derive an iterative algorithm to increase at each iteration the profit by changing the subregion where the effort is localized. Final comments are given as well concerning furtherdirections to extend the results and methods presented here.

Optimal traverses in spatial resource harvesting

Authors: Behringer, S., Upmann, T.
Friday, Dec. 1, session 2, 12:25-13:00.

Abstract:
In standard models of spatial harvesting, the resource is distributed over the complete domain and the agent is able to control the harvesting activity everywhere all the time. In some cases, though, it is more realistic to assume that the resource is located at a single point in space and that the agent is required to travel there in order to be able to harvest. The agent then faces a combined travelling-harvesting problem. We investigate this type of a two-stage optimal control problem and illuminate the interdependencies between the solution of travelling and that of the harvesting sub-problem. In particular, bounds on either controli. e. on the acceleration respectively on the harvesting capacity significantly affect the policies in both sub-problems. Since the model is parsimoniously parameterised, we are able to characterise the optimal policy of the complete travelling-harvesting problem analytically.

Necessary conditions for infinite horizon optimal control problems revisited

Author: Belyakov, A.
Friday, Dec. 2, session 5, 9:35-10:10.

Abstract:
Necessary optimality conditions in the form of the maximum principle for control problems with infinite time horizon are considered. Both finite and infinite values of objective functional are allowed since the concept of overtaking and weakly overtaking optimality is used. New form of optimality condition is obtained and compared with the transversality conditions usually used in the literature. The examples, where these transversality conditions may fail while the new condition holds, are presented. For Ramsey problem of capital accumulation a simple form of necessary optimality conditions is derived, which is also valid in the case of zero discounting.

Endogenous leadership and market inefficiency in differential game of R&D

Authors: Bondarev, A., Greiner, A.
Friday, Dec. 2, session 6, 11:50-12:25.

Abstract:
In this paper we analyze the dynamics of differential game in R&D under technological spillovers and convex-concave investment efficiency functions of multiproduct oligopolies. We demonstrate how the presence of this learning dynamics together with spillovers may generate the rich set of outcomes, varying from technology lockin to permanent innovation cycles. These types of dynamics are qualitatively different both from the single firm dynamics with learning and from duopoly situation with spillovers without learning. The learning dynamics is assumed over the range of technologies, thus making one or another of the firms more efficient in developing given technologies. We account for three possible regimes of this multi-modal differential game for the case of 2 players, arising from the convex-concave investment efficiency functions. These regimes are: constant leadership of one of the players, the regime where leadership changes once and where leadership changes twice. Two latter regimes cast the dynamics of the problem into the class of piecewise-continuous systems, where jumps of control trajectories and/or cyclical and sliding dynamics of optimal trajectories may be observed. The resulting dynamical picture combines the presence of Skiba-points, multimodal differential game with switching leadership and piecewise-continuous dynamical system of Filippov or impacting types, depending on the strength of imposed continuity constraints. We discuss main features of the emerging phase space of the system and discuss mathematical as well as economic implications of the model.

Averaged optimization and cyclicity

Author: Davidov, A.
Friday, Dec. 2, session 5, 10:10-10:45.

Abstract:
The averaged optimization along an admissible motion $x = x(t); t \in [0, \infty)$ of a control system is connected with maximization of functional

$$\overline{\lim} \frac{1}{T} \int_0^T f(x(t)) \ dt.$$

Here a continuous function f could be interpreted as a density of some resource which is collected by an object with this motion (see, for example, [1], [2], [3]).

Under some conditions in such situation any optimal motion could be approximated by a periodic one [1], or it could be just a cycle [4], [5], even in the presence of a discount in integrant [6], [7].

The talk is devoted to this results and some another related.

The work is done by partial financial support by Ministry of Education and Science of the Russian Federation under the project 1.638.2016/FPM and by Russian Fond for Basic Research under the grant 15-01-08075a.

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Optimal investment models with vintage capital and state constraints

Authors: Dupuis, X., Gozzi, F.
Friday, Dec. 2, session 6, 12:25-13:00.

Abstract:
This paper consider a family of optimal investment models with vintage capital (already studied in the literature by various authors) and studies how the presence of the natural state constraints (positivity of the capital) affects the optimal solution. We start from the "easy" one dimensional case (no vintage capital) where we can fully describe when and how state constraints matter. Then we pass to the more complicated case with vintage capital. First we prove ad hoc necessary conditions in this case and then we use them to get some initial insights on the optimal strategies.

Some recent developments on endogenous spatial growth models

Author: Fabbri, G.
Thursday, Dec. 1, session 3, 14:15-14:50.

Abstract:
In this presentation some recent developments on spatial economic growth models are presented.

Our benchmark is the model of endogenous growth with explicit capital accumulation presented and solved by Boucekkine Camacho and Fabbri (2013). We show how, weakening a series of assumptions of this setting, we can obtain a series of further insights about spatial distribution of economis activity.

First we present the results of Fabbri (2016) where an AK spatial growth model with a general geographical structure is considered. In this case the morphology becomes one determinant of the qualitative behavior of the economy. We show how it is possible to characterize the conditions on the geographical structure that guarantee convergence of the detrended capital across locations in the long run, and those inducing spatial capital agglomeration.

We will then describe another possibility (Boucekkine, Fabbri, Federico, 2016) of breaking the "spatial neutrality" of the benchmark explicitly supposing that the (exogenous) technological level and the population density are space dependent. The convergence result found in the benchmark does not hold anymore in this generalized setting. We analyze the dependence of the long run profile of the detrended capital on the technological and population distribution.

We will finally be interested at the impact of different welfare utility function on the found results.

Do intergenerational transfers increase wealth inequality?

Authors: Feichtinger, G., Fürnkranz-Prskawetz, A., Sanchez-Romero, M., Wrzaczek, S.
Thursday, Dec. 1, session 1, 9:00-9:35.

Abstract:
In this paper we study to what extent intergenerational wealth transfers, i.e. *bequest* and *inter-vivos*, may increase wealth inequality along the demographic transition. We propose a small-open economy model populated by overlapping generations in which altruistic individuals towards their heirs differ according to their age, year of birth, and their parent's age (*generational length*). Individuals face different mortality and fertility profiles and optimally decide their consumption path and the wealth transferred to their heirs while alive and at the time of death. In case that individuals do not have heirs, they decide to purchase annuities. Our main finding suggest that intergenerational transfers increase wealth inequality along the demographic transition. Moreover, in order to have a better understanding of the impact of demography to the increase of wealth inequality, we analyze the contribution of fertility and mortality, separately, to this process.

Life-cycle behavior in the face of large shocks to health: a modeling approach

Authors: Frankovic, I., Kuhn, M., Wrzaczek, S.
Thursday, Dec. 1, session 2, 11:15-11:50.

Abstract:
Typically life-cycle models of health behavior take an ex-ante stance, where a representative individual is subject to the depreciation of a health stock, subject to some mortality process, or subject to the accumulation of deficits. Life-cycle decisions are fixed with anticipation of these processes. However, in reality, health is not developing according to a smooth process which may be shaped to some extent by health expenditures, but is subject to smaller or greater health shocks. Some of them (life-threating diseases, accident, chronic diseases) have the potential to put the entire life-course on a different trajectory (permanently lower income stream, higher mortality risk, etc.). We formulate a model where individuals anticipate such a shock with a certain probability. We present optimality conditions, economic intuitions, and finally a transformation of the model, which facilitates the numerical solution of the model.

Planned obsolescence in a vintage model of the firm

Authors: Hartl, R.F., Kort, P.M., Wrzaczek, S.
Thursday, Dec. 1, session 4, 17:05-17:40.

Abstract:
We consider a firm that is producing a durable good. The disadvantage of a durable good that does not break down is that a consumer only needs to buy once. This in contrast with a durable good that breaks down often, because then every time the consumer has to decide whether to purchase this product again. If the consumer decides positively, the firms revenue goes up. On the other hand, if a product breaks down too often, the consumer is more inclined to decide not to buy this product anymore. The above makes clear that when in the production process the firm decides about the quality of the product it faces the following trade off: a high quality implies a high reputation but a low breakdown probability so that consumers will not repurchase too often, whereas a low quality means that the product breaks down soon, implying that consumers need to buy this product again but at the same time this damages the products reputation, which reduces demand. The paper investigates this problem in a dynamic model of the firm that explicitly takes account of the time the product is sold and the age of the product.

Life in shackles? The quantitative implications of reforming the educational financing system

Authors: Heijdra, B., Kindermann, F., Reijnders, L.S.M.
Thursday, Dec. 1, session 1, 9:35-10:10.

Abstract:
We conduct a quantitative analysis of educational financing systems in a stochastic overlapping generations model in which human capital can be enhanced through both formal schooling and learning-bydoing. The model is calibrated to the United States economy, including a stylized version of its mortgage loan system. We find that moving to an income-contingent educational financing system, whereby transfers to students are financed from taxes on labour income, generates aggregate welfare gains. Such a system improves risk-sharing among college graduates and incentivizes individuals to obtain more education. These positive effects overturn the negative impact from labour supply distortions. Reforming the educational financing system towards income contingency, however, generates a considerable amount of transitional dynamics, so that welfare gains and losses are distributed unevenly across generations.

Catastrophe and cooperation

Authors: Heijnen, P., Dam, L.
Thursday, Dec. 1, session 4, 16:30-17:05.

Abstract:
We study international environmental agreements in a setting that incorporates catastrophic climate change, and sovereign countries, who are heterogeneous in their exposure to climate change. This leads to a stochastic game with an absorbing state whose equilibrium structure is very different from the infinitely repeated games that are usually studied in the literature on environmental agreements. In particular there is no *folk theorem* that guarantees that the social optimum can be sustained in a Nash equilibrium as long as players are sufficiently patient. However, in most circumstances, it is feasible to implement an abatement scheme with a level of aggregate abatement that is close to the social optimum. Moreover, the discount rate has a non-monotonic effect on the optimal environmental agreement.

Towards a more realistic life-cycle model of the labor market

Author: Kerndler, M.
Thursday, Dec. 1, session 3, 14:50-15:25.

Abstract:
Life-cycle labor market models often abstract from important empirical regularities. Population economists and demographers mostly assume a frictionless labor market where wages always clear the market. The analysis typically focuses on issues of labor supply, consumption, and savings. The labor market literature, by contrast, acknowledges that job search is impeded by frictions. The main focus then lies on vacancy creation of firms, matching of unemployed workers to vacancies, and wage determination. At the same time, the worker side remains very stylized. Individuals are mostly assumed to be risk neutral and unable to accumulate savings. This paper provides an effort to bring these two strands of literature together, and at the same time explain several salient life-cycle features of the labor market that have not yet jointly been modeled:

- wages of job-stayers are increasing in tenure and hardly affected by productivity shocks,
- job-finding rates and starting wages of unemployed workers are decreasing in age,
- the wage loss after a separation is persistent and increasing in age and tenure.

The core of the model is a frictional labor market with directed search. Risk neutral firms post vacancies that pay different wage-tenure schedules. In every period, unemployed and employed individuals can apply to these vacancies and are matched with a probability that depends on the number of applicants. Additionally, the individuals decide on consumption and savings, which allows them to ensure against the risk of future job loss. They retire exogenously after T periods. In equilibrium, the optimal job search behavior of an individual depends on her asset stock and the characteristics of her current job. Despite this complexity, the model allows to derive insightful analytic expressions. Numerical tractability requires appropriate techniques as the lifespan T increases, because the current wage-tenure contract of the individual appears as a T-dimensional state variable.

Fertility, mortality and environmental policy

Authors: Palokangas, T., Lemijoki, U.
Thursday, Dec. 1, session 1, 10:10-10:45.

Abstract:
This paper examines pollution and environmental mortality in a model where fertility is endogenous and output is produced by two sectors, dirty and clean. To decrease environmental mortality, government curbs the dirty sector by posing a pollution tax. If the dirty sector is more capital intensive than the clean sector, then the environmental policy rises labor demand and thus wages. This tends to rise the opportunity costs of children, the demand of which decreases. But, if the clean sector is more capital-intensive, then pollution taxes decrease wages, generating the opposite development. In that case, the burden to provide capital to greater population will curb down per capita consumption.

The quest for status and R&D-based growth

Authors: Prettner, K., Hof, F.X.
Thursday, Dec. 1, session 3, 15:25-16:00.

Abstract:
We analyze the impact of status preferences on technological progress and long-run economic growth within an R&D-based framework. For this purpose, we extend the standard relative wealth approach by allowing the various assets held by households to differ with respect to their status relevance. Relative wealth preferences imply that the effective rate of return on saving in the form of a particular asset is the sum of its market rate of return and its status-related extra return. We show that the status relevance of shares issued by entrants to finance the purchase of new technologies is of crucial importance for long-run growth: First, an increase in the intensity of the quest for status raises the steady-state economic growth rate only if the status-related extra return of these shares is strictly positive. Second, for any given degree of status consciousness, the long-run economic growth rate depends positively on the relative status relevance of shares issued by entrants. Third, while the decentralized long-run economic growth rate is less than its socially optimal counterpart in the standard model, wealth externalities reduce this distortion.

Optimal harvesting of a size-structured forest under the perfect plasticity approximation assumption

Author: Rovenskaya, E.
Thursday, Dec. 1, session 4, 17:40-18:15.

Abstract:
Introduction

In this study, we derive optimal harvesting strategies of a forest using a size-structured population growth model under the assumption of stationarity. We model the dynamics of the size distribution of trees by means of the transport equation with non-linear growth and mortality rate functions, depending on the overall densitiy of trees. In these functions, asymmetric competition for resources, especially for light, is taken into consideration. This assymetry makes those trees which receive full light grow faster and live longer than trees which are shaded by others receiving less light. Optimal harvesting strategies aim to maximize the economic benefit from cutting the trees. Since the number of the newborn trees depends on the total number of the already existing mature trees, the boundary conditions describing the arrival of new trees in the model is non-local.

A large numnber of works, e.g., [1, 3, 4, 5], to name just a few, consider optimal control problems of this kind under quite general conditions on the growth, mortality and fecundity rate functions. These works mostly focus on the necessary conditions for optimality and on the principal structure of the optimal control.

Model

In this work we employ a special assumption on the growth, mortality and fecundity functions called "perfect plasticity approximation" (PPA; [6, 7]) in which a continuous dependency of the vital functions on the available light and through that on the density of trees is replaced by a piece-wise constant function with the boundary between two areas dynamically dependent on the density of trees:

$$G(D,N) = \begin{cases} G_c, D \in [0, D^*) \\ G_u, D \in [D^*, \bar{D}) \end{cases} , \mu(D,N) = \begin{cases} \mu_c, D \in [0, D^*) \\ \mu_u, D \in [D^*, \bar{D}) \end{cases} ,$$

$$F(D,N) = \begin{cases} 0, D \in [0, D^*) \\ F, D \in [D^*, \bar{D}) \end{cases} \quad (1)$$

where $G(D,N), \mu(D,N)$ $F(D,N)$ are the growth, mortality and fecundity rates of trees of diameter $D \geq 0$ given the density of trees $N = N(D)$; D^* is the critical diameter, which separates the area of the full light and the shaded area; $\bar{D}$ is the maximal size of a tree; G_c and G_u are constant positive growth rates in thefull light area and in the shaded area ($G_c > G_u$), μ_c μ_u are constant positive mortality rates in thefull light area and in the shaded area ($\mu_c < \mu_u$), F is the constant positive fecundity rate of trees in the full light area (here we assume that the trees in the shaded area do not contribute to the produciton of new trees). Despite its relative simplicity, this model proved to be a realistic representation of natural (undisturbed) forests [6, 7].

In this work, we consider cutting trees in such a way to preserve the stationarity of the size distribution; as shown in [8] this assumption is rather plausible for estimating

long-term effects of selective cutting. Given (1), the control system becomes

$$G_u \frac{dN_u(D)}{dD} = -(\mu_u + c_u(D))N_u(D) \text{ for } 0 \le D < D^*$$

$$G_c \frac{dN_c(D)}{dD} = -(\mu_c + c_c(D))N_c(D) \text{ for } D^* < D \le \bar{D}, \tag{2}$$

where $c_u(D)$ and $c_c(D)$ are cutting rates of trees of diameter D; $N(D) = N_u(D)$ for $0 \le D < D^*$ and $N(D) = N_c(D)$ for $D^* < D \le \bar{D}$. Additionally we assume the continuity of the trees' flow across the boundary:

$$\lim_{D \to D^*} G_u N_u(D) = \lim_{D \to D^*} G_c N_c(D) \tag{3}$$

Arrival of new trees is defined as

$$G_u N_u(0) = \int_{D^*}^{\bar{D}} F N_c(D) A(D) dD, \tag{4}$$

where $A(D)$ is the crown area of a tree of diameter D. Critical diameter D^* is defined as

$$\int_{D^*}^{\bar{D}} A(D) N_c(D) dD = 1,$$

which means that the tallest trees occupy all available space of full light, that is are perfectly plastic.

We assume that the optimal cutting of trees aims to maximize the economic benefit function defined as follows:

$$J = \int_0^{D^*} p(D) c_u(D) N_u(D) dD + \int_{D^*}^{\bar{D}} p(D) c_c(D) N_c(D) dD,$$

where $p(D)$ is the net marginal profit of cutting the trees of diameter D. Controls $c_u(\cdot)$ $c_c(\cdot)$ are assumed to be measurable and bounded functions over $[0, D^*]$ and $[D^*, \bar{D}]$ correspondingly.

Problem decomposition

Let $D^* > 0$ and $q > 0$. Let $c_c^*(\cdot|q, D^*)$ and $N_c^*(\cdot|q, D^*), y^*(\cdot|q, D^*)$ be an optimal control and an optimal trajectory in problem

$$J_c = \int_{D^*}^{\bar{D}} p(D) c_c(D) N_c(D) dD \to \max_{c_c(\cdot)} \tag{7}$$

$$G_c \frac{dN_c(D)}{dD} = -(\mu_c + c_c(D))N_c(D), N_c(D^*) = \frac{G_u}{G_c} q$$

$$\frac{dy(D)}{dD} = -A(D)N_c(D), y(D^*) = 1, y(\bar{D}) = 0$$

$$c_c(D) \in [0, \bar{c}] \ (D \in [D^*, \bar{D}]),$$

while $c_u^*(\cdot|q,D^*)$ $N_u^*(\cdot|q,D^*)$ be an optimal control and an optimal trajectory in problem

$$J_u = \int_0^{D^*} p(D)c_u(D)N_u(D)dD \to \max_{c_u(\cdot)} \tag{8}$$

$$G_u \frac{dN_u(D)}{dD} = -(\mu_u + c_u(D))N_u(D), N_u(0) = \frac{F}{G_u}, N_u(D^*) = q$$

$$c_u(D) \in [0, \bar{c}] \ (D \in [0, D^*]).$$

Then optimal control $c_u^*(\cdot), c_c^*(\cdot)$ and trajectory $N_u^*(\cdot), N_c^*(\cdot)$ can be found as follows

$$c_u^*(D) = c_u^*(D|q,D^{**}), N_u^*(D) = N_u^*(D|q,D^{**}) \ (D \in [0,D^*))$$

$$c_c^*(D) = c_c^*(D|q,D^{**}), N_c^*(D) = N_c^*(D|q,D^{**}) \ (D \in (D^*,\bar{D}]),$$

$$(q^*, D^{**}) = \mathrm{Arg} \max_{q \geq 0, D^* \in [0,\bar{D}]} J(q, D^*),$$

$$J(q,D^*) = \int_0^{D^*} p(D)c_u(D|q,D^*)N_u(D|q,D^*)dD + \int_{D^*}^{\bar{D}} p(D)c_c(D|q,D^*)N_c(D|q,D^*)dD.$$

Problems (7) and (8) can be solved by means of the Pontryagin maximum principle.

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Smoking kills: An economic theory of addiction, health, and longevity

Author: Strulik, H.
Thursday, Dec. 1, session 2, 11:50-12:25.

Abstract:
In this paper I unify the economic theories of addiction and health deficit accumulation and develop a life cycle theory in which individuals take into account that the consumption of addictive goods reduces their health and longevity. I distinguish two types of individuals: sophisticated types perfectly control their addiction, naive types, though otherwise rational and forward looking, fail to fully understand how their addiction develops. I argue that the life cycle consumption patterns predicted for naive types are more suitable to describe actually observed addictive behavior. I calibrate the model using U.S. data and the case of smoking as unhealthy behavior and use it to explain the socioeconomic gradients of smoking, health, and longevity.

Optimal fishery with coastal catch

Authors: Upmann, T., Uecker, H.
Friday, Dec. 2, session 6, 11:15-11:50.

Abstract:
In spatial resource models it is assumed that the agent is able to determine the harvesting activity over the complete spatial domain (outside possible reserve areas). However, agents frequently have only access to a resource at particular locations at which the moving biomass, such as sh or game, may be caught or hunted. To analyse this problem, we set up a simple optimal control model of boundary harvesting. Using the Pontryagin's Maximum Principle we derive the associated canonical system, and numerically compute canonical steady states and optimal time dependent paths. We characterise the optimal control and the associated stock of the resource, and explore the properties of these variables by means of an extensive numerical analysis. Finally, we extend our model to a predator-prey model of the Lotka-Volterra type, and show how the presence of two-species enriches the results of our basic model. For both models we illustrate the dependence of the optimal steady states and the optimal paths on the cost parameters.