Wittgenstein Centre
FOR DEMOGRAPHY AND GLOBAL HUMAN CAPITAL
IIASA ÖAW universität wien

Cheating Death: Beating the odds to longer survival

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Adrien Remund [1] Vanessa di Lego[2]

1 Population Research Centre - University of Groningen
2 Wittgenstein Centre (IIASA, OeAW, University of Vienna)
Vienna Institute of Demography at the Austrian Academy of Sciences

Background

Jeanne Calment and Creme Puff continuously cheated death, beating the odds of dying many times, given the average mortality experience of their cohort.

	Jeanne Calment	Creme Puff
		
Born	1875	1967
Died	1997	2005
Lifespan (LS)	122.4	38.0
e_0	48.59 (HMD 2019)	14 (O'Neill et al. 2015)
$\frac{LS}{e_0}$	2.52	2.71
Lives lived	?	9 (as all cats)

Background

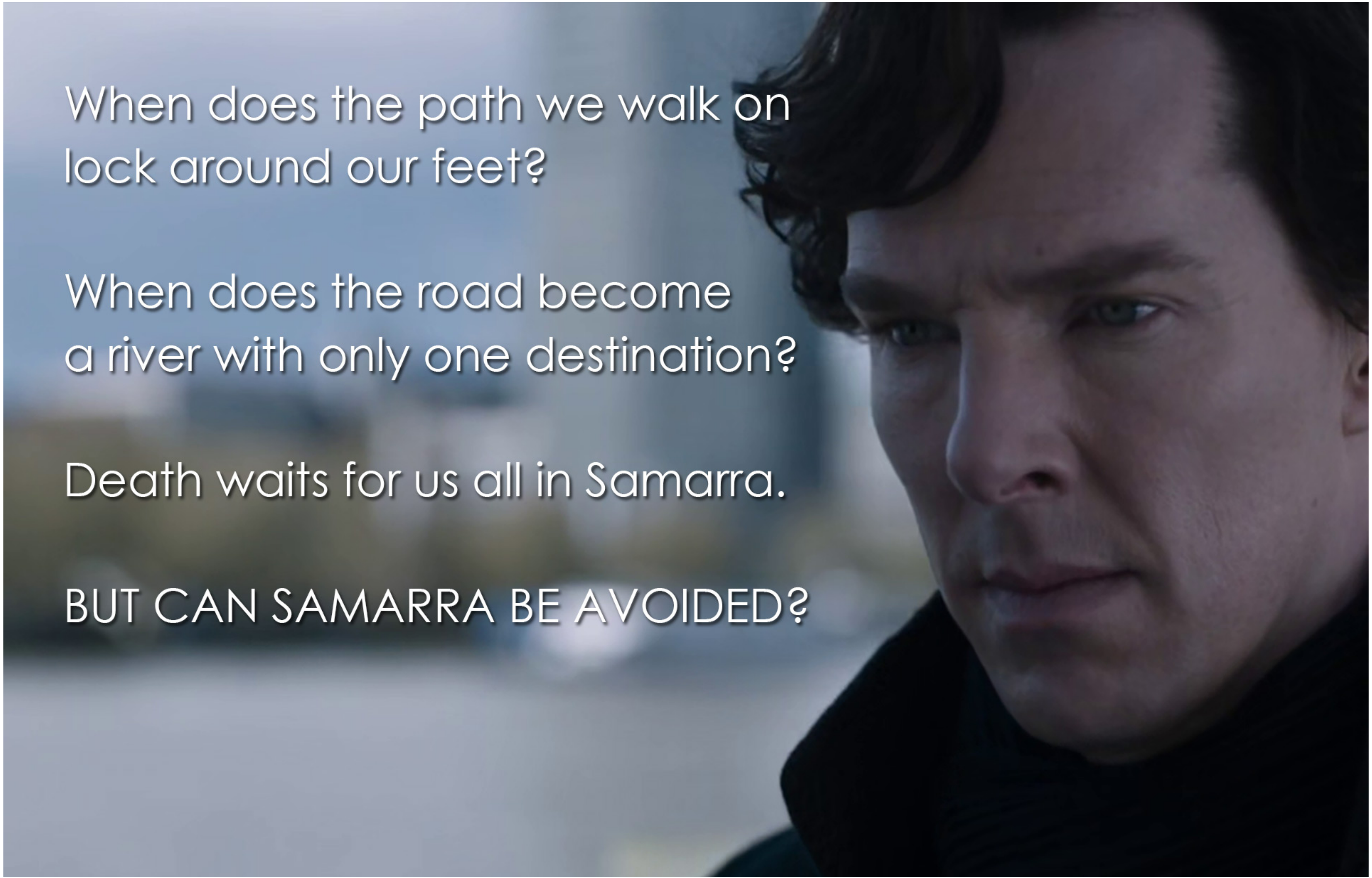
- “the average exhaustions of a man’s power to avoid death”, [...] “portion of his remaining power to oppose destruction”. Gompertz 1825
- Tybalt: What wouldst thou have with me? Mercutio: Good king of cats, nothing but one of your nine lives. Shakespeare, Romeo and Juliet, III, i.

The Appointment in Samarra

"There was once a merchant in the famous market at Baghdad. One day he saw a stranger looking at him in surprise. And he knew that the stranger was Death. Pale and trembling, the merchant fled the marketplace and made his way many, many miles to the city of Samarra. For there he was sure Death could not find him. But when, at last, he came to Samarra, the merchant saw, waiting for him, the grim figure of Death.

"Very well," said the merchant. "I give in. I am yours. But tell me, why did you look surprised when you saw me this morning in Baghdad?"

"Because," said Death, "I had an appointment with you tonight, in Samarra." (retold by W. Somerset Maugham [1933])


When does the path we walk on
lock around our feet?
When does the road become
a river with only one destination?
Death waits for us all in Samarra.
BUT CAN SAMARRA BE AVOIDED?

Background

- Longstanding effort to understand trends and patterns in **longevous lifespans** and extreme survival.(Lenart, Aburto, Stockmarr, and Vaupel, 2021)

- **Advancement of methods** to maximum lifespan and the form of the mortality hazard at extreme ages (Poulain, Herm, and Pes, 2013; Nepomuceno and Turra, 2020; Gampe, 2021; Jdanov, Shkolnikov, and Gellers-Barkmann, 2021)

- **Validating** centenarians and supercentenarians; The International Database on Longevity (IDL). (Poulain, Herm, and Pes, 2013; Jdanov, Shkolnikov, and Gellers-Barkmann, 2021; Robine and Cubaynes, 2017)

Background

- Our direction of research takes a different route and we ask:

> how many times does somebody need to **cheat death** in order to reach a specific target age and **beat the odds** of dying, given the average mortality experience of their cohort?

- How many times does someone need to beat the odds by **successively outliving his/her expectable age at death** in order to reach a given target age?
- In line with theory of ageing(Strehler and Mildvan, 1960), where in order to reach older ages one's intrinsic vitality must surpass extrinsic challenges to survival. We refer to this individual process as **cheating death**.
- Resembles "repeated resuscitation" (Vaupel and Yashin, 1987b) and "lifesaving" (Finkelstein, 2005), without the comparative dimension of contrasting different mortality scenarios.

The Mathematics of Cheating Death

The cornerstone of our measure is the *Expected Age at Death* (EAD) or the average age reached by individuals of a given birth cohort, provided that they have survived to a given age a. This is similar to the denominated *total expected longevity* (Canudas-Romo and Engelman, 2009)

$$EAD(a) = e(a) + a$$

Since $e(a)$ is the remaining life expectancy at age a (Canudas-Romo and Schoen, 2005; Keyfitz and Caswell, 2005)

$$e(a) = \left[\frac{1}{\ell(a)} \int_a^{\omega} \ell(t)dt\right]$$

$$EAD(a) = \left[\frac{1}{\ell(a)} \int_a^{\omega} \ell(t)dt\right] + a$$

Definitions

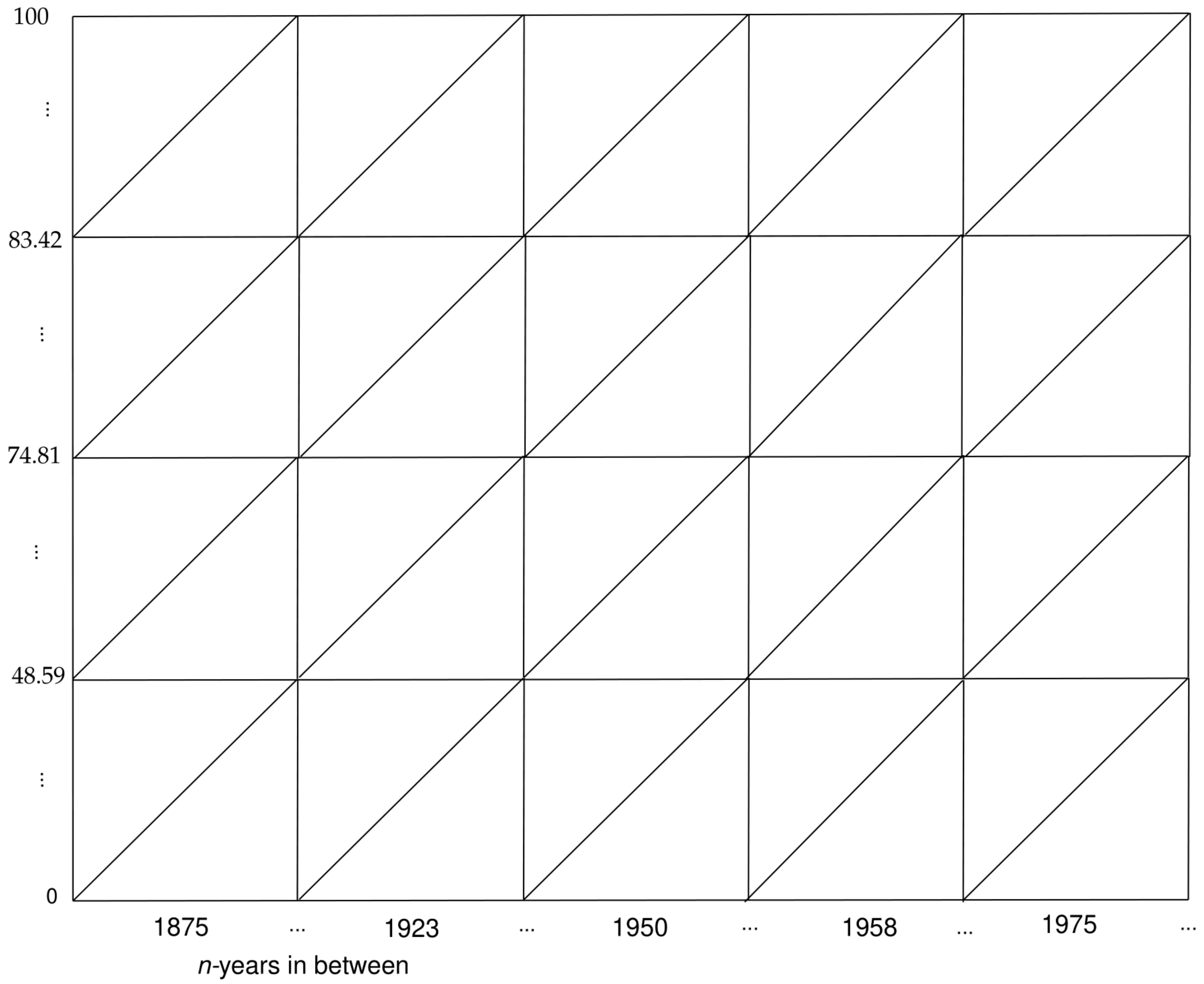
Given an arbitrary target age $\tilde{a}$, we define n as the number of times that $EAD(a)$ needs to be successively applied to itself until $EAD(a) \geq \tilde{a}$ or an interated function such that $n \text{ such that } EAD(a)^{\circ n} \geq \tilde{a}$ (Kuczma, Choczewski, and Ger, 1990).

$$
\begin{aligned}
i = 1 &: EAD(0) = e(0) + 0 < \tilde{a} \\
i = 2 &: EAD(EAD(0)) = EAD \circ EAD = EAD^{\circ 2} = e(e(0)) + e(0) < \tilde{a} \\
i = n &: EAD(EAD(\ldots(EAD(0)))) = EAD^{\circ n} \geq \tilde{a}
\end{aligned}
$$

We call the number of iterations needed to reach a target age as the number of times one has beaten the odds of dying, or the number of lives lived

Find $\tilde{a}^* < \tilde{a}$, such that $EAD(\tilde{a}^*) = \tilde{a}$ exactly.

$n(100) \cong 7.95$ Figure 2. The pathway of the effort required to become a centenarian (France 1875)



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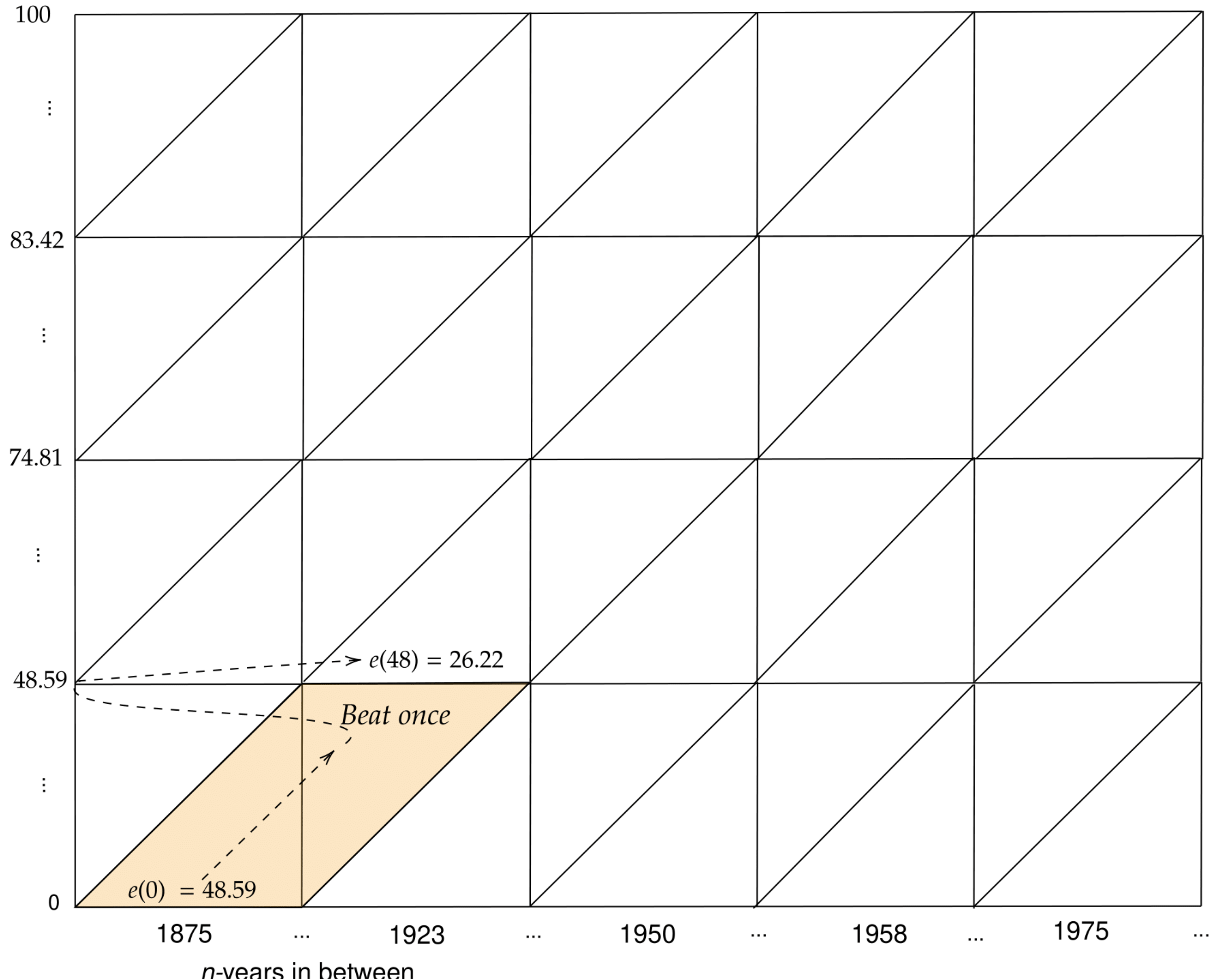

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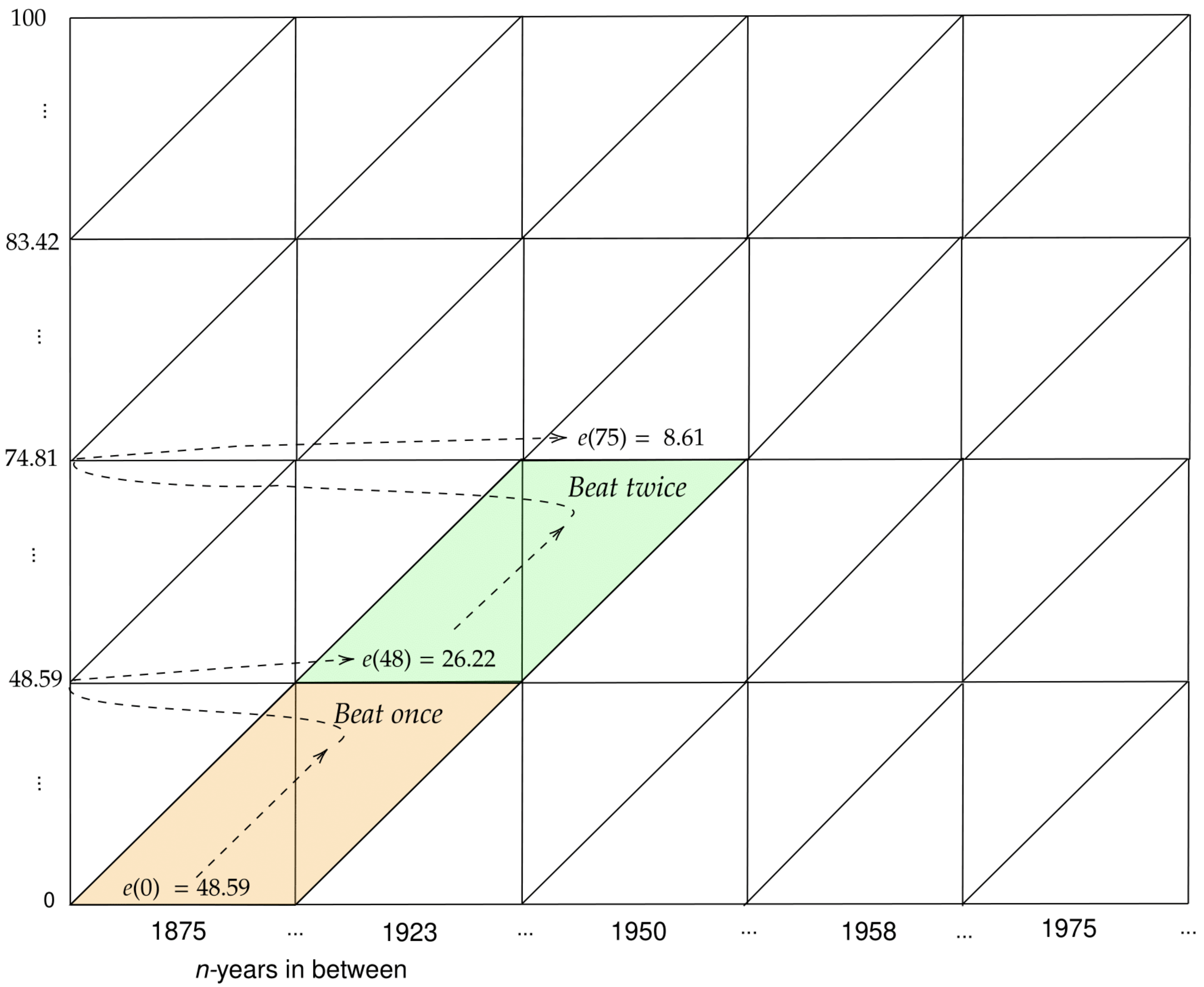

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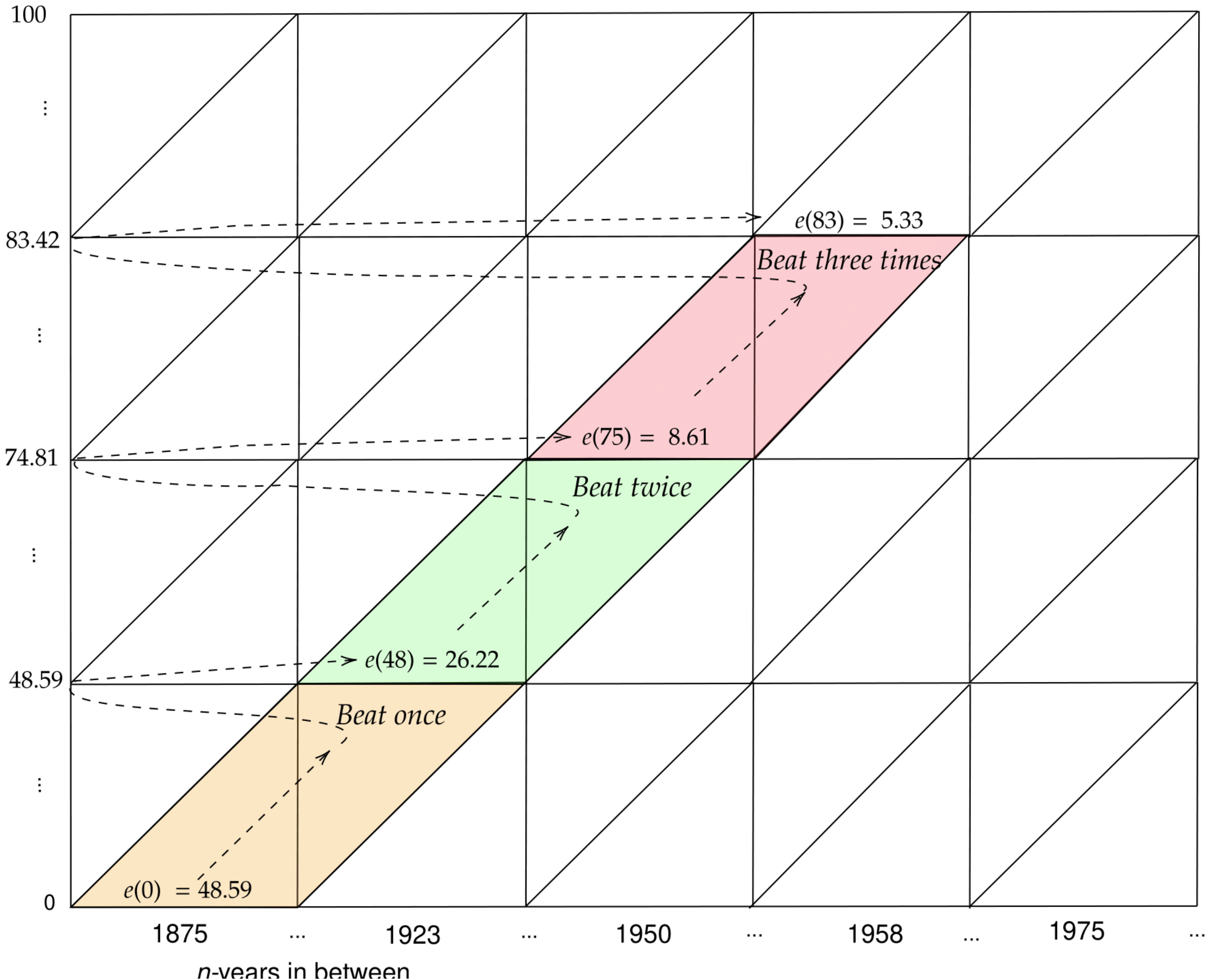

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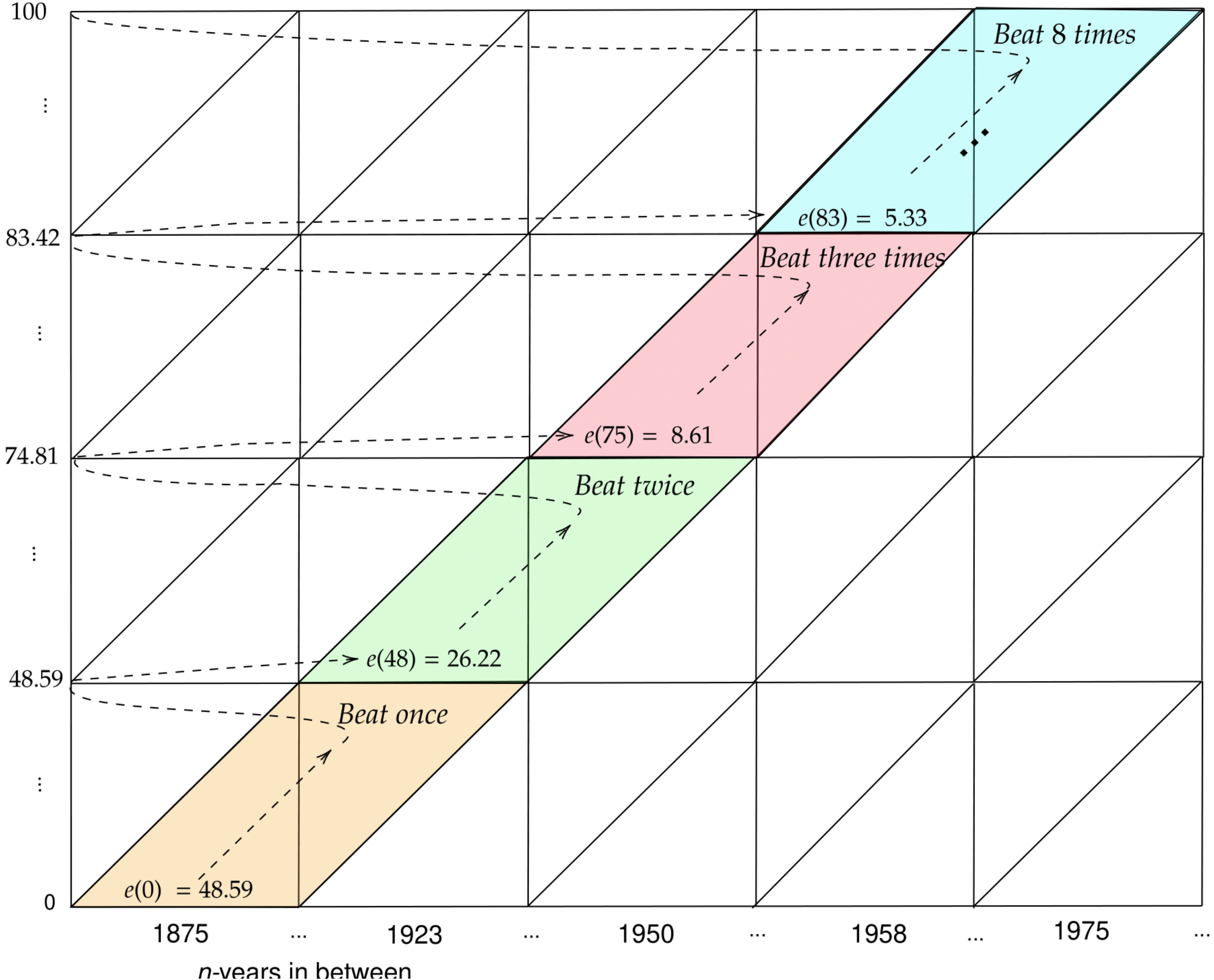


Relationships

We define a relationship between the probability $\ell(\tilde{a})$ to reach the target age $\tilde{a}$ and the probability to reach each successive *new life* $p(i)$ as:

$$\ell(\tilde{a}) = \prod_{i=1}^{n} p(i) = p(1)p(2)p(3)\cdots p(n)$$

Or as a series of i iterations to reach $\tilde{a}$, such that $n \in \mathbb{N} : EAD^{\circ n} \geq \tilde{a}$:

$$\ell(\tilde{a}) = \ell(EAD(0)) \cdot \left(\frac{\ell(EAD^{\circ 2})}{\ell(EAD(0))}\right) \cdot \left(\frac{\ell(EAD^{\circ 3})}{\ell(EAD^{\circ 2})}\right) \cdots \left(\frac{\ell(EAD^{\circ n})}{\ell(EAD^{\circ n-1})}\right)$$

or, in terms of iteration i instead of age a,

$$\ell(n) = \ell(0) \cdot \ell(1) \cdot \left(\frac{\ell(2)}{\ell(1)}\right) \cdot \left(\frac{\ell(3)}{\ell(2)}\right) \cdots \left(\frac{\ell(n)}{\ell(n-1)}\right)$$

the consecutive $\ell(i)$ values are the probabilities to reach the $i-th$ iteration, conditioned on surviving to $i-1$

Relationships

Consider the life table survival at a given age a, as a function of the force of mortality, is defined as (Keyfitz 2005):

$$\ell_a = \mathrm{e}^{-\int_0^a \mu(s)ds}$$

And the probability to outlive one's expected age at death the $n-th$ time depends on having outlived the expected age at death at $n-1$:

$$\frac{\ell(n)}{\ell(n-1)} = \frac{\mathrm{e}^{-\int_0^n \mu(s)ds}}{\mathrm{e}^{-\int_0^{n-1} \mu(s)ds}} = \mathrm{e}^{-\int_{n-1}^n \mu(s)ds}$$

Proof

Standard assumption in the last open-age interval a_∞ = $a_{x,\infty} = 1/m_x$ (Preston 2001). Then, at the limit, $\mu = \bar{\mu}$, and the remaining life expectancy is $\frac{1}{\bar{\mu}}$. Hence:

$$\frac{\ell(n)}{\ell(n-1)} = \mathrm{e}^{-\int_{n-1}^{n-1+1/\bar{\mu}} \bar{\mu} ds}$$

As $\bar{\mu}$ is a constant, we can take it out of the integral and solve, yielding:

$$\frac{\ell(n)}{\ell(n-1)} = \mathrm{e}^{-\bar{\mu}\int_{n-1}^{n-1+1/\bar{\mu}} ds} = \mathrm{e}^{-\bar{\mu}\frac{1}{\bar{\mu}}} = \mathrm{e}^{-1}$$

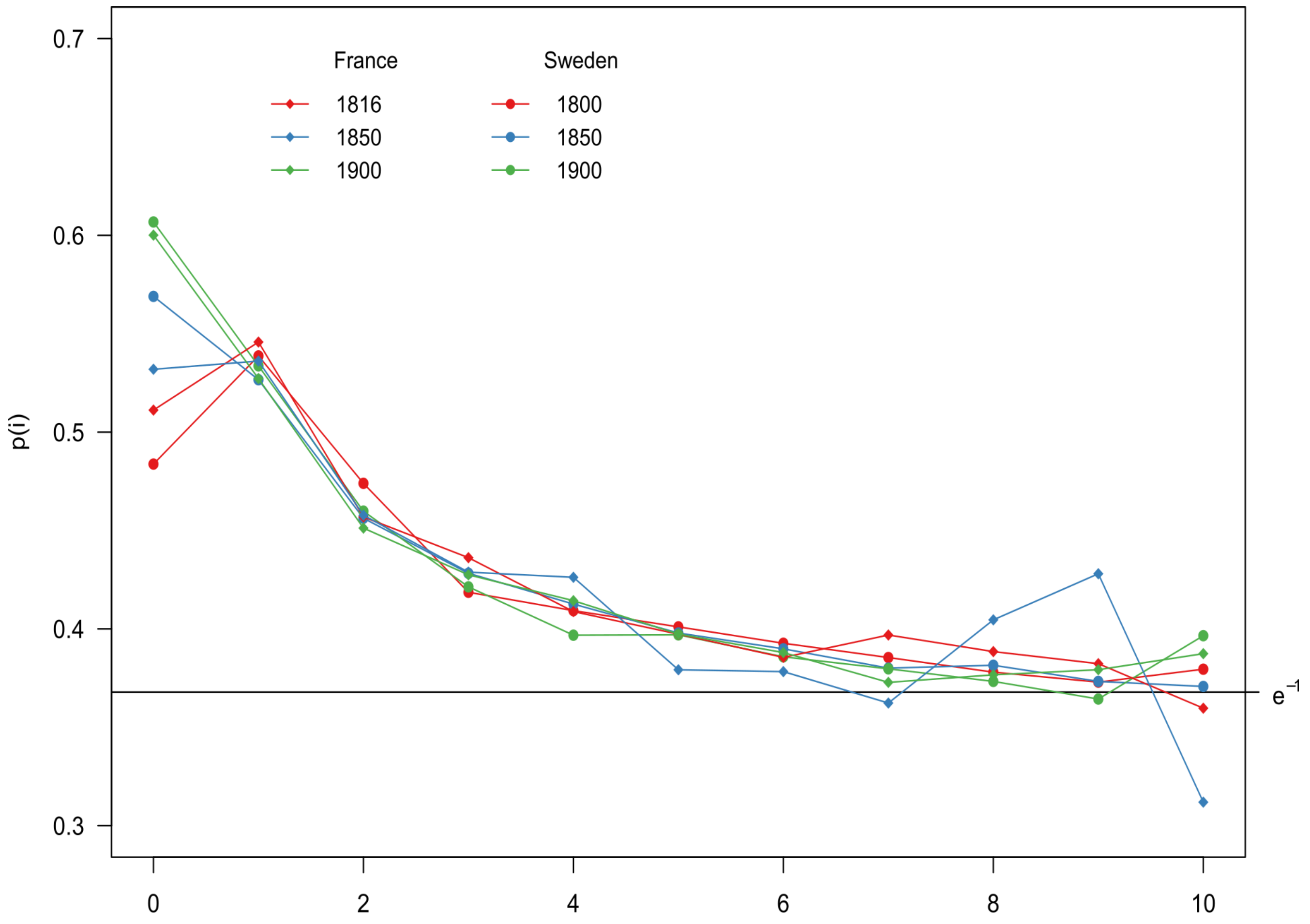
Corollary

Further claim that $\ell(\tilde{a})$ is asymptotic as $\tilde{a} \to \infty$ (or equivalently $\ell(i)$ when $i \to \infty$). Capture the shift generated on $\ell(n)$ by the progressive convergence of $lim_{i\to\infty} p(i) = \mathrm{e}^{-1}$ through error term ε.

$$\lim_{i\to\infty} \ell(n) = \varepsilon \cdot \prod_{i=1}^{n} p(i) = \varepsilon \cdot \mathrm{e}^{-n} \Rightarrow \lim_{i\to\infty} \ln(\ell(n)) = \ln(\varepsilon) - n$$

Application

Figure 3. Different Mortality Schedules



Application

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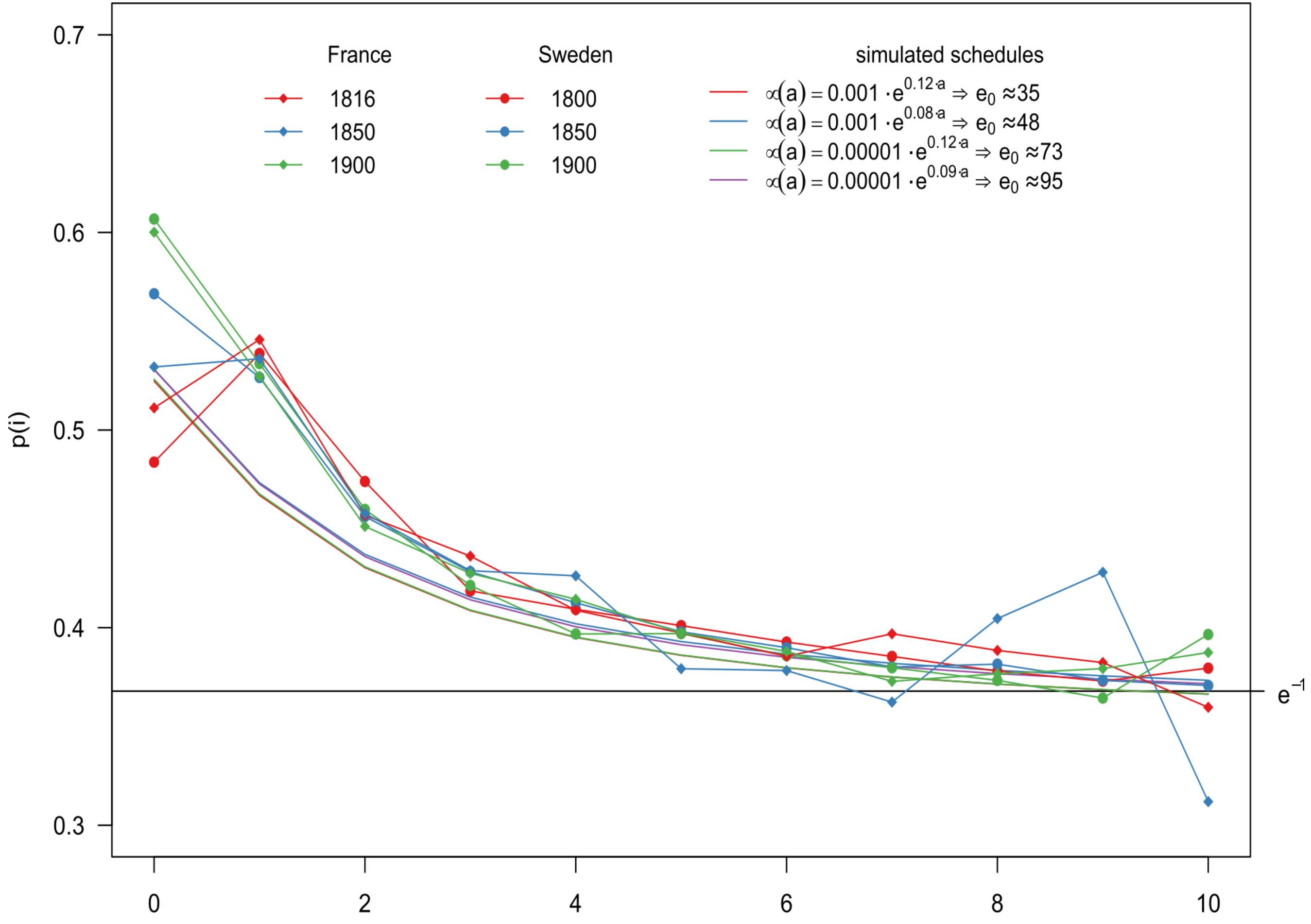

Application

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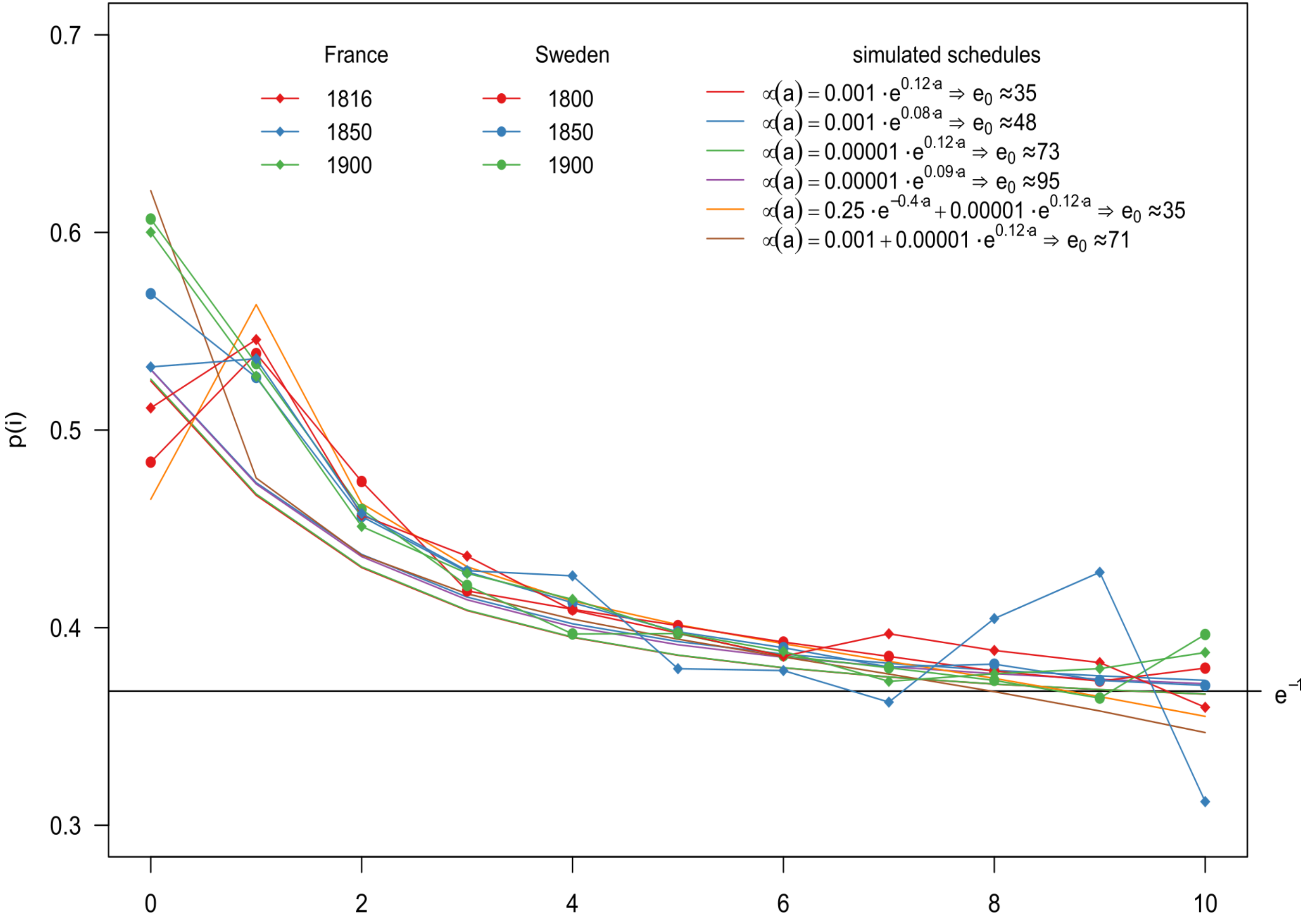
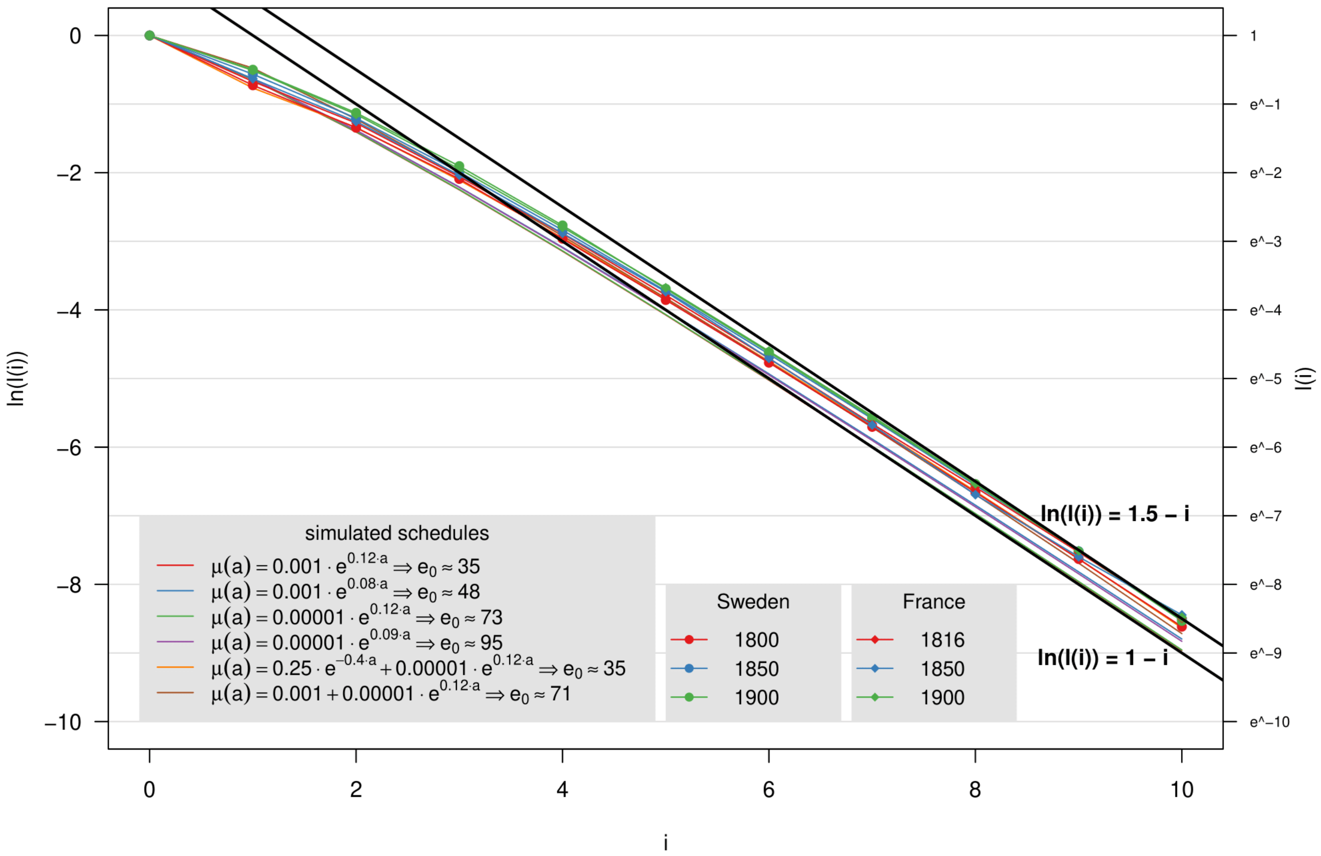


Application

Figure 4. Cumulative Probability to successively outlive one's expected age at death in a set of historical and theoretical mortality schedules



Consequence

Number of *lives* needed to reach age $\tilde{a}$ can be expressed as a simple logarithm transform of the probability to reach age $\tilde{a}$.

$$n(\tilde{a}) \cong 1.5 - \ln(\ell(\tilde{a}))$$

- Since $n(\tilde{a})$ is not a function of age, but only of the probability to survive to that age, the number of *lives lived* can be computed from the probability itself.

- When an individual survives to an age reached by only 1% of the members of their birth cohort, they have *cheated death* about $1.5 - \ln(0.01) \cong 1.5 + 4.6 \cong 6$ times.

Discussion

- **Result is independent from the age $\tilde{a}$** that corresponds to this survival level, which becomes a relative measure of time to the specific population to which the individual belongs.

- $p(i)$ is **increadibly stable** across mortality contexts

- *Lives lived* **a universal measure of time** that is comparable across extremely diverse populations, human or not, and probably even across other kingdoms of life.

- In line with the finding that mortality rates measured at ages defined by fixed **levels of survival are much more stable over time** (Alvarez and Vaupel 2023).

Discussion

- A measure based on this definition of time becomes independent from the necessarily arbitrary measures of time used to compute the risk of death (e.g. single or grouped years of age, shorter or longer units of time, depending on the context).

- It is likely that $n(100)$, because it reflects processes that are particularly intense at old age, is more strongly correlated for instance to $e(60)$ than to $e(0)$, but, contrarily to the former, it includes the mortality experience of all ages, and contrarily to the latter, it is not heavily dependent on infant mortality.

Thank you!

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Appendix

Figure 1. Empirical computation of the number of lives required to become centenarian (France 1875). $n(100) \cong 7.95$