Prepare or react? Integrating large health shocks into life-cycle models

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Outline of the talk

1. Definitions and examples
2. Model structure
3. Extensions to the VOL concept
4. Optimality conditions
5. Numerical solution (cancer diagnosis)
1 Definitions and examples

2 Model structure

3 Extensions to the VOL concept

4 Optimality conditions

5 Numerical solution (cancer diagnosis)
A large health shock has a significant impact on the decision of an individual in the present and future and can also incentivise precautionary measures. These behavioural changes after the health shock can result from

- new constraints (e.g. changes in income, assets),
- affected personal characteristics (e.g. increased mortality risk),
- changes of the individuals preferences and/or objectives.
A cancer diagnosis has several long-run impacts on an individual:

- Effects of the disease on general well-being decreases the utility.
- Cancer potentially limits the ability to generate working income.
- Cancer imposes an additional mortality risk.
Figure 1: SEER*Explorer: An interactive website for SEER cancer statistics. Surveillance Research Program, National Cancer Institute.
• However whether or when cancer is diagnosed for an individual cannot be perfectly foreseen.
Example: Cancer diagnosis

- However whether or when cancer is diagnosed for an individual cannot be perfectly foreseen.
- Nevertheless the majority of models in the literature take an ex-ante stance, with individuals being able to foresee the development of their health perfectly,
However whether or when cancer is diagnosed for an individual cannot be perfectly foreseen.

Nevertheless the majority of models in the literature take an ex-ante stance, with individuals being able to foresee the development of their health perfectly,

This can somehow be rationalized in a macro perspective for a representative individual,
• However whether or when cancer is diagnosed for an individual cannot be perfectly foreseen.

• Nevertheless the majority of models in the literature take an ex-ante stance, with individuals being able to foresee the development of their health perfectly,

• This can somehow be rationalized in a macro perspective for a representative individual,

• however health shocks with significant impacts on the individual life (such as cancer) should not be averaged into a mean value in the analysis of individual behaviour.
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To describe large health shocks in general, we set up a two stage dynamic optimal control framework including:

- A stochastically appearing health shock. We thereby allow for different kinds of shocks with different characteristics.
To describe large health shocks in general, we set up a two stage dynamic optimal control framework including:

- A stochastically appearing health shock. We thereby allow for different kinds of shocks with different characteristics.
- Multiple kinds of health investments:
  - General health investments
  - Health shock specific preventive care, acute care, and chronic care.
Dynamics of the health status/survival:

\[ \dot{S}_1(t) = -\mu_1(t, S_1(t), b_1(t))S_1(t) \]
\[ S_1(0) = 1 \]

- \( S_1(t) \)... Survival at time \( t \), also used as a proxy for the health status.
- \( b_1(t) \)... General health expenditure at time \( t \).
- \( \mu_1(\cdot) \)... Mortality rate at time \( t \).
Dynamics of the probability of not suffering a health shock:

\[ \dot{Z}_1(t) = -\eta(t, S_1(t), h_1(t))Z_1(t) \]

\[ Z_1(0) = 1 \]

- \( Z_1(t) \)... Probability of not suffering a health shock until \( t \).
- \( h_1(t) \)... Shock specific preventive care at time \( t \).
- \( \eta(\cdot) \)... Hazard rate of shock occurrence.
Dynamics of the assets:

\[ \dot{A}_1(t) = [r(t) + \bar{\mu}(t)]A_1(t) + w^1(t) - c_1(t) - p^b(t)b_1(t) - p^1(t)h_1(t) \]

\[ A_1(0) = 0, \quad A_1(T) = 0 \]

- \( A_1(t) \)... Assets.
- \( w^1(t) \)... Income flow.
- \( \bar{\mu} \)... Annuity rate.
Before the shock - Objective function

\[ \mathbb{E}_s \left[ \int_0^s e^{-\rho t} S_1(t) u^1(c_1(t)) \, dt + e^{-\rho s} V^* (S_1(s), A_1(s), s) \right] \]

- $c_1(t)$... Final good consumption.
- $s$... Time of the health shock (stochastic).
- $V^*(\cdot)$... Optimised utility over the remaining life-course after the shock.
After the shock - Deficits from the shock $E(t, s)$

Dynamics of the shock specific deficits:

$$\dot{E}(t, s) = f(t, s, E(t, s), h_2(t, s))$$
$$E(s, s) = B(S_1(s), d(s))$$

- $h_2(t, s)$... Chronic care to reduce the deficits.
- $B(\cdot)$... Initial deficits.
- $d(s)$... Acute care to reduce initial deficits.
Dynamics of the health status/survival:

\[
\dot{S}_2(t, s) = -[\mu^1(t, S_2(t, s), b_2(t, s)) + \mu^2(t, s, E(t, s))]S_2(t, s)
\]

\[S_2(s, s) = S_1(s)\]

- \(\mu^2(\cdot)\)... Additional mortality risk resulting from deficits \(E\).
After the shock - Asset dynamics $A_2(t, s)$

Dynamics of the assets:

\[ \dot{A}_2(t, s) = [r(t) + \bar{\mu}(t)]A_2(t, s) + w^2(t, s, E(t, s)) - c_2(t, s) - p^b(t)b_2(t, s) - p^2(t)h_2(t, s) \]

\[ A_2(s, s) = A_1(s) - p^d(s)d(s) \]

\[ A_2(T, s) = 0 \]

- $A_2(t, s)$... Assets.
- $w^2(t, s, E(t, s))$... Income flow depending on time of shock and the deficit level.
Before the shock - Objective function

\[ V^*(s, A_1(s), S_1(s)) = P(S_1(s), d(s)) \times \int_s^T e^{-\rho t} S_2(t, s) u^2(c_2(t, s), E(t, s)) \, dt \]

- \( P(\cdot) \)... Probability of surviving the health shock depending on acute care and the health state at the shock \( \Longrightarrow \) Continuation probability.
- \( u^2(\cdot) \)... utility after the shock, depending on the shock deficits.
Stochastic model formulation

\[
\max_{c_1(t), h_1(t), b_1(t) \geq 0} \mathbb{E} \left[ \int_0^s e^{-\rho t} S_1(t) u_1(c_1(t)) \, dt + e^{-\rho s} V^*(S_1(s), A_1(s), s) \right]
\]

\[
\dot{S}_1(t) = -\mu_1(t, S_1(t), b_1(t)) S_1(t)
\]

\[ S_1(0) = 1, \quad S_1(T) = 0 \]

\[
\dot{A}_1(t) = (r(t) + \bar{\mu}(t)) A_1(t) + w_1(t) - c_1(t) - \rho^b(t) b_1(t) - \rho^1(t) h_1(t),
\]

\[ A_1(0) = 0, \quad A_1(T) = 0 \]

\[
V^*(S_1(s), A_1(s), s) := \max_{c_2(t,s), h_2(t,s), b_2(t,s), d(s) \geq 0} P(S_1(s), d(s)) \times \int_s^T e^{-\rho t} S_2(t, s) u_2(c_2(t, s), E(t, s)) \, dt
\]

\[
\dot{S}_2(t, s) = -\left[ \mu_1(t, S_2(t, s), b_2(t, s)) + \mu^2(t, s, E(t, s)) \right] S_2(t, s),
\]

\[ S_2(s, s) = S_1(s), \]

\[
\dot{A}_2(t, s) = (r(t) + \bar{\mu}(t)) A_2(t, s) + w_2(t, s, E(t, s)) - c_2(t, s) - \rho^b(t) b_2(t, s) - \rho^2(t) h_2(t, s)
\]

\[ A_2(s, s) = A_1(s) - p^d(s) d(s), \quad A_2(T, s) = 0 \]

\[
\dot{E}(t, s) = f(t, s, E(t, s), h_2(t, s)),
\]

\[ E(s, s) = B(S_1(s), d(s)) \]
max $c_1(t), h_1(t), b_1(t) \geq 0$
$\int_0^T e^{-\rho t} \left[ Z_1(t) S_1(t) u_1^1(c_1(t)) + Q(t) \right] dt$

s.t.

$\dot{S}_1(t) = -\mu^1(t, S_1(t), b_1(t)) S_1(t), \quad S_1(0) = 1$

$\dot{A}_1(t) = (r(t) + \bar{\mu}(t)) A_1(t) + w^1(t) - c_1(t) - p^b(t) b_1(t) - p^1(t) h_1(t), \quad A_1(0) = 0, A_1(T) = 0$

$\dot{Z}_1(t) = -\eta(t, S_1(t), h_1(t)) Z_1(t), \quad Z_1(0) = 1$

$\frac{dS_2(t, s)}{dt} = -\left[ \mu^1(t, S_2(t, s), b_2(t, s)) + \mu^2(t, s, E(t, s)) \right] S_2(t, s) \quad S_2(s, s) = S_1(s)$

$\frac{dA_2(t, s)}{dt} = (r(t) + \bar{\mu}(t)) A_2(t, s) + w^2(t, s, E(t, s)) - c_2(t, s) - p^b(t) b_2(t, s) - p^2(t) h_2(t, s)$

$A_2(s, s) = A_1(s) - p^d(s) d(s), \quad A_2(T, s) = 0, \quad \forall s \geq 0$

$\frac{dZ_2(t, s)}{dt} = 0, \quad \frac{dE(t, s)}{dt} = f(t, s, E(t, s), h_2(t, s)), \quad E(s, s) = B(S_1(s), d(s))$

$Q(t) = \int_0^t Z_2(t, s) S_2(t, s) u_2^2(c_2(t, s), E(t, s)) ds$. 
1 Definitions and examples

2 Model structure

3 Extensions to the VOL concept

4 Optimality conditions

5 Numerical solution (cancer diagnosis)
We generalise the value of life concept by defining the willingness to pay to

- decrease of the mortality rate
  - before the shock $\psi_{life}(t)$
  - after the shock $\psi_{life}(t, s)$
Extensions to the VOL concept

We generalise the value of life concept by defining the willingness to pay to

- decrease of the mortality rate
  - before the shock $\psi_{\text{life}}^1(t)$
  - after the shock $\psi_{\text{life}}^2(t, s)$
- decrease of the hazard rate of the health shock $\psi_{\text{shock}}(t)$
Extensions to the VOL concept

We generalise the value of life concept by defining the willingness to pay to

- decrease of the mortality rate
  - before the shock $\psi_{life}^1(t)$
  - after the shock $\psi_{life}^2(t, s)$
- decrease of the hazard rate of the health shock $\psi_{shock}(t)$
- decrease of the impact of the health shock
  - at the time of the shock (change in the continuation probability) $\psi_P(t)$
  - in the long run through the deficits. $\psi_E(t, s)$
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Intuitive FOCs

Stage 1:
\[
[-\mu_{b_1}(t)] \cdot \psi_{\text{life}}(t) = p^b(t)
\]
\[
[-\eta_{h_1}(t)] \cdot \psi_{\text{shock}}(t) = p^1(t)
\]

Stage 2:
\[
[-\mu_{b_2}(t, s)] \cdot \psi_{\text{life}}(t, s) = p^b(t)
\]
\[
[-f_{h_2}(t, s)] \cdot \psi_{E}(t, s) = p^2(t)
\]

Shock time s:
\[
[-B_d(s)] \cdot \psi_{E}(s, s) + P_d(s) \cdot \psi_{P}(s) = p^d(s)
\]
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Data-sources and specifications

Data sources for calibration for the US in 2010:

- Human Mortality Database (general mortality rates and survival data)
- NTA-Database (general consumption, health expenditure and income)
- SEER-Database (cancer-specific incidence and mortality)
- healthdata.org (cancer specific expenditure data)

Other Specifications for cancer disease:

- No mortality risk at diagnosis
- No explicit preventive care
- No impact on income

$r(t) = \rho$ and all prices equal to one.
Data-sources and specifications

Data sources for calibration for the US in 2010:

- Human Mortality Database (general mortality rates and survival data)
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Other Specifications for cancer disease:

- No mortality risk at diagnosis
- No explicit preventive care
- No impact on income
- \( r(t) = \rho \) and all prices equal to one.
Calibration - model vs. data

Expected Health Expenditures

Survival

$S_1$ and $Z_1$

Cancer survival after diagnosis

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Health shocks in life-cycle models
Control variables

Distributed Control $c_2$

First stage Control $c_1(t)$
Initial second stage values $c_2(t,t)$

Distributed Control $b_2$

First stage Control $b_1(t)$
Initial second stage values $b_2(t,t)$

Distributed Control $h_2$

Age

0
10
20
30
40
50
60 in 1000$

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State variables

Distributed State $S_2$

Distributed State $A_2$

Age distribution of cancer diagnosis

Distributed State $E$

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Health shocks in life-cycle models
Euler equation dynamics - consumption

Distributed Control $c_2$

First stage Control $c_1(t)$

Initial second stage values $c_2(t,t)$

\[
\dot{c}_1 = u_1 c_1 - u_2 c_1 c_2 \begin{bmatrix} r - \rho + \bar{\mu} - \mu_1 - \eta + \eta \end{bmatrix} u_2 c_2
\]
Euler equation dynamics - consumption

\[ \frac{\dot{c}_1}{c_1} = \frac{u^1_{c_1}}{u^1_{c_1}c_1} \left[ r - \rho + \bar{\mu} - \mu^1 - \eta + \eta P(S_1, d) \frac{u^2_{c_2}}{u^1_{c_1}} \right] \]
Euler equation dynamics - consumption

Distributed Control $c_2$

First stage Control $c_1(t)$

Initial second stage values $c_2(t,t)$

\[
\dot{c}_2 = u_2 c_2 - u_2 c_2 c_2 + r - \rho + \mu - \mu_{1} - \mu_{2} + u_2 E c_2 f
\]
Euler equation dynamics - consumption

\[
\frac{\dot{c}_2}{c_2} = \frac{u^2 c_2}{-u^2 c_2 c_2 c_2} \left[ r - \rho + \tilde{\mu} - \mu^1 - \mu^2 + \frac{u^2 c_2 E}{u^2 c_2} f \right]
\]

In 1000$

Distributed Control $c_2$

First stage Control $c_1(t)$

Initial second stage values $c_2(t,t)$

C2 shaping forces

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Health shocks in life-cycle models
Valuations of Health

- Value of Life (cancer free)
- A-priori (averaged) VOL
- Value of reducing the shock risk
- Value of increasing the survival - probability

Values in 1000$

Age

Values in 1000$

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Valuations of Health

- Value of Life (cancer free)
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- Value of reducing the shock risk
- Value of increasing the survival - probability

Value of Life - Stage 1
Conventional VOL
Impact on hazard rate
Impact on initial deficits
Impact on S2
TOTAL

Value in 1000$
Valuations of health

**Valuations of Health**

- Value of Life (cancer free)
- A-priori (averaged) VOL
- Value of reducing the shock risk
- Value of increasing the survival - probability

**Value of Life - Stage 1**

- Conventional VOL
- Impact on hazard rate
- Impact on initial deficits
- Impact on S2
- TOTAL

**Value of Life First and Second Stage**

- Value of life after the cancer diagnosis
- Value of Life (cancer free)
- A-priori (averaged) VOL

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Valuations of health

Valuations of Health
- Value of Life (cancer free)
- A-priori (averaged) VOL
- Value of reducing the shock risk
- Value of increasing the survival - probability

Value of Life First and Second Stage
- Value of life after the cancer diagnosis
- Value of Life (cancer free)
- A-priori (averaged) VOL

Value of Life - Stage 1
- Conventional VOL
- Impact on hazard rate
- Impact on initial deficits
- Impact on S2
- TOTAL

Value of Reducing the shock
- TOTAL

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Health shocks in life-cycle models
Conclusions

• We have introduced stochastic health shocks into health economic life-cycle models in a highly general fashion.
• We used a transformation into a vintage-structured model for the solution.
• We have extended the VOL concept to other aspects of health and can assess the several contributing factors analytically.
• We calibrated and solved the model numerically for cancer in the US in 2010.
• Evaluating the valuations of health and Euler equations numerically gives further insight into the impacting factors for the life-cycle profiles.
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Vintage Workshop 2019 (Vienna, December 5th)
Additional material
Hazard rate $\eta$

Assets
Health stock / survival
Consumption
Preventive care
Health investment
Utility
Working income

Chronic care
Health investment
Consumption
Working income

Acute care

Health shocks in life-cycle models
Freiberger, Kuhn, Wrzaczek

Hazard rate $\eta$
Assets
Health stock / survival
Consumption
Preventive care
Health investment
Utility
Working income

Acute care
S
T
A
G
E

Hazard rate $\eta$
Assets
Health stock / survival
Consumption
Preventive care
Health investment
Utility
Working income

Chronic care
S
H
O
C
K

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Hazard rate $\eta$

Assets

Health stock / survival

Consumption

Preventive care

Health investment

Utility

Working income

Deficits from the shock

Acute care

Health stock / survival

Assets

Chronic care

Health investment

Consumption

Working income
We assume, that the providers of annuities cannot observe if an individual has suffered a health shock or not. Consequently the annuity rate follows the expected mortality rate:

\[
\bar{\mu}(t) = \frac{Z_1(t)(-\dot{S}_1(t)) + \int_0^t Z_2(t, s)(-\dot{S}_2(t, s))ds}{Z_1(t)S_1(t) + \int_0^t Z_2(t, s)S_2(t, s)ds}
\]

\[
= \frac{Z_1\mu^1 S_1 + \int_0^t Z_2 \left[ \mu^1 + \mu^2 \right] S_2(t, s)ds}{Z_1(t)S_1(t) + \int_0^t Z_2(t, s)S_2(t, s)ds}
\]
Euler equations for consumption

\[
\frac{\dot{c}_1}{c_1} = \frac{u^1_{c_1}}{-u^1_{c_1c_1} c_1} \left[ r + \bar{\mu} - \rho - \mu^1 - \eta + \eta P(S_1, d) \frac{u^2_{c_2}}{u^1_{c_1}} \right]
\]

\[
\frac{\dot{c}_2}{c_2} = \frac{u^2_{c_2}}{-u^2_{c_2c_2} c_2} \left[ r + \bar{\mu} - \rho - \mu^1 - \mu^2 + \frac{u^2_{c_2} E}{u^2_{c_2}} f \right]
\]
Euler equations for health investments

\[ \frac{\dot{b}_1}{b_1} = \frac{-\mu_{b_1}^1}{\mu_{b_1 b_1}^1 b_1} \left[ r + \bar{\mu}^1 - \frac{p^b}{p^b} + \frac{\mu_{b_1}^1}{p^b} \frac{u^1}{u_{c_1}} + \mu_{S_1}^1 S_1 + \frac{\mu_{b_1 t}^1}{\mu_{b_1}^1} - \frac{\mu_{b_1 S_1}^1}{\mu_{b_1}^1} \right] \]

\[ + \eta P \frac{u_{c_2}^2}{u_{c_1}^1} + \frac{\mu_{b_1}^1 S_1 \eta S_1}{\eta h_1} \frac{p^1}{p^b} + \eta \mu_{b_1}^1 S_1 P S_1 \frac{\xi Z / S_1}{u_{c_1}^1 p^b} - \eta P \frac{\mu_{b_1}^1}{\mu_{b_2}^1} \frac{u_{c_2}^2}{u_{c_1}^1} + \eta P S_1 \frac{u_{c_2}^2}{u_{c_1}^1} \frac{p^2}{p^b} \frac{B_{S_1}}{f_{h_2}} \right] \]

\[ \frac{\dot{b}_2}{b_2} = \frac{-\mu_{b_2}^1}{\mu_{b_2 b_2}^1 b_2} \left[ r + \bar{\mu}^2 - \frac{p^b}{p^b} + \frac{\mu_{b_2}^1}{p^b} \frac{u^2}{u_{c_2}^1} + \mu_{S_2}^1 S_2 + \frac{\mu_{b_2 t}^1}{\mu_{b_2}^1} - \frac{\mu_{b_2 S_2}^1}{\mu_{b_2}^1} (\mu^1 + \mu^2) S_2 \right] \]
Euler equation dynamics - health expenditure

B1 shaping forces

\[ r + \mu - \frac{p^b}{p^s} \]

\[ \frac{u_{b}}{u_{p}} \cdot \frac{u_{2}^{2}}{u_{1}^{2}} \]

\[ \frac{\mu_{b}}{\mu_{p}} \]

\[ \rho_{b} \cdot u_{2} \cdot u_{2}^{2} \]

\[ -\mu_{b} \cdot \frac{\rho_{b}}{\rho_{b} + 1} \cdot (\lambda_{z} - P_{z}) \]

\[ -\eta \cdot \frac{u_{b}}{u_{p}} \cdot \frac{u_{2}^{2}}{u_{1}^{2}} \]

\[ \tau \rho S_{1} \cdot \frac{u_{2}^{2} \cdot p^{b} \cdot w_{1}}{u_{1}^{2} \cdot p^{s} \cdot w_{1}} \]

Total

B2 shaping forces

\[ r + \mu - \frac{p^b}{p^s} \]

\[ \frac{u_{b}}{u_{p}} \cdot \frac{u_{2}^{2}}{u_{1}^{2}} \]

\[ \frac{\mu_{b}}{\mu_{p}} \]

\[ \rho_{b} \cdot u_{2} \cdot u_{2}^{2} \]

\[ -\mu_{b} \cdot \frac{\rho_{b}}{\rho_{b} + 1} \cdot (\lambda_{z} - P_{z}) \]

\[ -\eta \cdot \frac{u_{b}}{u_{p}} \cdot \frac{u_{2}^{2}}{u_{1}^{2}} \]

\[ \tau \rho S_{1} \cdot \frac{u_{2}^{2} \cdot p^{b} \cdot w_{1}}{u_{1}^{2} \cdot p^{s} \cdot w_{1}} \]

Total
Euler equation for preventive care

\[
\frac{\dot{h}_1}{h_1} = \frac{-\eta h_1}{\eta h_1 h_1} \left[ r + \bar{\mu}^1 - \frac{\dot{p}_1}{p^1} + \eta P \frac{u_{c_2}^2}{u_{c_1}^1} + \frac{\eta h_1 t}{\eta h_1} - \frac{\eta h_1 s_1 \mu^1 S_1}{\eta h_1} + \eta h_1 \frac{(u^1 - Pu^2)}{u_{c_1}^1 p^1} + \right.
\]
\[
+ \eta h_1 \frac{(P_d \dot{d} - \mu^1 S_1 P S_1) \xi Z}{S_1 u_{c_1}^1 p^1} + \eta h_1 \frac{P \frac{\partial \xi Z}{\partial s}(t, t)}{S_1 u_{c_1}^1 p^1} \right]
\]

Euler equation for chronic care

\[
\frac{\dot{h}_2}{h_2} = \frac{-f h_2}{f h_2 h_2} \left[ r + \bar{\mu}^2 - \frac{\dot{p}_2}{p^2} + \frac{f h_2 t}{f h_2} + \frac{f h_2 E f}{f h_2} - f_E - \frac{\mu^2}{\mu_{b_2}} f h_2 p^b - \frac{w_E^2}{p^2} f h_2 - \frac{f h_2 u_E^2}{u_{c_2}^2 p^2} \right]
\]
H2 shaping forces

\[ r + \dot{\mu} - \frac{p^2}{\rho^2} \]

\[ f_h \cdot \frac{f_E}{f_h} \]

\[ -f_h \]

\[ -\frac{\partial f_h}{\partial f_h} f_{hp} \]

\[ \text{TOTAL} \]
Valuations of health

\[
\psi_{\text{life}}^1(t) = -\frac{\partial V}{\partial \mu_1} = \frac{S_1(t)\lambda_S(t)}{\lambda_A(t)}
\]

\[
\psi_{\text{life}}^2(t, s) = -\frac{\partial V}{\partial (\mu_1 + \mu_2)} = -\frac{\partial V}{\partial \mu_1} = \frac{S_2(t, s)\xi_S(t, s)}{\xi_A(t, s)}
\]

\[
\psi_{\text{shock}}(t) = -\frac{\partial V}{\partial \eta} = \frac{Z_1(t)\lambda_Z(t) - Z_1(t)P(S_1(t), d(t))\xi_Z(t, t)}{\lambda_A(t)}
\]

\[
\psi_P(t) = \frac{\partial V}{\partial P} = \frac{Z_1(t)\eta(S_1(t), h_1(t))\xi_Z(t, t)}{\xi_A(t, t)}
\]

\[
\psi_E(t, s) = -\frac{\partial V}{\partial E} = \frac{-\xi_E(t, s)}{\xi_A(t, s)}
\]
Stage 2-valuations:

\[ \widetilde{\psi}^{2}_{\text{life}}(t, s) = \int_{t}^{T} R(t, \tau) \frac{u^{2}(\tau, s)}{u^{2}_{c2}(\tau, s)} \, d\tau \]

\[ \psi_{E}(t, s) = \int_{t}^{T} R^{2}_{\text{def}}(t, \tau, s) \times \left\{ \mu^{2}_{E}(\tau, s)\psi^{2}_{\text{life}}(\tau, s) - w^{2}_{E}(\tau, s) - \frac{u^{2}_{E}(\tau, s)}{u^{2}_{c2}(\tau, s)} \right\} \, d\tau \]
Split-up valuations of health

Stage 1-valuations:

\[\psi_P(t) = \frac{\tilde{\psi}^2_{\text{life}}(t, t)}{P(t)}\]

\[\psi_{\text{life}}^1(t) = \int_t^T R_{\text{life}}^1(t, \tau) \left\{ \frac{u_1(\tau)}{u_{c_1}^1(\tau)} - \eta S_1(\tau) S_1(\tau) \psi_{\text{shock}}(\tau) + \eta(\tau) P(\tau) \frac{u_{c_2}^2(\tau, \tau)}{u_{c_1}^1(\tau)} \left[ \psi_{\text{life}}^2(\tau, \tau) + S_1(\tau) \frac{P S_1(\tau)}{P(\tau)} \tilde{\psi}^2_{\text{life}}(\tau, \tau) - S_1(\tau) B S_1(\tau) \psi_E(\tau, \tau) \right] \right\} d\tau\]

\[\psi_{\text{shock}}(t) = \int_t^T R_{\text{shock}}^1(t, \tau) \left\{ \frac{u_1(\tau)}{u_{c_1}^1(\tau)} + \eta P \frac{u_{c_2}^2(\tau, \tau)}{u_{c_1}^1(\tau)} \tilde{\psi}^2_{\text{life}}(\tau, \tau) \right\} d\tau - P \frac{u_{c_2}^2(t, t)}{u_{c_1}^1(t)} \tilde{\psi}^2_{\text{life}}(t, t)\]
\[ R(t, \tau) := \exp \left( - \int_{t}^{\tau} r(\tau') + \bar{\mu}(\tau') d\tau' \right) \] (1)

\[ R^1_{\text{shock}}(t, \tau) := R(t, \tau) \exp \left( - \int_{t}^{\tau} \eta(\tau') P(\tau') \frac{u^2_{c_2}(\tau', \tau')}{u^1_{c_1}(\tau')} d\tau' \right) \] (2)

\[ R^1_{\text{life}}(t, \tau) := R(t, \tau) \exp \left( - \int_{t}^{\tau} \mu^1_{S_1}(\tau')S_1(\tau') + \eta(\tau') P(\tau') \frac{u^2_{c_2}(\tau', \tau')}{u^1_{c_1}(\tau')} \right) \] (3)

\[ R^2_{\text{life}}(t, \tau, s) := R(t, \tau) \exp \left( - \int_{t}^{\tau} \mu^1_{S_2}(\tau', s)S_2(\tau', s) d\tau' \right) \] (4)

\[ R^2_{\text{def}}(t, \tau, s) := R(t, \tau) \exp \left( - \int_{t}^{\tau} f_E(\tau', s) d\tau' \right) \] (5)
Other interesting indicators

- A-priori value of life

\[ \psi_{\text{life}}(t) = Z_1(t)\psi_1^{\text{life}}(t) + \int_0^t Z_2(t, s)\psi_2^{\text{life}}(t, s)ds \]

- Realised lifetime utility

\[ V_R^R(s) := \int_0^s e^{-\rho t}S_1(t)u^1(c_1(t))dt + e^{-\rho s}\xi_Z(s, s) \]

\[ \xi_Z(t, s) = \int_t^T e^{-\rho(\tau-t)}S_2(\tau, s)u^2(c_2(\tau, s), E(\tau, s))d\tau \]