Taking into account perception of environmental quality: how space matters?

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Heterogeneous dynamic models of economics and population systems

The starting point

- Representation of environmental quality
  - Many ways to measure environmental quality (Eurobarometre 295)
    - Specific pollutant concentration (22%)
    - Climate change (18%)
    - Global index (Environmental Performance Index)

The starting point

- How do individuals experiment environmental quality?
    - Bloom(95), Mariani: Many scales
        - perception of the local environment
        - perception of the surrounding localities
        - perception of global information
- Subjective, imperfect and uncertain perception (Eurobaromètre 295)

The starting point

- Amenity value of environmental quality
    - Empirical studies
    - Theoretical studies
        - Le Kama Schubert (2004): uncertain future preferences
        - Le Kama, Pommeret, Prieur (2014): irreversibility
- Green preferences
    - Ott Soretz (2018)

The benchmark model and the questions raised

$$
\begin{aligned}
J(c) &= \int_0^\infty \frac{c^{\left(1-\frac{1}{\sigma}\right)} Q^{\theta\left(1-\frac{1}{\sigma}\right)}}{1-\frac{1}{\sigma}} e^{-\rho t} dt \\
dQ &= (\gamma Q - c)\, dt + Q\nu dWt
\end{aligned}
$$

- θ: preference for the environment
- Distaste effect (Rotillon and Michel (1995))

  $\nearrow Q \Rightarrow \nearrow$ desire to consume

The benchmark model and the questions raised

$$
\begin{aligned}
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dQ &= (\gamma Q - c)\, dt + Q\nu dWt
\end{aligned}
$$

- $c = \alpha^* Q$ where $\alpha^*(1+\theta)$ is equal to

$$\sigma\rho - \gamma(\sigma-1)(1+\theta) - \frac{\nu^2}{2}(1+\theta)(\sigma-1)\left((1+\theta)\left(\frac{\sigma-1}{\sigma}\right) - 1\right)$$

- $E[Q(t)] = Q_0 e^{gt}$, with $g = \gamma - \alpha^* + \frac{\nu^2}{2}$ thus g is equal to

$$\frac{-\sigma\rho + \gamma\sigma(1+\theta) + \frac{\nu^2}{2}(1+\theta)\left[\left((1+\theta)\frac{(\sigma-1)^2}{\sigma} + (1-\sigma)\right) + 1\right]}{(1+\theta)}$$

The questions raised

- Growth rate g may be negative if
  - If ρ is high
  - if γ is small
- Growth rate g increases with ν.
- The questions raised
  - When space is taken into account, does it enable to increase global environmental quality growth rate?
    $\Rightarrow$ Policy recommendation to nudge people behavior?
  - What are the main consequences between regional versus global externality on environmental dynamics?
    - Global externality: Lacker and Zariphopoulou (2018)
    - Regional externality (Augeraud and Ducrot (2019), Augeraud and Hupkes (2019))

The global externality- A multiplayer approach

$i \in [1..n]$ locations

$$J_i\left(c_i, c_{-i}\right) = \int_0^\infty \frac{c_i^{\left(1-\frac{1}{\sigma_i}\right)} Q_i^{\varepsilon_i^1 \phi_i \left(1-\frac{1}{\sigma_i}\right)} \widetilde{Q}^{\varepsilon_i^2 \phi_i \left(1-\frac{1}{\sigma_i}\right)}}{1-\frac{1}{\sigma_i}} e^{-\rho t} dt$$

$$dQ_i = \left(\gamma_i Q_i - c_i\right) dt + Q_i \nu_i dWt$$

where $\widetilde{Q} = \left(\prod_{j=1}^{n} Q_j\right)^{1/n}$

- Why Geometric mean?
  - Great values of environmental quality have less impact that the small.
  - Technical reasons....

The global externality- Rewriting the objective

$\theta_i = \varepsilon_i^1 \phi_i, \eta_i = \varepsilon_i^2 \phi_i$ et $\mu_i = \theta_i + \frac{\eta_i}{n}$.

$$J_i\left(c_i, c_{-i}\right) = \int_0^\infty \frac{c_i^{\left(1-\frac{1}{\sigma_i}\right)} Q_i^{\mu_i\left(1-\frac{1}{\sigma_i}\right)} \widehat{Q}^{\eta_i\left(1-\frac{1}{\sigma_i}\right)}}{1-\frac{1}{\sigma_i}} e^{-\rho t} dt$$

$$dQ_i = \left(\gamma_i Q_i - c_i\right) dt + Q_i \nu_i dWt$$

where $\widehat{Q} = \left(\prod_{j \neq i} Q_j\right)^{1/n}$

- What is the dynamics of $\widehat{Q}$?

Global externality- Global EQ dynamics

- Let $\widehat{Q} = \left(\prod_{j\neq i} Q_j\right)^{1/n}$ and let $\widehat{Y} = \ln\left(\widehat{Q}\right)$
- We assume $c_i = \alpha_i Q_i$.
- For $Y_i = \ln\left(Q_i\right)$,

$$\begin{aligned} dY_j &= \frac{dQ_j}{Q_j} - \frac{\nu_j^2}{2}dt \\ &= \left(\gamma_j - \alpha_j - \frac{\nu_j^2}{2}\right)dt + \nu_j dWt \end{aligned}$$

- Thus: $d\widehat{Y} = \frac{1}{n}\sum_{j\neq i}\left(\gamma_j - \alpha_j - \frac{\nu_j^2}{2}\right)dt + \frac{1}{n}\sum_{j\neq i}\nu_j dWt$
- Letting $\widehat{\nu} = \frac{1}{n}\sum_{j\neq i}\nu_j$, $\widehat{\varphi} = \frac{1}{n}\sum_{j\neq i}\left(\gamma_j - \alpha_j - \frac{\nu_j^2}{2}\right)$
- Thus: $d\widehat{Q} = \left(\widehat{\varphi} + \frac{\widehat{\nu}^2}{2}\right)\widehat{Q}dt + \widehat{\nu}\widehat{Q}dWt$

The global externality- A multiplayer approach

- We have to solve the $n-$ players game:

$$J_i\left(c_i, c_{-i}\right) = \int_0^\infty \frac{c_i^{\left(1-\frac{1}{\sigma_i}\right)} Q_i^{\mu_i\left(1-\frac{1}{\sigma_i}\right)} \widehat{Q}^{\eta_i\left(1-\frac{1}{\sigma_i}\right)}}{1-\frac{1}{\sigma_i}} e^{-\rho t} dt$$

$$dQ_i = \left(\gamma_i Q_i - c_i\right) dt + Q_i \nu_i dWt$$

$$d\widehat{Q} = \left(\widehat{\varphi} + \frac{\widehat{\nu}^2}{2}\right)\widehat{Q} dt + \widehat{\nu}\widehat{Q} dWt$$

The global externality- A multiplayer approach-Resolution

- HJB equation

$$\rho V\left(Q,\widehat{Q}\right) = \max_c \begin{pmatrix} \frac{c_i^{\left(1-\frac{1}{\sigma_i}\right)} Q^{\mu_i\left(1-\frac{1}{\sigma_i}\right)} \widehat{Q}^{\eta_i\left(1-\frac{1}{\sigma_i}\right)}}{1-\frac{1}{\sigma_i}} \\ +V'_Q\left(Q,\widehat{Q}\right)\left(\gamma_i Q_i - c_i\right) \\ +V'_{\widehat{Q}}\left(Q,\widehat{Q}\right)\left(\left(\widehat{\varphi}+\frac{\widehat{\nu}^2}{2}\right)\widehat{Q}\right) \end{pmatrix}$$
$$+\frac{1}{2}\widehat{\nu}^2\overline{Q}^2 V'_{\widehat{Q}\widehat{Q}}\left(Q,\widehat{Q}\right)+\frac{1}{2}\nu_i^2 Q_i^2 V'_{QQ}\left(Q,\widehat{Q}\right)$$

- We look for $V\left(Q,\widehat{Q}\right)=\frac{1}{1-\frac{1}{\sigma_i}}A_i Q^{(1+\mu_i)\left(1-\frac{1}{\sigma_i}\right)}\widehat{Q}^{\eta_i\left(1-\frac{1}{\sigma_i}\right)}$

The global externality- A multiplayer approach-Resolution

$$A_i^{-\sigma_i} = \frac{\rho\sigma_i - (\sigma_i - 1)\left[\eta_i\left[\widehat{\phi} - \overline{\alpha} + \frac{\alpha_i}{n} + \frac{\widehat{\nu}^2}{2}\eta_i\left(1 - \frac{1}{\sigma_i}\right)\right] + (1+\mu_i)\left[\gamma_i + \right.\right.}{(1+\mu_i)^{1-\sigma_i}}$$

- As at the optimum $c_i = (1+\mu_i)^{-\sigma_i} A_i^{-\sigma_i} Q_i$, then $\alpha_i = (1+\mu_i)^{-\sigma} A_i^{-\sigma}$,

$$\overline{\alpha} = \frac{\frac{1}{n}\sum_{i=1}^{n} \frac{\rho\sigma_i - (\sigma_i - 1)\left[\eta_i\left[\widehat{\phi} + \frac{\widehat{\nu}^2}{2}\eta_i\left(1 - \frac{1}{\sigma_i}\right)\right] + (1+\mu_i)\left[\gamma_i + \frac{\nu_i^2}{2}\left((1+\mu_i)\left(1 - \frac{1}{\sigma_i}\right) - 1\right)\right]\right]}{(1+\mu_i)\left[1 + \frac{(\sigma_i - 1)\eta_i}{(1+\mu_i)n}\right]}}{1 + \frac{1}{n}\sum_{i=1}^{n} \frac{(1-\sigma_i)\eta_i}{(1+\mu_i)\left[1 + \frac{(\sigma_i - 1)\eta_i}{(1+\mu_i)n}\right]}}$$

The global externality- n tend to infinity

- We let n tend to infinity
- The ratio $\overline{\alpha}\left(1+\overline{\left[\frac{(1-\sigma)\eta}{1+\theta}\right]}\right)$ satisfies

$$\rho\overline{\left[\frac{\sigma}{1+\theta}\right]}+\left(\overline{\gamma}-\frac{\overline{\nu^2}}{2}\right)\overline{\left[\frac{(1-\sigma)\,\eta}{1+\theta}\right]}+\frac{\overline{\nu}^2}{2}\overline{\left[\frac{(1-\sigma)\,\eta^2\left(1-\frac{1}{\sigma}\right)}{(1+\theta)}\right]}$$

$$+\overline{[(1-\sigma)\,\gamma]}+\overline{\left[(1-\sigma)\,\frac{\nu^2}{2}\,(1+\theta)\left(1-\frac{1}{\sigma}\right)\right]}-\overline{\left[(1-\sigma)\,\frac{\nu^2}{2}\right]}$$

- Mean consumption only depends on mean values and variance value of η and ν.

The global externality- infinity number of agents

- We assume heterogeneity only on η and ν. Then $\overline{\alpha}\left(1+\theta+(1-\sigma)\overline{[\eta]}\right)$ is given by

$$\rho\sigma+\left(\gamma-\frac{\overline{\nu^2}}{2}\right)(1-\sigma)\overline{[\eta]}+(1-\sigma)\left(1-\frac{1}{\sigma}\right)\frac{\overline{\nu}^2}{2}\overline{[\eta^2]}$$
$$+(1+\theta)(1-\sigma)\gamma+(1+\theta)^2(1-\sigma)\left(1-\frac{1}{\sigma}\right)\overline{\left[\frac{\nu^2}{2}\right]}$$
$$-(1+\theta)(1-\sigma)\overline{\left[\frac{\nu^2}{2}\right]}$$

- For given η, there exists values of $(\sigma,\overline{\eta},V(\eta))$ such that the growth rate of the environmental quality is greater that when the agents do not care of global environment

The Regional externality

$i \in \mathbb{R}$ locations

$$J_i(c_i) = \int_0^\infty \frac{c_i^{\left(1-\frac{1}{\sigma}\right)} Q_i^{\varepsilon^1 \phi_i \left(1-\frac{1}{\sigma}\right)} \widetilde{Q}^{(1-\varepsilon)\phi\left(1-\frac{1}{\sigma}\right)}}{1-\frac{1}{\sigma}} e^{-\rho t} dt$$

$$dQ_i = (\gamma Q_i - c_i)\, dt$$

where

$$\widetilde{Q}_i(t) = \alpha \sum_{j \in \mathbb{Z}} p_j Q_{i-j}(t),$$

The Regional externality-continuous space

$x \in \mathbb{R}$ locations

$$J(c) = \int_0^\infty \frac{c^{\left(1-\frac{1}{\sigma}\right)} Q^{\varepsilon^1 \phi\left(1-\frac{1}{\sigma}\right)} \overline{Q}^{(1-\varepsilon)\phi\left(1-\frac{1}{\sigma}\right)}}{1-\frac{1}{\sigma}} e^{-\rho t} dt$$

$$\frac{\partial Q(x,t)}{\partial t} = (\gamma Q - c)$$

where

$$\overline{Q}(x,.) = \int_{\mathbb{R}^2} \varphi(x-y)\, Q(y,.)\, dy$$

where φ is a non negative integrable function such that $\int_{\mathbb{R}^2} \varphi(x)\, dx = 1$. Such a φ are choosen to take into account the weight due to geographical distance among agents (Augeraud-Veron and Ducrot

The Regional externality-resolution

$$c(x,t) = \frac{\lambda(x,t)^{-\sigma}}{\left(Q(x,t)^{\varepsilon}\,\overline{Q}(x,t)^{1-\varepsilon}\right)^{\phi(1-\sigma)}}$$

and the equilibrium is thus a solution of

$$\left\{ \begin{array}{r} \frac{\partial\lambda(x,t)}{\partial t} = (\rho-\gamma)\,\lambda - \varepsilon\phi\lambda^{1-\sigma}Q^{-(\varepsilon\phi(1-\sigma)+1)}\left[\int_{\mathbb{R}^2}\varphi(x-y)\,Q(y,t)\,dy \right. \\ \frac{\partial Q(x,t)}{\partial t} = \gamma Q - \lambda^{-\sigma}Q^{-\phi\varepsilon(1-\sigma)}\left[\int_{\mathbb{R}^2}\varphi(x-y)\,Q(y,t)\,dy\right]^{-(1-\varepsilon} \end{array} \right.$$

The Regional externality- resolution-steady state

- The homogenous steady state is such that $C = \alpha^* Q$ with

$$\alpha^* = \frac{\sigma\rho + (\phi + 1)(1 - \sigma)\gamma}{(1 + \phi) - (1 - \varepsilon)\sigma\phi} > 0$$

- The growth rate is

$$g_\varepsilon = \frac{\sigma((1 + \varepsilon\phi)\gamma - \rho)}{(1 + \phi) - (1 - \varepsilon)\sigma\phi}$$

The Regional externality- resolution-dynamics

- Fourier transform of w, defined by

$$\widehat{w}\left(\xi, t\right) = \int_{\mathbb{R}^p} w\left(x, t\right) e^{-2i\pi x\xi} dx, \text{ for all } \xi \in \mathbb{R}^p$$

- For every $\xi \in \mathbb{R}^p$, the characteristic matrix is defined by

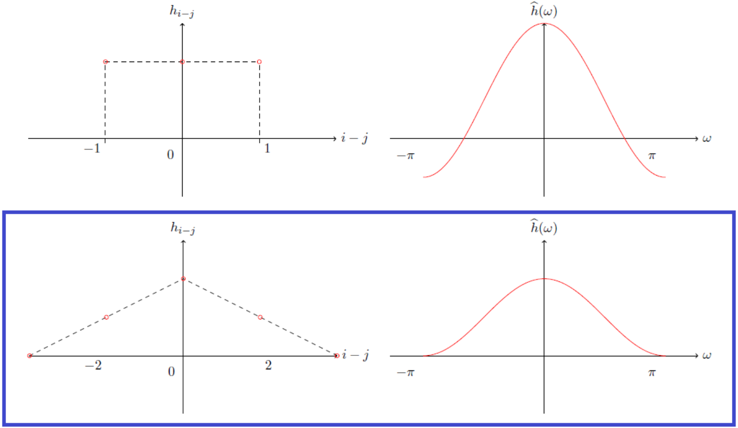
$$\left[\begin{array}{cc} \left(\varepsilon\phi + 1\right) + \left(1-\varepsilon\right)\phi\left(1-\sigma\right)\widehat{\varphi}\left(\xi\right) & -\left(1-\varepsilon\right)\phi\left(1-\sigma\right)\left(\frac{\gamma}{z^*}-1\right)\widehat{\varphi} \\ -1+\widehat{\varphi}\left(\xi\right) & -\left(\frac{\gamma}{z^*}-1\right)\widehat{\varphi}\left(\xi\right) \end{array} \right.$$

- For every $\xi \in \mathbb{R}^p$, the characterics polynomial is given by

$$\begin{aligned} P\left(X;\xi\right) \;=\; & X^2 - \Bigg(\left(\varepsilon\phi+1\right) + \left(1-\varepsilon\right)\phi\left(1-\sigma\right)\widehat{\varphi}\left(\xi\right) - \left(\frac{\gamma}{z^*}-1\right) \\ & - \left(\frac{\gamma}{z^*}-1\right)\widehat{\varphi}\left(\xi\right)\left[1+\left[1-\sigma\left(1-\varepsilon\right)\right]\phi\right] \end{aligned}$$

Regional interactions- dynamics consequences

- The nature of the roots depend on the sign of $\widehat{\varphi}$
  - If $\widehat{\varphi}(\xi) > 0$, characteristic roots have opposite sign.
- An example of interaction kernel



Conclusion

- If only regional spatial externality matters
  - Steady state value of the BGP is the same as the one in the non spatial model
  - The dynamics exhibit transitionnary dynamics
- If only global spatial externality matters
  - The dynamics follows the BGP dynamics
  - The growth rate of environmental quality may increase according to the preference weight on the global partial term
- Policy recommendations?