Planned Obsolescence: a Dynamic Analysis

Richard F. Hartl[1], Peter M. Kort[2], Stefan Wrzaczek[3]

[1] University of Vienna, Austria
[2] Tilburg University, the Netherlands; University of Antwerp, Belgium
[3] Vienna Institute of Demography, Austria

UVienna,TilburgU,VID

November 27, 2019

The Economist ESPRESSO

Friday January 4th

Sour taste: Apple

Apple's profit warning will continue to preoccupy markets. The technology giant's announcement that it was revising down its earnings expectations in the fourth quarter of 2018, largely because of lower iPhone sales and signs of economic weakness in China, has already hit it hard: Apple's share price fell by 10% when the market opened on January 3rd. Investors will be asking two questions. The first is the extent of the economic slowdown in China. Tim Cook, Apple's boss, blamed "deceleration" there for most of the firm's revenue miss and pointed the finger at escalating trade tensions between America and China. The second is slowing consumer demand for smartphones. Mr Cook also mentioned weaker-than-expected iPhone upgrades, the result of longer battery lives and dearer prices. If you had to pick two forces propelling the world economy over the past decade, you would probably choose China and phones. Apple's travails grab attention for good reason.

Introduction

firm produces durable good.

disadvantage durable good that does not break down: consumer only needs to buy once.

durable good that breaks down often: every time consumer has to decide whether to purchase product again.

If consumer decides positively: firm's revenue goes up.
if product breaks down too often: consumer more inclined to decide not to buy this product anymore.

when deciding about product quality firm faces trade off:
1. high quality implies high reputation but low breakdown probability so that consumers will not repurchase too often
2. low quality means product breaks down soon, implying consumers need to buy this product again but this damages product's reputation, which reduces demand.

The paper investigates this problem in a dynamic model of the firm that explicitly takes account of the time the product is sold and the age of the product.

light bulb: originally lasted much longer

women's tights made from nylon: engineers used less of the chemicals that make the nylon immune against sunlight (UV)

IPod: originally batteries lasted 18 months but cannot be replaced
Someone sued, the media talked about it and finally Apple had to offer a battery repair service and 2 years warranty

Research questions:

1.

Effect length warranty period on control variable "(average) time to breakdown"

too long: large costs and firm does not want to spend too much on this product ⇒low time to breakdown

2.

Implications when social planner advertises existence warranty period

3.

Effect of the variance of the average time to breakdown

Kreiss (2015): lifetime can be planned and controlled very precisely by engineers

Eduard Sailer (Miele): questions this

Literature

Seminal paper: Bulow (1986, QJE): two-period setting: monopolist has incentive to reduce durability to enhance future demand.

Fethke and Jagannathan (2002, JEDC): multi-period model, but just two vintages

Agrawal et al. (2016, MSOM): planned obsolescence does not work with conspicuous goods: then offer high quality and long-lasting goods against high price

Our paper: the first with a fully dynamic framework.

Model

Notation:
t : time
a : age of the (durable) good
v : building year or vintage

$$a = t - v$$

State variables:
$C(t)$: stock of potential consumers
$q(t, a)$: number of products in use of age a at time t
$R(t)$: stock of reputation

Control variables:
$s(t)$: quantity to be sold at time t
$b(v)$: expected product life time of a product built in year v

$b(v)$: firm may not be able to control breakdown completely
Kreiss (2015): lifetime can be planned and controlled very precisely by engineers
Eduard Sailer (Miele): questions this

$f(a,b)$: density function that stands for the breakdown probability
$F(a,b)$: cumulative distribution function
$h(a,b)$: hazard rate where we adopt the gamma distribution:

$$h(a,b) = \frac{f(a,b)}{1 - F(a,b)} = \frac{a^{\frac{b^2}{\sigma}-1} e^{-\frac{ab}{\sigma}} \left(\frac{\sigma}{b}\right)^{-\frac{b^2}{\sigma}}}{\Gamma\left(\frac{b^2}{\sigma}, \frac{ab}{\sigma}\right)},$$

with
σ : variance
So the assumption is that the firm can control the mean $b = b(v)$, given the variance σ.

$B(t)$: number of breakdowns at time t :

$$B(t) = \int_0^\infty h(a, b(t-a))\, q(t,a)\, da$$

Policy:

ω : length warranty period

$\alpha \in (0,1)$: fraction of consumers making use of their warranty rights

$W(t)$: number of breakdowns during warranty period at time t:

$$W(t) = \int_0^\omega h(a, b(t-a))\, q(t,a)\, da$$

$$q(t,0) = s(t) + \alpha W(t)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial a}\right) q(t,a) = -h(a, b(t-a))\, q(t,a)$$

Output price

$$\frac{dC(t)}{dt} = B(t) - \alpha W(t) - s(t)$$

$Q(t)$: total number of products in use at time t :

$$Q(t) = \int_0^\infty q(t,a)\,da$$

$R(t)$: stock of reputation at time t :

$$\frac{dR(t)}{dt} = \delta\left(\left(1 - \frac{B(t)}{Q(t)}\right) - R(t)\right)$$

$p(R, C, s)$: output price:

$$p(R, C, s) = \theta_1 R + \theta_2 C - s$$

Cost:

$c_1 q(t, 0)$: production cost

$c_2 b$: cost of setting average time to breakdown equal to b

The full model

$$\max_{s(t),b(t)} \int_0^\infty e^{-rt} \left(\left(\theta_1 R(t) + \theta_2 C(t) - s(t) - c_1 \right) s(t) - c_1 \alpha W(t) - c_2 b \right) dt$$

subject to

$$q(t,0) = s(t) + \alpha W(t)$$

$$\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial a} \right) q(t,a) = -h(a, b(t-a))\, q(t,a)$$

$$Q(t) = \int_0^\infty q(t,a)\, da$$

$$B(t) = \int_0^\infty h(a, b(t-a))\, q(t,a)\, da$$

$$W(t) = \int_0^\omega h(a, b(t-a))\, q(t,a)\, da$$

$$\frac{dC(t)}{dt} = B(t) - \alpha W(t) - s(t)$$

$$\frac{dR(t)}{dt} = \delta \left(\left(1 - \frac{B(t)}{Q(t)} \right) - R(t) \right)$$

Remark

The model implicitly assumes stationary "population":

$$C(t) + Q(t) = C_{\max}$$

The Benchmark: Deterministic Variant

$$\max_{s(t),b(t)} \int_0^\infty e^{-rt} \left(\left(\theta_1 R(t) + \theta_2 C(t) - s(t) - c_1 \right) s(t) - c_1 \alpha W(t) - c_2 b \right) dt$$

$$q(t,0) = s(t) + \alpha W(t)$$

$$q(t,a) = \begin{cases} q(t-a,0) & \text{if} \quad a < b(t-a) \\ 0 & \text{if} \quad a \geq b(t-a) \end{cases}$$

$$Q(t) = \int_0^\infty q(t,a)\, da$$

$$B(t) = \int_0^\infty q(t-a,0)\, |_{b(t-a)=a}\, da$$

$$W(t) = \int_0^\omega q(t-a,0)\, |_{b(t-a)=a}\, da$$

$$\frac{dC(t)}{dt} = B(t) - \alpha W(t) - s(t)$$

$$\frac{dR(t)}{dt} = \delta \left(\left(1 - \frac{B(t)}{Q(t)} \right) - R(t) \right)$$

OSSP Formulation

$$\max_{s,b}\left(\left(\theta_1 R + \theta_2 C - s - c_1\right) s - c_1 \alpha W - c_2 b\right)$$

subject to

$$q\left(0\right) = s + \alpha W = B$$

$$q\left(a\right) = \begin{cases} q\left(t-a, 0\right) & \text{if} \quad a < b\left(t-a\right) \\ 0 & \text{if} \quad a \geq b\left(t-a\right) \end{cases}$$

$$Q = bq\left(0\right)$$

$$W = \begin{cases} q\left(0\right) & \text{if} \quad \omega > b \\ 0 & \text{if} \quad \omega \leq b \end{cases}$$

$$C_{\max} = C + Q$$

$$R = 1 - \frac{1}{b}$$

Theorem

1. No breakdowns in the warranty period iff

$$\omega < \sqrt{\frac{\theta_1}{\theta_2 s + c_2 \frac{1}{s}}} = b$$

in which

$$s = \frac{1}{2\left(\theta_2 b + 1\right)} \left(\theta_1 \left(1 - \frac{1}{b}\right) + \theta_2 C_{\max} - c_1\right)$$

Theorem

2. All breakdowns in the warranty period otherwise with

$$b_\alpha = \sqrt{\frac{\theta_1}{\frac{\theta_2 s}{1-\alpha} + c_2 \frac{1}{s}}}$$

$$s = \frac{1}{2\left(\frac{\theta_2 b}{1-\alpha} + 1\right)} \left(\theta_1 \left(1 - \frac{1}{b}\right) + \theta_2 C_{\max} - c_1 \frac{1}{1-\alpha}\right)$$

Effect of policy ω and α on b

We have:

$$\begin{aligned}
\omega &< b : b = \sqrt{\frac{\theta_1}{\theta_2 s + c_2 \frac{1}{s}}} \\
\omega &\geq b : b_\alpha = \sqrt{\frac{\theta_1}{\frac{\theta_2 s(\alpha)}{1-\alpha} + c_2 \frac{1}{s(\alpha)}}} \\
b &> b_\alpha
\end{aligned}$$

Effect of ω :

As long as $\omega < b$: no effect

If ω is increased beyond b : b jumps down to b_α as soon as ω hits b.

Effect of α :

Calculate:

$$\frac{db_\alpha}{d\alpha} = \frac{\partial b_\alpha}{\partial \alpha} + \frac{\partial b_\alpha}{\partial s} \frac{\partial s}{\partial \alpha}$$

No warranty period better in this case?

The Full Model Solution

Numerical analysis

Benchmark parameter values:

$\theta_1 = \theta_2 = 0.3$

$C_{\max} = 1$

$c_1 = 0.01$

$c_2 = 0.001$

$\omega = 2$

$\alpha = 0.5$

$\sigma = 6$

OSSP

$$\max_{s,b}\left(\left(\theta_1 R + \theta_2 C - s - c_1\right)s - c_1 \alpha W - c_2 b\right)$$

subject to

$$q\left(0\right) = s + \alpha W = B$$

$$\frac{\partial}{\partial a} q\left(a\right) = -h\left(a, b\right) q\left(a\right)$$

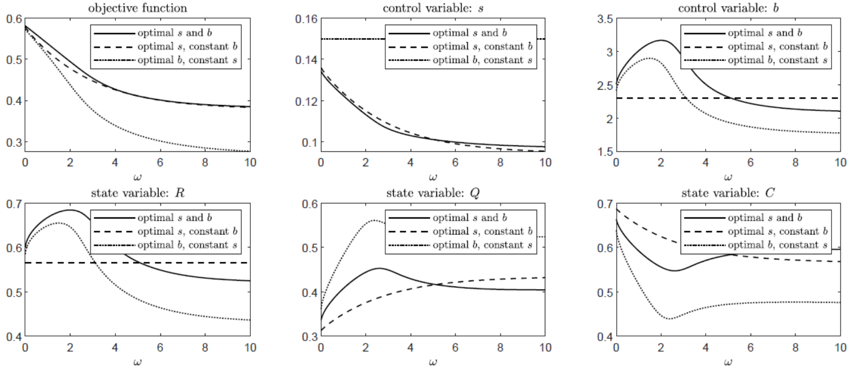
$$Q = \int_0^{\infty} q\left(a\right) da$$

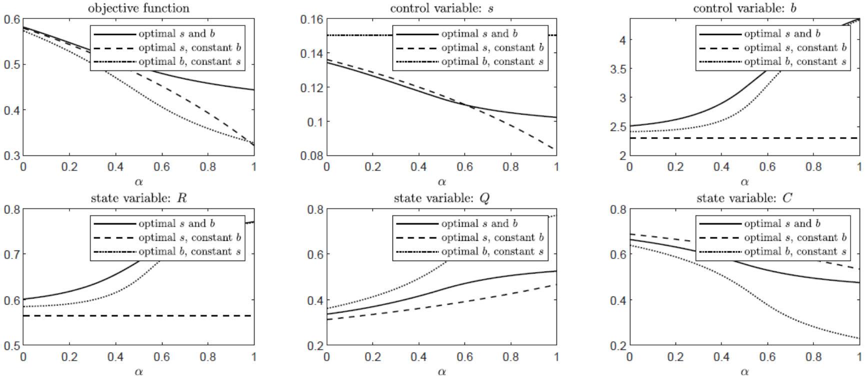
$$B = \int_0^{\infty} h\left(a, b\right) q\left(a\right) da$$

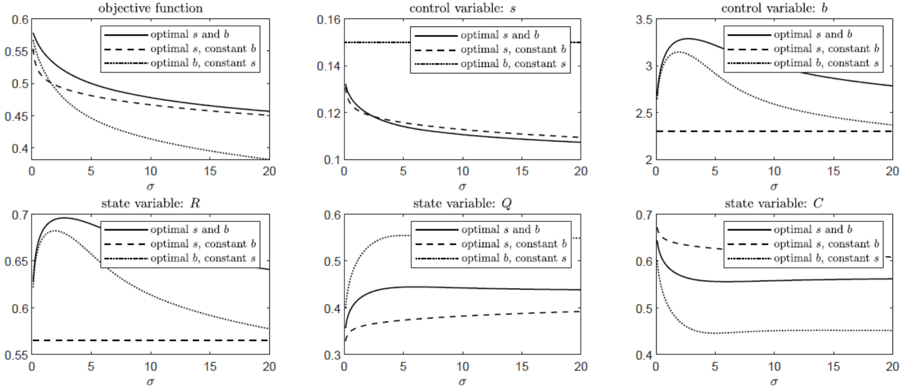
$$W = \int_0^{\omega} h\left(a, b\right) q\left(a\right) da$$

$$C + Q = C_{\max}$$

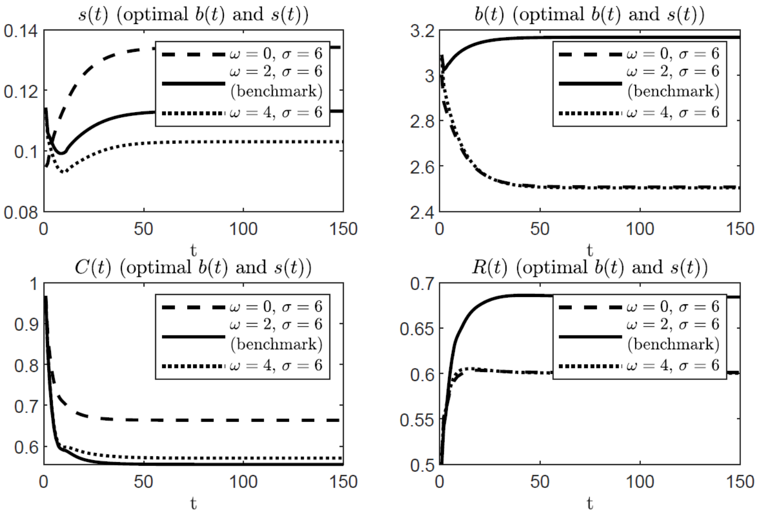
$$R = 1 - \frac{1}{b}$$

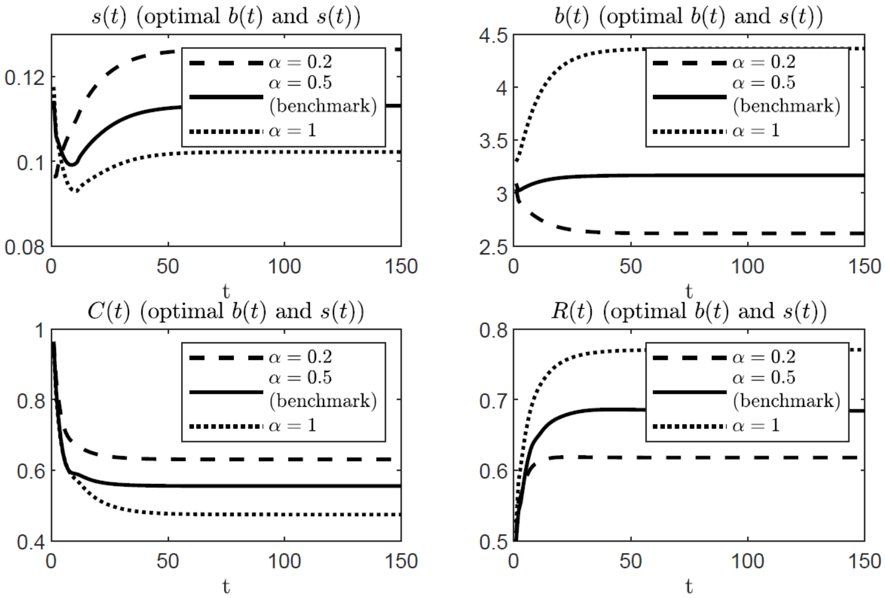

objective function
control variable: s
control variable: b
state variable: R
state variable: Q
state variable: C
optimal s and b
optimal s, constant b
optimal b, constant s
ω

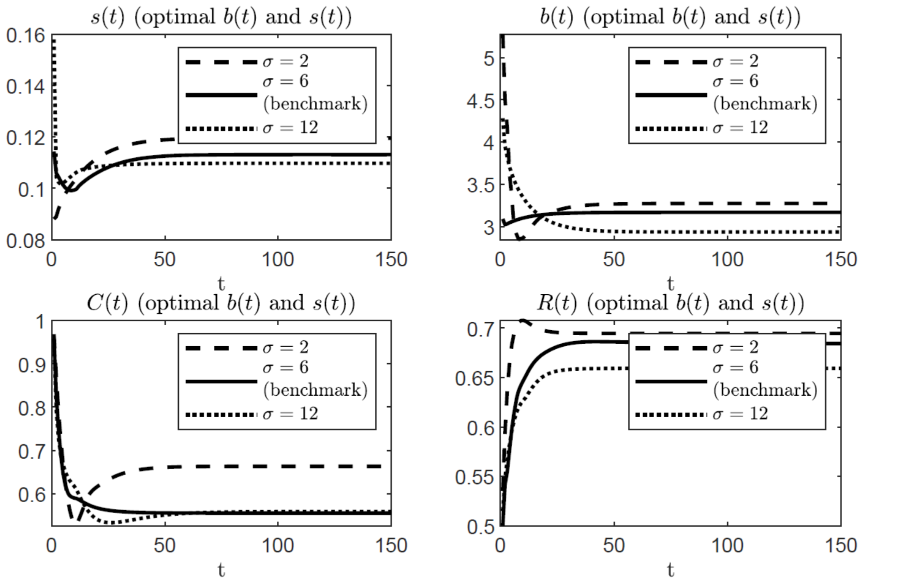

objective function
control variable: s
control variable: b
state variable: R
state variable: Q
state variable: C
optimal s and b
optimal s, constant b
optimal b, constant s
α


objective function
optimal s and b
optimal s, constant b
optimal b, constant s
σ
control variable: s
control variable: b
state variable: R
state variable: Q
state variable: C

The optimal dynamic solution




s(t) (optimal b(t) and s(t))
b(t) (optimal b(t) and s(t))
C(t) (optimal b(t) and s(t))
R(t) (optimal b(t) and s(t))
α = 0.2
α = 0.5 (benchmark)
α = 1
t
0
50
100
150
0.08
0.1
0.12
2.5
3
3.5
4
4.5
0.4
0.6
0.8
1
0.5
0.6
0.7
0.8


$s(t)$ (optimal $b(t)$ and $s(t)$)
$b(t)$ (optimal $b(t)$ and $s(t)$)
$C(t)$ (optimal $b(t)$ and $s(t)$)
$R(t)$ (optimal $b(t)$ and $s(t)$)
$\sigma = 2$
$\sigma = 6$ (benchmark)
$\sigma = 12$
t
0.16
0.14
0.12
0.1
0.08
5
4.5
4
3.5
3
1
0.9
0.8
0.7
0.6
0.7
0.65
0.6
0.55
0.5
0
50
100
150

Conclusions

Answers to research questions:

1.

Effect length warranty period on control variable "(average) time to breakdown"

time to breakdown first increases, then decreases

sales are negatively affected by existence warranty period

2.

Implications when social planner advertises existence warranty period

positive for time to breakdown: make consumers aware of warranty period

negative for sales

3.

Effect of the variance of the average time to breakdown

negative: do not spend money on something that cannot be influenced

also very limited uncertainty leads to a small time to breakdown