Optimal Taxation with Endogenous Population Growth and the Risk of Environmental Disaster

Tapio Palokangas
University of Helsinki, Helsinki GSE, IIASA and IZA

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Contents

Abstract

- This study considers a market economy where firms produce goods from labor and capital and households supply labor, rear children, save in capital, protect themselves against mortality by health care and derive utility from their consumption and children.
- There is a risk that population growth and capital accumulation trigger a deadly environmental disaster.
- Optimal policy is solved by a game where the government is the leader and the representative household the follower.
- The solution yields precautionary taxes on both capital income and the demand for health care.

Specification

- For the sake of clarity, the environmental disaster is taken as a random regime shift that occurs only once, with the post-event regime holding indefinitely.
  - As pointed out by de Zeeuw and Zemel (2012), this restriction is not essential and models of recurrent events, where several shifts occur at random times with independent intervals, can be analyzed using the same methodology.
- Because the construction of different mortality rates for different cohorts would excessively complicate the analysis, then, following Becker (1981), it is assumed that the whole population has a uniform mortality rate, for simplicity.

Literature: damage functions

Van der Ploeg and de Zeeuw (1992), and Dockner and Long (1993)

- assume smooth convex damage functions, which ignores the effect of a potential regime shift on the optimal policy
- then, there is no need for precautionary measures against pollution: the policy maker should respond at the moment pollution occurs, but not beforehand

de Zeeuw and Zemel (2012)

- examine the need for precautionary environmental policy
- consider the management of a system that is subject to the risk of an abrupt and random jump in pollution damage.

This document applies precautionary environmental policy for a market economy with pollution-related mortality as the damage.

Literature: models of the central planner

Haurie and Moresino (2006), Polasky et al. (2011) and de Zeeuw and Zemel (2012)

- ignore population growth and consider only the central planner that can fully control all resources of the economy.

In contrast, this study examines

- a market economy where households and firms determine production, fertility and capital accumulation,
- constructs a dynamic Stackelberg game where the government is the leader, using only linear taxes
- has the the benefit that the suggested policy rules can be presented directly in terms of observable variables (e.g. prices and the interest rate).

Population and labor supply

- Time t is continuous.
- Population L grows at the rate that is equal to the fertility rate f minus the mortality rate m:

$$\frac{1}{L}\frac{dL}{dt} = f - m, \quad L(0) = L_0.$$

- It takes one unit of labor to rear one newborn. The remainder of the population, $N = L - fL$, works in production.

The goods market

- There is only one good.
- The depreciation of capital is included in the production function, so that saving equals the accumulation of capital.
- The government uses the exogenous amount g of the good per each person to maintain the infrastructure.
- The output of the good, Y, is used in consumption C, health care H, saving $\frac{dK}{dt}$ and public expenditures gL:

$$Y = C + H + \frac{dK}{dt} + gL.$$

Accumulation of capital

- Define consumption C, health care H and capital K in proportion to population L:

$$c \doteq C/L, \quad h \doteq H/L, \quad k \doteq K/L.$$

- *Investment per head* is

$$s \doteq \frac{dk}{dt} = \frac{d}{dt}\left(\frac{K}{L}\right) = \frac{1}{L}\frac{dK}{dt} - \frac{dL}{dt}\frac{K}{L^2} = \frac{1}{L}\frac{dK}{dt} + (m - f)k.$$

- Because investment per head $s \doteq \frac{dk}{dt}$ is used as a control in dynamic programming, then private saving is

$$\frac{dK}{dt} = [s + (f - m)k]L.$$

Production

- The firms produce output Y from capital K and labor input N according to neoclassical technology $Y = F(K, N)$.
- Output per head, $y = \frac{Y}{L}$, can be defined as a function of capital per head, k, and the fertility rate f as follows:

$$1 - f = n \doteq N/L, \quad Y/L = F(k, n) = F(k, 1 - f) \doteq y(k, f),$$
$$y_k \doteq \frac{\partial y}{\partial k} = F_K(k, n) > 0, \quad y_f \doteq \frac{\partial y}{\partial f} = -F_N(k, n) = -w.$$

- In equilibrium, the marginal products of capital and labor, F_K and F_N, are equal to the interest rate r and the wage w:

$$r = F_K(k, n) = y_k, \quad w = F_N(k, n), \quad y_f = -F_N(k, n) = -w.$$

Environmental disaster

- Aggregate capital K and aggregate population L pollute according to
$$P = K^{\gamma} L^{1-\gamma} = k^{\gamma} L,$$
where $0 < \gamma < 1$ is a constant.
- The probability of the environmental disaster, π, is an increasing function of pollution P. Then, the disaster is a random shock q with mean $\pi(P)$ as follows:

$$q = \begin{cases} 1 & \text{with probability } \pi(P), \\ 0 & \text{with probalility } 1 - \pi(P), \end{cases} \quad \pi' > 0.$$

Externality

- The environmental shock q increases every individual's mortality rate m simultaneously, but each individual can decrease its personal mortality rate m by spending on its personal health care h with increasing marginal costs:

$$m = \chi(\delta q - h), \quad \chi' > 0, \quad \chi'' \text{ exists}, \quad \delta > 0 \text{ constant.}$$

- Health care h can be replaced by the mortality rate m as the household's control in the model, for convenience. Denoting the inverse function of χ by $z(m) \doteq \chi^{-1}(m)$ yields

$$h = \delta q - z(m), \quad z' \doteq \frac{1}{\chi'(m)} > 0, \quad z'' \doteq -\frac{\chi''}{(\chi')^2} \text{ exists.}$$

Public policy

- The government sets a poll tax a per head and the tax τ on capital income rK, the parental tax x on the number of children, fL, and the tax b on health care, H.
- The representative household is the ***follower*** that decides its consumption per head, c, its mortality rate m through its spending on health care and its fertility rate f, taking the taxes (a, x, τ, b) and the environmental shock q as given.
- The benevolent government is the ***leader*** that maximizes the representative household's utility, determines the taxes (a, x, τ, b), observing the follower's behavior, the behavior of the firms and the risk of the regime shift due to pollution.

Utility

- The representative household's utility over time $t \in [\zeta, \infty)$ is

$$\int_{\zeta}^{\infty} u(t)^{\sigma} e^{(\rho+m)(\zeta-t)} dt \ \text{ with } \ u(t) \doteq c(t)f(t)^{\alpha}_{,}$$

where ρ is the constant rate of time preference, $\sigma \in (0,1)$ a parameter and m the mortality rate.

- The household's budget constraint can be expressed so that consumption per head, c, is a function of the household's controls (s, f, m), capital per head, k, taxes (a, x, τ, b), the wage w, the interest rate r and the shock q:

$$c = \widetilde{c}(s, f, m, k, a, x, \tau, b, w, r, q).$$

Transformation from real into virtual time

- The mortality rate m is eliminated from the discount factor of the utility function by Uzawa's (1968) transformation:

$$\theta(t) = (\rho + m)t \text{ with } dt = \frac{d\theta}{\rho + m}.$$

- Then, the utility function and the constraint $s = \frac{dk}{dt}$ can be transformed into virtual time θ as follows:

$$\int_{\zeta}^{\infty} \frac{c(\theta)^{\sigma} f(\theta)^{\alpha\sigma}}{\rho + m(\theta)} e^{\zeta - \theta} d\theta,$$

$$\frac{dk}{d\theta} = \frac{s(\theta)}{\rho + m(\theta)}, \quad k(0) = k_0.$$

The household's value function

- The household maximizes its utility

$$\int_{\zeta}^{\infty} \frac{c^{\sigma} f(\theta)^{\alpha\sigma}}{\rho + m(\theta)} e^{\zeta-\theta} d\theta$$

by investment per head, s, the fertility rate f and the mortality rate m subject to

consumption per head $\quad c = \widetilde{c}(s, f, m, k, a, x, \tau, b, w, r, q),$

capital accumulation $\quad \frac{dk}{d\theta} = \frac{s(\theta)}{\rho + m(\theta)}.$

- This defines the value function at initial time ζ as

$$\Phi(k, a, x, \tau, b, w, r, q, \zeta) \doteq \max_{s,f,m} \int_{\zeta}^{\infty} \frac{c(\theta)^{\sigma} f(\theta)^{\alpha\sigma}}{\rho + m(\theta)} e^{\zeta-\theta} d\theta.$$

Bellman equation for the household

Bellman equation for that problem is

$$\Phi = \max_{s,f,m} \Lambda \;\text{ with }\; \Lambda \doteq \frac{c^\sigma f^{\alpha\sigma}}{\rho + m} + \frac{\partial \Phi}{\partial k}\frac{dk}{d\theta} = \frac{1}{\rho + m}\left(c^\sigma f^{\alpha\sigma} + \frac{\partial \Phi}{\partial k} s\right).$$

This yields the first-order conditions

$$\frac{\partial \Lambda}{\partial s} = 0 \;\Leftrightarrow\; \frac{\partial \Phi}{\partial k} = \sigma c^{\sigma-1} f^{\alpha\sigma} \quad \text{trade off between consumption and saving} \qquad (1)$$

$$\frac{\partial \Lambda}{\partial f} = 0 \;\Leftrightarrow\; \frac{f}{c} = \frac{\alpha}{w + k + x + b} \quad \text{the fertility rate} \qquad (2)$$

$$\frac{\partial \Lambda}{\partial m} = 0 \;\Leftrightarrow\; k + b + z' = \frac{1}{\sigma c^{\sigma-1}}\frac{\Lambda}{\rho + m}$$

$$\Leftrightarrow\; m = M(k, w, r, q, a, x, \tau, b, \zeta) \quad \text{the mortality rate.} \qquad (3)$$

Solution

- To solve the problem, let's try the specification

$$\Phi = \vartheta \frac{c^{\sigma} f^{\alpha\sigma}}{\rho + m}, \quad \text{where } \vartheta \text{ is constant.}$$

- Inserting this into the Bellman equation and the first-order condition for s yields $\vartheta = 1$ and

$$f = (1 - \tau)r - \rho. \tag{4}$$

Setup for public policy

- The change of pollution $v \doteq \frac{dP}{dt}$ is in virtual time θ as

$$\frac{dP}{d\theta} = \frac{v(\theta)}{\rho + m(\theta)}, \quad P(0) = P_0.$$

- Because the government can control saving per head, s, the change of pollution, v, and the mortality rate m by the taxes on capital income, τ, the number of children, x, and the demand for health care, b, then
  - the former (s, v, m) can be treated as the government's controls in the model and
  - consumption per head, c, can be derived as a function of these, capital per head, k, and pollution P:

$$c(s, v, m, k, P, q).$$

Policy optimization

- The government maximizes the representative household's welfare by its controls (s, v, m) subject to the occurrence of the environmental shock, q, the accumulation of capital per head, k, and pollution P, and the determination of the household's fertility rate f and consumption per head, c.
- Thus, its value function at initial time ζ is defined by

$$\Psi(k, P, q, \zeta) \doteq \max_{s(\zeta), v(\zeta), m(\zeta)} \int_{\zeta}^{\infty} \frac{c(\theta)^{\sigma} f(\theta)^{\alpha\sigma}}{\rho + m(\theta)} e^{\zeta - \theta} d\theta,$$

where $q = 0$ holds true before and $q = 1$ after the shock.
- Then, one can define the ***relative damage*** of the shock in terms of welfare as follows:

$$D(k, P, \zeta) \doteq \frac{\Psi(k, P, 0, \zeta) - \Psi(k, P, 1, \zeta)}{\Psi(k, P, 0, \zeta)}.$$

Bellman equation for the government

- The Bellman equation for the government's program is

$$\Psi = \max_{s(\zeta),v(\zeta),m(\zeta)} \Upsilon(s,f,k,P,q,\zeta) \text{ with}$$

$$\Upsilon(s,f,k,P,q,\zeta) \doteq \frac{c^{\sigma} f^{\alpha\sigma}}{\rho+m} + \frac{\partial\Psi}{\partial k}(k,P,q,\zeta)\frac{dk}{d\theta}$$
$$+\frac{\partial\Psi}{\partial P}(k,P,q,\zeta)\frac{dP}{d\theta} + \pi(P)\big[\Psi(k,P,1,\zeta) - \Psi(k,P,q,\zeta)\big]$$
$$= \frac{1}{\rho+m}\left[c^{\sigma} f^{\sigma\alpha} + \frac{\partial\Psi}{\partial k}(k,P,1,\zeta)s + \frac{\partial\Psi}{\partial P}(k,P,1,\zeta)v\right]$$
$$+\pi(P)\big[\Psi(k,P,1,\zeta) - \Psi(k,P,q,\zeta)\big],$$

where $\pi(P)$ is the probability of the environmental shock and $\Psi(k,P,1,\zeta) - \Psi(k,P,q,\zeta)$ is the immediate change of welfare due to that shock.

Solution

- To find a solution, let's try the specification

$$\Psi(k,P,q,\zeta)=\frac{\varpi c^{\sigma}f^{\alpha\sigma}}{\rho+m}>0,\quad \text{where } \varpi \text{ is a constant.}$$

- Noting this and the definition of the relative damage D, the Bellman equation becomes

$$1=\frac{\Upsilon}{\Psi}=\begin{cases}\frac{1}{\varpi}+\frac{1}{\Psi}\frac{\partial\Psi}{\partial k}\frac{dk}{d\theta}+\frac{1}{\Psi}\frac{\partial\Psi}{\partial P}\frac{dP}{d\theta}-\pi(P)D(k,P,\zeta) & \text{for } q=0,\\ \frac{1}{\varpi}+\frac{1}{\Psi}\frac{\partial\Psi}{\partial k}\frac{dk}{d\theta}+\frac{1}{\Psi}\frac{\partial\Psi}{\partial P}\frac{dP}{d\theta} & \text{for } q=1,\end{cases}$$

where $\pi(P)$ is the probability of the shock q that causes the damage $D(k,P,\zeta)$.

The jump of the value function due to the shock

- Because there are different steady states before ($q = 0$) and after ($q = 1$) the shock, the multiplier ϖ jumps at the occurrence of the shock:

$$\varpi\big|_{q=0} = \frac{1}{1 + \pi^* D^*} < 1, \quad \varpi\big|_{q=1} = 1,$$

  where superscript (*) denotes the steady state, $\pi^* \doteq \pi(P^*|_{q=0})$ is the probability of the disaster, $D^* \doteq D(k^*|_{q=0}, P^*|_{q=0}, \xi)$ the relative damage, and $\pi^* D^*$ the expected relative damage in the steady state before the occurrence of the shock.

Taxing the number of children

- The maximization by υ in the Bellman equation yields

$$\frac{\partial \Upsilon}{\partial \upsilon} = 0 \quad \Leftrightarrow \quad f = \frac{\alpha c}{w + k}.$$

- Comparing the optimal fertility rate f for the household, (2), and the government yields the optimal parental tax x:

$$\frac{\alpha c}{w + k + x} = f = \frac{\alpha c}{w + k} \quad \Leftrightarrow \quad x = 0.$$

Proposition

The parental tax per child can be eschewed, i.e., its optimal value is zero, $x = 0$.

This tax is not needed, because the other taxes can alone eliminate the externality though pollution and the mortality rate.

Taxing capital income, 1

- The maximization by τ in the Bellman equation yields

$$\frac{\partial \Upsilon}{\partial s} = 0 \quad \Leftrightarrow \quad r = f - m + \frac{\rho + m}{\varpi} = f - m + (\rho + m)(1 + \pi^* D^*).$$

- Because the difference between the fertility and mortality rates, $f - m$, is very small relative to the sum of the rate of time preference and the mortality rate, $\rho + m$, this result can be approximated as follows:

$$\frac{r}{\rho + m} = \frac{f - m}{\rho + m} + 1 + \pi^* D^* \approx 1 + \pi^* D^*.$$

Taxing capital income, 2

- Inserting this approximation into the household's response (4) and noting the value of the multiplier ϖ lead to

$$\tau = \frac{r-f-\rho}{r} = \left(\frac{1}{\varpi}-1\right)\frac{\rho+m}{r}$$
$$= \begin{cases} \frac{\rho+m}{r}\pi^* D^* = \frac{\pi^* D^*}{1+\pi^* D^*} & \text{for } q=0, \\ 0 & \text{for } q=1. \end{cases}$$

Proposition

Before the disaster, the tax on capital income increases with the expected loss for the disaster, $\pi^* D^*$, and it can be approximated by $\tau|_{q=0} \approx \frac{\pi^* D^*}{1+\pi^* D^*} \in (0,1)$. After the disaster, that tax can be eschewed, $\tau|_{q=1} = 0$.

Taxing the demand for health care, 1

- The ratio of the household's and the government's value functions is given by $\Upsilon/\Lambda = \Psi/\Phi = \varpi/\vartheta = \varpi$.
- With $\Upsilon = \varpi\Lambda$, $1/z' = \chi'$ and the household's response (3), the maximization by m in the Bellman equation yields

$$\frac{\partial\Upsilon}{\partial m} = 0 \quad \Leftrightarrow$$

$$k + z' = \frac{1}{\sigma c^{\sigma-1} f^{\alpha\sigma}} \frac{\Upsilon}{\rho + m} = \frac{1}{\sigma c^{\sigma-1} f^{\alpha\sigma}} \frac{\varpi\Lambda}{\rho + m} = \varpi(bz' + k + z')$$

$$\Leftrightarrow$$

$$b = \left(\frac{1}{\varpi} - 1\right)\left(\frac{k}{z'} + 1\right) = \begin{cases} (k\chi' + 1)\pi^* D^* > 0 & \text{for } q = 0, \\ 0 & \text{for } q = 1. \end{cases}$$

Taxing the demand for health care, 2

Proposition

Before the disaster, the tax on health care should be in proportion $(k\chi' + 1)$ *to the expected loss for the disaster,*

$$b|_{q=0} = (k\chi' + 1)\pi^* D^* > 0,$$

where k *is capital per head and* χ' *the marginal efficiency of personal health care* h *in decreasing the mortality rate* m*. After the disaster, that tax can be eschewed,* $b|_{q=1} = 0$*.*

Conclusions

- It is necessary to set the taxes on capital income and the demand for health care before the disaster to implement Pareto optimality, i.e., to internalize the externality though pollution and mortality. All other taxes, except the revenue raising-poll tax, can be eschewed.
- There are two reasons for this sharp result.
    1. There is no incremental contribution of pollution to the mortality rate. Thus, there is only the precautionary motive, but no maintenance motive for the government to intervene.
    2. The mortality rate can be decreased by spending on health care. Then, the mortality shock turns into a negative income effect that is equivalent to an increase in the cost of health care.