Vintages of windmills

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Motivation: Learning-by-doing & spillovers

- On- and offshore wind energy: One of the most important pillars of the transition to renewable electricity in many countries.
- With learning-by-doing, later vintages are more productive. Spillovers constitute a positive externality.

(Nykvist & Nilsson 2015, Rubin et al. 2015, Wright 1936)
Motivation: Heterogenous site quality & site scarcity


Newly installed capacity in Germany

from: Tagesschau, 25.10.2019, “Ökostrom auf Rekordkurs”
Motivation: Repowering

“Repowering of a wind farm is the process of replacing existing wind turbines with new turbines [...] resulting in a net increase of the power generated.”

(del Río 2011)
Motivation: Repowering

“Repowering of a wind farm is the process of replacing existing wind turbines with new turbines [...] resulting in a net increase of the power generated.”
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For us: “Premature” replacement, before the old turbine’s productivity goes to zero due to depreciation.
Research questions:

1. What drives the optimal repowering path over time and how does it look like?
2. How does the optimal repowering age differ from the market induced repowering age under learning-by-doing?
3. (How) Will this gap between private and public optimum repowering age depend on site quality?
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• static model: Learning-by-doing under site scarcity gives rise to interesting subsidy differentiation between technologies
  (Lancker & Quaas (2019))

• dynamic view: Vintage capital modeling with land as resource
  ▶ Mitra & Wan, 1985: Faustmann periodic solution and the sustained yield
  ▶ concave utility/ profit function, discounting, discrete time

• technology vintage modeling in the energy sector:
  ▶ Cai et al, 2018: energy efficiency of household appliance vintages
  ▶ Jacobsen, 2000: input efficiency of power plant vintages
  ▶ Conrad & Nostbakken, 2018: exogenous technical progress, site heterogeneity: derive condition for saving up high quality sites for later vintages, but abstract from depreciation and reinvestment
Model: Two vintages & exogenous technical progress

\[ \sum_{t=1}^{\infty} b^t (\alpha_t x_{1t} + \alpha_{t-1} x_{2t} - \psi x_{1,t+1}) \]

- linear utility/ profit function
- full depreciation only after period 2
- \( x_{1t} + x_{2t} = 1 \)
- start with full young vintage \( x_{11} = 1 \)

\( t, s \) time, vintage
\( x_{st} \) land used for vintage \( s \) at \( t \)
\( \alpha_t \) productivity
\( \psi \) marginal turbine cost
\( b^t \) discount factor

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Model: Two vintages & exogenous technical progress

Objective function

\[ \sum_{t=1}^{\infty} b^t (\alpha_t x_{1t} + \alpha_{t-1} x_{2t} - \psi x_{1,t+1}) \]

Constraints

\[ x_{2t} + z_t = x_{1,t+1} \]
\[ x_{1t} - z_t = x_{2,t+1} \]
\[ z_t \geq 0 \]
\[ z_t - x_{1t} \leq 0 \]

- \( t, s \) time, vintage
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- \( \alpha_t \) productivity
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- \( b^t \) discount factor
- \( z_t \) repowered land
Model: Two vintages & exogenous technical progress

Objective function

\[ \sum_{t=1}^{\infty} b^t (\alpha_t x_{1t} + \alpha_{t-1} x_{2t} - \psi x_{1,t+1}) \]

Specify

\[ \alpha_t = 1 - \gamma^t \] with \( 0 < \gamma < 1 \),

\( \gamma \): speed of technical progress
(faster for smaller \( \gamma \))

Steady state: no repowering.

t, s: time, vintage
x_{st}: land used for vintage s at t
\( \alpha_t \): productivity
\( \psi \): marginal turbine cost
\( b^t \): discount factor
\( z_t \): repowered land
Model: Two vintages & exogenous technical progress

Objective function

$$\sum_{t=1}^{\infty} b^t (\alpha_t x_{1t} + \alpha_{t-1} x_{2t} - \psi x_{1,t+1})$$

Specify

$$\alpha_t = 1 - \gamma^t \text{ with } 0 < \gamma < 1,$$

Stop repowering at:

$$t^* = \frac{\ln \left( \frac{1 + b \gamma}{1 + b} \frac{\psi}{b(1-\gamma)} \right)}{\ln(\gamma)}$$

$t, s$ time, vintage

$x_{st}$ land used for vintage $s$ at $t$

$\alpha_t$ productivity

$\psi$ marginal turbine cost

$b^t$ discount factor

$z_t$ repowered land
Results: Two vintages & exogenous technical progress

$t^*$: Time at which repowering stops (stylized ex.)

- $\psi = 0.2$
- $\psi = 0.4$
- $\psi = 0.6$
- $\psi = 0.8$
Model: \( N \) vintages & exogenous technical progress

**Objective function**

\[
\sum_{t=1}^{\infty} b^t \left( \sum_{s=1}^{N} \alpha_{t+1-s} x_{st} - \psi x_{1,t+1} \right)
\]

Stop repowering at:

\[
t^*_N = \ln \left( \frac{\psi \left( \frac{1-b}{1-b^N} \right) (1 - b^N \gamma^N)}{b \gamma \left( \frac{1-\gamma^{N-1}}{\gamma^{N-1}} - \frac{b(1-\gamma)}{1-b} (1 - b^{N-1}) \right)} \right) / \ln(\gamma)
\]

**Symbols:**

- \( t, s \) time, vintage
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1. What drives the optimal repowering path over time and how does it look like?
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Model: 2 vintages & Learning-by-doing

**Objective function**

\[
\sum_{t=1}^{\infty} b^t \left( \sum_{s=1}^{2} \alpha(Y_{t+1-s}) x_{st} - \psi(z_t + x_{2t}) \right)
\]

*with*

\[
Y_{t+1} = \sum_{i=1}^{t} x_{1i}
\]

\[\Rightarrow \text{assume } \alpha_t = 1 - \gamma^{1+Y_t}\]

With symmetric sites, \(z_t < x_{1t}\) (incomplete repowering) can only occur in one marginal period.

\(t, s\) time, vintage

\(x_{st}\) land used for vintage \(s\) at \(t\)

\(\alpha(Y_t)\) productivity

\(\psi\) marginal turbine cost

\(b^t\) discount factor

\(z_t\) repowered land
Model: 2 vintages & Learning-by-doing

**Objective function**

$$\sum_{t=1}^{\infty} b^t \left( \sum_{s=1}^{2} \alpha(Y_{t+1-s}) x_{st} - \psi(z_t + x_{2t}) \right)$$

At $t^*_{LBD}$: Shadow price differential $\varphi_t = \psi$:

$$\varphi_{t-1} = b \left( \gamma^{Y_{t-1}} - \gamma^{1+Y_t} + \psi - \varphi_t \right)$$

$$+ b (-\ln \gamma) \sum_{i=t}^{\infty} b^{i+1-t} \gamma^{1+Y_{i+1}} (x_{1,i+1} + b x_{2,i+2})$$

- $t, s$ time, vintage
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Model: 2 vintages & Learning-by-doing

**Objective function**

\[
\sum_{t=1}^{\infty} b^t \left( \sum_{s=1}^{2} \alpha(Y_{t+1-s}) x_{st} - \psi(z_t + x_{2t}) \right)
\]

Stop repowering at:

\[
t^*_{LBD} = \ln \left( \frac{\frac{\psi}{1 + b} + \frac{b \gamma}{b \gamma}}{1 - \gamma + \frac{b(1 + b)(-\ln \gamma)\gamma}{1 - b^2 \gamma}} \right) \frac{1 + b \gamma}{b \gamma} \ln \gamma
\]

- \(t, s\) time, vintage
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### Learning-by-doing, market solution and subsidy

**FOCs**

**$SP : \alpha(Y_t) - \frac{\lambda_{1,t-1}}{b} + \lambda_{2t}$**

\[
+ \sum_{i=t}^{\infty} b^{i+1-t} (\alpha'(Y_{i+1})(x_{1,i+1} + bx_{2,i+2})) = 0
\]

**$F : \alpha(Y_t) - \frac{\lambda_{1,t-1}}{b} + \lambda_{2t} = 0$**

**Consequences:** $t^*_{LBD} - t^*$  
Reduce $\psi$ by subsidy $s_t$ in the meantime:

\[
s_t = \frac{\psi}{1+b} - \frac{b(1-\gamma)}{1+b\gamma}\gamma^t \text{ with } \frac{\partial s_t}{\partial t} > 0, \frac{\partial s_t}{\partial b} < 0, \frac{\partial s_t}{\partial \gamma} \text{ amb.}
\]
Learning-by-doing, market solution and subsidy

discount rate $r = 3\%$, marginal cost $ψ = 0.015$

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Learning-by-doing, market solution and subsidy

discount rate $r = 3\%$, marginal cost $\psi = 0.015$
Conclusions

- we use a model from forest economics to model optimal land-use for wind turbines with technical progress
- learning-by-doing (LBD) in a two vintage model
  - increases the time until repowering stops
  - demands a subsidy to internalize the spillover externality
  - speed of technical progress has a non-monotonous impact on the repowering subsidy
- next steps:
  - heterogeneous quality of sites
  - decreasing marginal utility of energy consumption
Thank you for your attention!
Cai, W., Grant, H., Pandey, M., 2018. Vintage Capital, Technology Adoption and 
Electricity Demand-Side Management. The Energy Journal 39 (2). 
del Río, P., Silvosa, A. C., Gómez, G. I., 2011. Policies and design elements for 
the repowering of wind farms: A qualitative analysis of different options. Energy 
Mitra, T., Wan, H. Y., 1985. Some theoretical results on the economics of fore-
Salo, S., Tahvonen, O., 2002. On equilibrium cycles and normal forests in optimal 
harvesting of tree vintages. Journal of Environmental Economics and Manage-
ment 44 (1), 1 – 22. 
Salo, S., Tahvonen, O., 2003. On the economics of forest vintages. Journal of 
Economic Dynamics and Control 27 (8), 1411 – 1435. 
Salo, S., Tahvonen, O., 2004. Renewable resources with endogenous age clas-
ses and allocation of land. American Journal of Agricultural Economics 86 (2), 
513–530.