A time-domain kernel independent fast multipole method based on strong recursive skeletonization using deterministic and randomized interpolative decomposition

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Abstract

Chebyshev Interpolation based fast multipole method (ChebyshevFMM) approximate far-field interactions through a tensor-product representation requiring p^3 interpolation points in three dimensions for order p, and are particularly effective for low frequency kernels[1, 2]. For high frequency oscillatory kernels, such as Helmholtz kernel, far-field interactions are no longer low rank, and their numerical rank increases with wavenumber and cluster size, necessitating higher interpolation order to maintain a prescribed accuracy [3, 4]. This is particularly relevant in convolution quadrature boundary element formulations, where the integral equations are solved at a sequence of complex Laplace frequencies, for which a single interpolation order is possibly insufficient.

In contrast, recursive skeletonization based fast multipole method (SkelFMM) approximates far-field interactions, within prescribed tolerance, by an equivalent set of skeleton points selected through interpolative decomposition (ID) to reproduce the effect of original sources in the far-field [5]. In this work, ChebyshevFMM and SkelFMM are investigated within Runge-Kutta convolution quadrature (CQ) boundary element framework for three dimensional transient acoustics, with particular emphasis on their low rank approximation behavior across CQ generated complex frequency spectrum, computational time, and storage requirements. Furthermore, deterministic and randomized ID within SkelFMM framework are compared to assess the efficiency of the skeletonization process. Numerical results demonstrate lower computational time and storage requirements for SkelFMM than for ChebyshevFMM.

Keywords: fast multipole method; interpolative decomposition; acoustics; boundary element method; convolution quadrature

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