Johann Radon Institute for Computational and Applied Mathematics Austrian Academy of Sciences (ÖAW)

PhD defence

Symbolic Computation

Tensor reduction systems for rings of linear operators

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Abstract

In order to facilitate symbolic computations with systems of linear functional equations (e.g. integro-differential, differential time-delay, recurrence, ...), we require an algebraic framework for such systems which enables effective computations in corresponding rings of operators. For finding such frameworks, instead of using parametrized Gröbner bases in free algebras as has been done so far in the literature, we exploit and generalize Bergman's basis-free analog in tensor rings, which often allows for a finite reduction system with unique normal forms. In short, we propose a general algorithmic approach to noncommutative operator algebras generated by additive operators.

In this thesis, a self-contained treatment of reduction systems in tensor rings including preliminaries such as tensor product of bimodules, tensor rings, and basics of term rewriting is presented. We apply our generalization of Bergman's setting to construct the ring of integro-differential operators (IDO) having (noncommutative) matrix coefficients. Moreover, we extend this ring to the ring of integro-differential operators with linear substitutions (IDOLS). As a new instance of the tensor setting, we construct the ring of inversive sum-difference operators (SDO). For finding normal forms in these rings, we complete their defining reduction systems to obtain confluent ones. These normal forms allow to solve operator equations by ansatz.

We show that, by applying tensor reduction systems, results like the method of variation of constants in the ring of IDO and the method of steps in the ring of IDOLS can be found and proven in an automated way. Using normal forms in the ring of IDO, we illustrate how to compute Green's operators for first-order systems of linear ordinary boundary problems. Moreover, in the ring of IDOLS, we partly automatize certain computations related to differential time-delay systems, e.g. Artstein's transformation and its generalization. Using the Mathematica package TenReS, we implement the rings of IDO, IDOLS, SDO, and corresponding normal forms. We also use these implementations to perform computations for the applications treated in this thesis.