Johann Radon Institute for Computational and Applied Mathematics
der Österreichischen Akademie der Wissenschaften

Group Seminar

Group: Multivariate Algorithms and Quasi-Monte Carlo Methods

Tuesday, March 20, 2018, 13:30

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Composite polynomials in second order linear recurrence sequences

Various questions about the possible ways of writing a polynomial as the composition of lower-degree polynomials $f = g \circ h$ have been studied, starting with Ritt in the 1920's. On the other hand, (polynomial) linear recurrence sequences often appear in connection with Diophantine problems. In the talk we shall discuss some results from joint work with Clemens Fuchs and Dijana Kreso, combining these two areas. Let $(G_n)_{n=0}^{\infty} \in \mathbb{C}[x]$ be a minimal non-degenerate simple binary linear recurrence sequence of polynomials, defined by $A_0, A_1, G_0, G_1 \in \mathbb{C}[x]$ and the relation

$$G_{n+2}(x) = A_1(x)G_{n+1}(x) + A_0(x)G_n(x), \ \ n \in \mathbb{N}.$$

Under an additional assumption, we show that if $G_n(x) = g \circ h(x)$ holds for some $n \in \mathbb{N}$ and h is indecomposable, then either h is of special shape or $\deg g$ is bounded by a constant not depending on n. Moreover, we give sufficient conditions on A_0, A_1, G_0, G_1 such that the assumption in question is satisfied. The proof of the main result relies on a theorem by Brownawell and Masser, giving an upper bound on the height of solutions of certain S-unit-equations over function fields, whereas the second statement takes a Galois-theoretic approach to decomposition questions by using information about the monodromy group of a polynomial.