THE DISCRETIZATION PROBLEM FOR CONTINUOUS FRAMES

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Abstract. A frame for an infinite dimensional separable Hilbert space H is a sequence of vectors $(x_n)_{n=1}^{\infty}$ in H such that there exists constants $0 < A \leq B < \infty$ with

$$A\|x\|^2 \leq \sum_{n\in\mathbb{N}} |\langle x, x_n\rangle|^2 \leq B\|x\|^2 \qquad \text{for all } x \in H.$$

A continuous frame is the natural analougue where we integrate the frame coefficients instead of summing. That is, a continuous frame for an infinite dimensional separable Hilbert space H is a collection of vectors $(x_t)_{t\in M}$ in H indexed by a σ-finite measure space (M, μ) such that there exists constants $0 < A \leq B < \infty$ with

$$A\|x\|^2 \leq \int_M |\langle x, x_t\rangle|^2 d\,\mu(t) \leq B\|x\|^2 \qquad \text{for all } x \in H.$$

The discretization problem asks when a continuous frame may be sampled to obtained a discrete frame. We solve this by proving that if $(x_t)_{t\in M}$ is a continuous frame for H which is bounded in norm, then there exists a sequence $(t_n)_{n=1}^{\infty}$ in M such that $(x_{t_n})_{n=1}^{\infty}$ is a frame for H.

We will discuss as well Marcus, Spielman, and Srivastava's solution of the Kadison-Singer problem and how we use it to solve the discretization problem as well as possible further avenues for research. This is joint work with Darrin Speegle.

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