Quantum Information Experiments with Multiple Photons in One and High-Dimensions: Concepts and Experiments

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Abstract: Starting with the first experimental generation of a Greenberger-Horne-Zeilinger entangled state in three dimensions we show how to describe photonic quantum experiments using graph theory. This novel link promises exciting new applications ranging from multi-photon high-dimensionally entangled states to special purpose quantum simulation.

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1. Introduction

In this short paper, I will outline the main content of my planned talk at QIM 5 in Rome from 4-6 April 2019. I will present our recent experimental as well as conceptual and theoretical results on multi-photonic quantum experiments including one- and high-dimensional systems. We start with the first experimental realization of a three-dimensionally entangled Greenberger-Horne-Zeilinger quantum state [1]. In the course of this work, we discovered a novel link between photonic quantum experiments and graph-theory. Exploiting this link leads to exciting new concepts. For example, how to produce multi-photon high-dimensionally entangled quantum states experimentally or an entirely new scheme to perform special purpose quantum simulation similar to Boson-Sampling.

2. Experimental Greenberger-Horne-Zeilinger Entanglement Beyond QuBits

Entanglement is the workhorse of quantum technologies today, with applications ranging from fault-tolerant quantum computation to device-independent quantum communication. The entanglement of more than two quantum particles, commonly known as GHZ entanglement, not only opened the door to the strongest test of local-realism, but also forms a key ingredient of such technologies. Since the discovery of the GHZ theorem, experimental research on multi-particle entanglement has mainly focused on two dimensional quantum systems or qubits, with realisations in a diverse range of physical systems including ions, photons and super-conducting circuits. In all of these systems, a general procedure for increasing the number of entangled particles exists. For example, for photons, a particularly simple experimental scheme uses polarising beam-splitters in combination with post-selection to produce arbitrarily high numbers of photons entangled in a GHZ manner.

However, no experiment till date has been able to create entanglement that is simultaneously high-dimensional and multipartite. This is mainly because we are still lacking a conceputal understanding of how to create such states experimentally. In part, this can also be attributed to the lack of technologies for manipulating high-dimensional quantum states experimentally. Going beyond qubit GHZ entanglement thus poses a significant challenge, overcoming which would lead to exciting new applications for quantum technologies and fundamental tests of quantum mechanics, such as the recent theoretical results on GHZ-like contradictions of local-realism in higher dimensions.

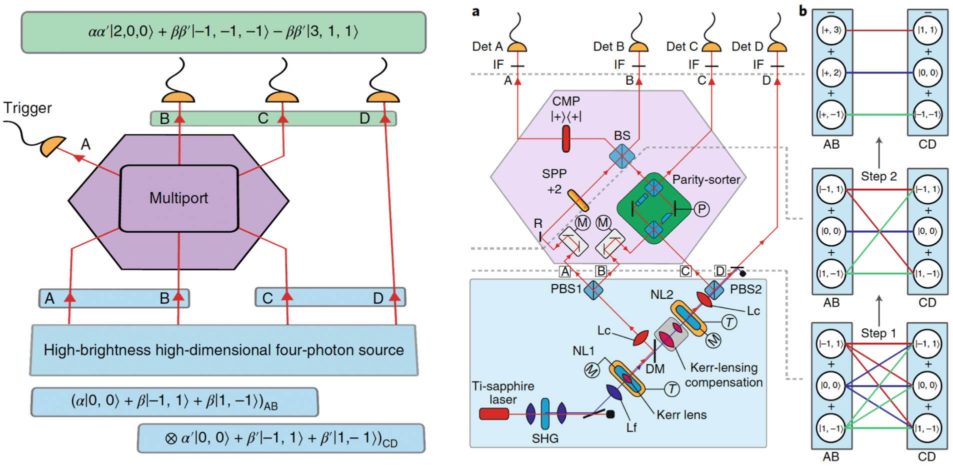

Fig. 1. **Concept of three-dimensional GHZ entanglement creation:** A high-brightness four-photon source creates two pairs of three-dimensionally entangled photons. The multi-port transforms three incoming photons coherently and simultaneously in its 27-dimensional operational space. Simultaneous detection of one photon in each detector results in the creation of a three-dimensional GHZ state, as depicted on top.

We show the first experimental realisation of a multi-particle entangled state using the orbital angular momentum of light, where all photons are genuinely entangled in a high-dimensional manner. Our experimental scheme employs a new type of multi-port (see Fig.1) that operates coherently and simultaneously on three photons in three dimensions each, forming a 27-dimensional operational space. In fact, the experimental generation of a three-dimensional GHZ state necessitates the manipulation of more than two photons in a higher-dimensional space, as we show by exploiting a link between quantum experiments and graph theory. Interestingly, the design of our experiment was originally suggested by the computer algorithm Melvin, which was designed for finding new experimental techniques in quantum mechanics.

3. Quantum Experiments and Graphs

We present a conceptually new approach to describe state-of-the-art photonic quantum experiments using Graph Theory [2]. There, the quantum states are given by the coherent superpositions of perfect matchings. The crucial observation is that introducing complex weights in graphs naturally leads to quantum interference. The new viewpoint immediately leads to many interesting results, some of which we present here. Firstly, we identify a new and experimentally completely unexplored multiphoton interference phenomenon. Secondly, we find that computing the results of such experiments is P-hard, which means it is a classically intractable problem dealing with the computation of a matrix function Permanent (see Fig.2) and its generalization Hafnian. Thirdly, we explain how a recent no-go result applies generally to linear optical quantum experiments, thus revealing important insights to quantum state generation with current photonic technology. The uncovered bridge between quantum experiments and Graph Theory offers a novel perspective on a widely used technology, and immediately raises many follow-up questions.

References

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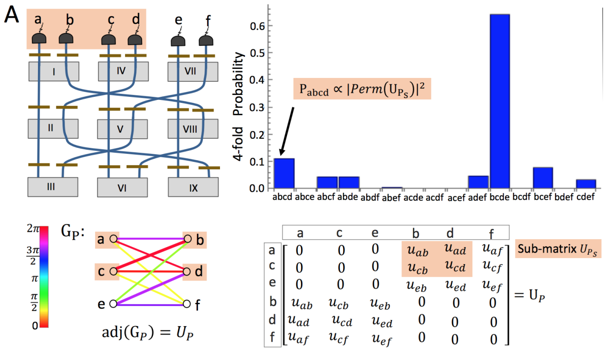

$$\begin{array}{c|cccccc} & a & c & e & b & d & f \\ \hline a & 0 & 0 & 0 & u_{ab} & u_{ad} & u_{af} \\ c & 0 & 0 & 0 & u_{cb} & u_{cd} & u_{cf} \\ e & 0 & 0 & 0 & u_{eb} & u_{ed} & u_{ef} \\ b & u_{ab} & u_{cb} & u_{eb} & 0 & 0 & 0 \\ d & u_{ad} & u_{cd} & u_{ed} & 0 & 0 & 0 \\ f & u_{af} & u_{cf} & u_{ef} & 0 & 0 & 0 \end{array} = U_P$$

Fig. 2. An experiment consisting of 9 nonlinear crystals (with labels I-IX) and 18 phase shifters (gold lines). They are arranged such that paths a, c and e are parallel. All the crystals are pumped coherently and can produce indistinguishable photon pairs. The pump power is set in such a way that two crystals can produce photon pairs. One can adjust the phase shifters and pump power to change the phases and transition amplitudes. The corresponding graph GP and its adjacency matrix adj(GP) for the setup are at the bottom. The ordering of the column and row are (a, c, e, b, d and f). Calculating four-fold coincidences in one specific subset path (a, b, c and d) of four outputs relates to summing the weights of perfect matchings of the sub-graph with related vertices, which corresponds to computing the matrix function Permanent of sub-matrix UPs highlighted in orange. Thus, the probability that a certain arrangement of detectors click Pabcd is proportional to the $|Perm(UPs)|^2$. All the combinations for the four-fold coincidences are depicted in the histogram.