# EINSTEIN-PODOLSKY-ROSEN CORRELATIONS IN HIGHER DIMENSIONS

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## ABSTRACT

Using multiport beam splitters it will be possible to study Einstein-Podolsky-Rosen correlations in higher dimensional Hilbert space. As an explicit example we present the design and theory of a tritter, which is a multiport beam splitter with three input ports and three output ports, such that any amplitude incident at one input port is distributed equally over the output ports. We will then show the results for a two-photon, two-tritter experiment, where novel Einstein-Podolsky-Rosen correlations occur.

## 1. Introduction

All experimental work concerning the Einstein-Podolsky-Rosen Paradox[1] and Bell's theorem[2] thus far is restricted to two-particle (in most cases two-photon) entangled states where the correlations can effectively be described by restricting the analysis to a Hilbert space of dimension 2 for each particle. These states can be two polarization states as proposed initially by Bohm[3] and first employed in an experiment by Freedman and Clauser[4], they can be two momentum eigenstates as in the experiment proposed by Horne and Zeilinger[5] and performed first by Rarity and Tapster[6], or, they can be two states which took beam paths of markedly different length on their way from the source to the detector as proposed by Franson[7]. This latter experiment has now been performed by various groups[8], the most conclusive experiment which showed a striking violation of a Bell-type inequality is due to Kwiat, Steinberg and Chiao[9].

There are two obvious routes for generalization. One is to consider more than two particles, the other is to analyze the case of more than two states available to each particle. The generalization to more than two particles has led to some new insight into the difference

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between quantum mechanics and local realistic theories[10]. But, due to the unavailability of coherent multi-particle sources this has not as yet resulted in an experiment.

In the present paper we would like to focus our analysis on another generalization. This is the case where each particle has more than two states available. The correlations are then defined in Hilbert spaces of higher dimension[11]. It is obvious that a possible route to generalizing EPR correlations to systems of higher dimension would be to investigate spin correlations between particles with spin-1 or higher (with the obvious and notable exception of the photon or other massless particles which have only 2 polarisation states.) Again, since at present there exist no sources for correlated particles of higher spin, such investigations based on spin correlations are purely theoretical to date[12].

This paper shows how to obtain such EPR correlations in more than two dimensions in real experiments. Such experiments are based on both the availability of parametric down-conversion as a source for highly correlated two-photon states[13] and on the use of multi-port devices[14]. Finally, we present some theoretical predictions for the novel correlations expected.

## 2. The Beam Splitter as a Four-Port Device

The beam splitter is a central element of many experiments in quantum optics. A general beam splitter has two input ports and two output ports (Fig. 1). Formally it may be described by a unitary operator in a two-dimensional Hilbert space. We should note here that for the present paper we deliberately adopt an explicit Hilbert space formalism because it is equally well suited for describing a beam splitter operating for any type of particle, be it electrons, photons, atoms or neutrons, to name just those types of radiation for which quantum interference experiments with beam splitters have been performed so far.

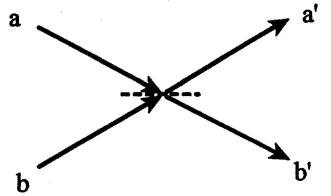


Fig. 1: A general beam splitter has two input ports and two output ports.

The general beam splitter pure input state is a superposition

$$|\psi\rangle = \psi_a|a\rangle + \psi_b|b\rangle \tag{1}$$

where $|a\rangle$ and $|b\rangle$ describe a particle in beam $a$ or $b$ (see Fig. 1) respectively. We assume the normalization $\psi_a\ \psi_a{}^* + \psi_b\ \psi_b{}^* = 1$. Likewise the general output state is the superposition

$$|\psi'\rangle = \psi'_a|a'\rangle + \psi'_b|b'\rangle \tag{2}$$

in obvious notation. Input and output states may equally well be written in matrix notation as

$$\psi = \begin{pmatrix} \psi_a \\ \psi_b \end{pmatrix}, \quad \psi' = \begin{pmatrix} \psi'_a \\ \psi'_b \end{pmatrix}. \tag{3}$$

The general beam splitter operator $U$ then couples $\psi'$ to $\psi$, $\psi' = U\psi$ with $U^{+}U = I$.

We restrict ourselves now to 50-50 beam splitters. This means that a particle incident at any of the two input ports of a symmetric beam splitter has the same probability $p = 1/2$ to be found in any of the two output ports. It is well known that such a beam splitter is defined only up to arbitrary phase factors in the input and output ports[15].

Two possible 50-50 beam splitter operators are for example

$$U_t = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad \text{or} \quad U_s = \frac{1}{\sqrt{2}}\begin{pmatrix} i & 1 \\ 1 & i \end{pmatrix} \tag{4}$$

where $U_t$ represents a time-symmetric beam splitter and $U_s$ represents a spatially symmetric one. The two beam splitters can be converted into each other using $\pi$ phase shifts in one input and one output port, i.e.

$$U_t = \begin{pmatrix} -i & 0 \\ 0 & 1 \end{pmatrix} U_s \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}. \tag{5}$$

The two beam splitter operators imply different transition rules for incident beams. These are

$$\begin{aligned} |a\rangle &\Rightarrow \frac{1}{\sqrt{2}}\{|a'\rangle + |b'\rangle\} \qquad |b\rangle \Rightarrow \frac{1}{\sqrt{2}}\{|a'\rangle - |b'\rangle\} \text{ for } U_t, \\ |a\rangle &\Rightarrow \frac{1}{\sqrt{2}}\{i|a'\rangle + |b'\rangle\} \qquad |b\rangle \Rightarrow \frac{1}{\sqrt{2}}\{|a'\rangle + i|b'\rangle\} \text{ for } U_s. \end{aligned} \tag{6}$$

The first beam splitter implies no phase change upon reflection from one side while reflection from the other side implies a phase change of $\pi$. The second beam splitter operator implies that both reflected beams acquire a phase shift of $\pi/2$ upon reflection.

We note here that beam splitters are just special cases of 4-port devices. Another example of a 4-port device would be a Mach-Zehnder interferometer.

## 3. Two-Particle Two-State Systems

Using these rules it is now easily possible to calculate the results of a two-particle two-state EPR-Bell experiment as shown in Fig. 2. A source emits two particles in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\{|a\rangle|c\rangle + |b\rangle|d\rangle\}. \tag{7}$$

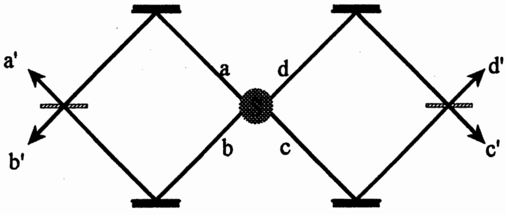


Fig. 2: Principle of a two-particle, two-state EPR-Bell experiment using beam splitters.

Here and below the first ket in a product always refers to particle 1 and the second to particle 2. Also, e.g., $|a\rangle|c\rangle$ implies the tensor product $|a\rangle\otimes|c\rangle$ etc. The beams $a$, $b$, $c$, $d$ may then be subject to the phase shifts $\alpha$, $\beta$, $\chi$, $\delta$ such that the state becomes

$$|\psi\rangle=\frac{1}{\sqrt{2}}e^{i(\alpha+\gamma)}\left\{|a\rangle|c\rangle+e^{i\chi}|b\rangle|d\rangle\right\} \tag{8}$$

with $\chi=\beta+\delta-\alpha-\gamma$. Applying now the beam splitter rules (6) and, analogously,

$$|c\rangle\Rightarrow\frac{1}{\sqrt{2}}\left\{|c'\rangle+|d'\rangle\right\} \qquad |d\rangle\Rightarrow\frac{1}{\sqrt{2}}\left\{|c'\rangle-|d'\rangle\right\} \tag{9}$$

one obtains for the joint probabilities for two detectors to register the particles in coincidence

$$\begin{aligned} p(a',c')=p(b',d')=\frac{1}{2}\cos^2(\chi/2) \\ p(a',d')=p(b',c')=\frac{1}{2}\sin^2(\chi/2)\cdot \end{aligned} \tag{10}$$

Thus, perfect correlations arise for

$$\chi=n\pi\cdot \tag{11}$$

For odd $n$ detector $a'$ fires in coincidence with detector $d'$ and detector $b'$ fires in coincidence with detector $c'$ while for even $n$ the coincidences are $a'-c'$ and $b'-d'$. These two different types of coincidences are represented in Fig. 3. In other words, for these parameter settings the path taken by a particle after its beam splitter is an Einstein-Podolsky-Rosen element of reality, i.e. firing of any one individual detector for one particle allows one to predict with certainty which detector will register the other particle.

These perfect correlations can be characterized via a value-assignment procedure introduced by Bell. The possible results obtained on either side are named $A$ and $B$, and they are assigned the values $\pm 1$. It then follows that the two cases of perfect correlation are signified by $AB=+1$ and $AB=-1$ respectively. We call these values Bell numbers. We notice that one of the beam splitter operator representations ($U_t$) just contains Bell numbers (+1 and -1 for the two dimensional case). It will be seen later that for multiports the generalization of

Bell's value assignment procedure is quite interesting. Furthermore, in any dimension there are always multiports whose unitary representation contains only Bell numbers.

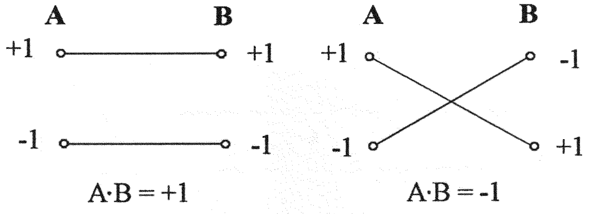


Fig. 3: Possible perfect correlations for the case of an experiment as shown in Fig. 2. The results $A, B$ on either side can be +1 or -1, depending on which detector in which outgoing beam registers a particle. The perfect correlations can be signified by either $A \cdot B = +1$ or $A \cdot B = -1$.

We should mention that the results of this section are basically known. They were repeated here in order to prepare the reader for the less familiar situations in the following sections. An experimentally available source which prepares the two particles in the entangled state of Eq. (7) is a non-linear crystal where through the process of spontaneous parametric down-conversion an incident photon may split into 2 photons of lower energy.

## 4. The Tritter as an Example of a Multiport Device

In this section we first introduce the general concept of multiports and then we give some explicit examples. A general multiport has $L$ input ports and $M$ output ports* and is called $N$-port $(N = M + L)$ . For simplicity we restrict our considerations to symmetric $N$-ports which are defined as having an equal number of input ports and output ports $(L = M = N/2)$ and, furthermore, which operate such that a single particle incident on any individual input port has equal probability (i.e. $p = 1/M = 2/N$) to be found in any specific output port. This is the generalization of the generic beam splitter discussed in section 2 above. We propose to call symmetric multiports "Critters" and specifically a critter with $L = M = 3$ is called a Tritter, one with $L = M = 4$ is a Quitter[16] etc.

Lossless symmetric multiports (critters) can be represented by unitary operators in an $M$-dimensional Hilbert space. Again, as was the case for the conventional beam splitter, there are many physically possible critters, but, as opposed to the beam splitter case, it is not always possible to transform all types of a specific critter (i.e. symmetric $N$-port with a given $N$) into each other by merely supplying external phase shifters or relabelling output ports[17].

Let us consider explicitly the tritter. The general input and output states are (Fig. 4)

$$\begin{aligned} |\psi\rangle &= \psi_a|a\rangle + \psi_b|b\rangle + \psi_c|c\rangle \\ |\psi'\rangle &= \psi'_a|a'\rangle + \psi'_b|b'\rangle + \psi'_c|c'\rangle. \end{aligned} \tag{12}$$

or, in matrix notation,

*In general some physical ports can work both as input and output ports (viz. the Michelson interferometer).

$$\psi = \begin{pmatrix} \psi_a \\ \psi_b \\ \psi_c \end{pmatrix} \quad \text{and} \quad \psi' = \begin{pmatrix} \psi'_a \\ \psi'_b \\ \psi'_c \end{pmatrix}. \tag{13}$$

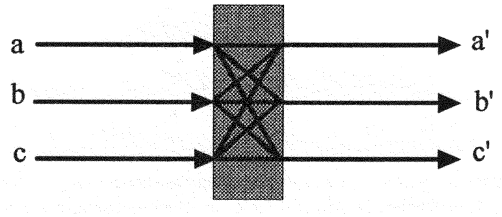


**Fig. 4: A generic tritter is devised with three input ports and three output ports such that an amplitude incident on any one of the input port excites any of the output ports equally.**

Again, a unitary operator couples the output state to the input state,

$$\psi' = U\psi. \tag{14}$$

This unitary operator can now be represented by a 3 x 3 matrix where the modulus of each matrix element is $1/\sqrt{3}$. Here again and for all critters it is possible to absorb any phase factors of the first row into phases of the input beams and to absorb any phase factors of the first column into phases of the output beams. Such a representation of a multiport only contains "1" in both the first column and the first row. We will call such a representation canonical. Thus, the general tritter operator can be written as

$$U = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \varphi & \varphi^* \\ 1 & \varphi^* & \varphi \end{pmatrix} \tag{15}$$

with $|\varphi| = 1$ and $\varphi + \varphi^* = -1$. The only *two* possible choices for $\varphi$ are $\varphi = \alpha$ and $\varphi = \alpha^2$ with $\alpha = e^{2\pi i/3}$.

Thus the tritter operator has two canonical representations, either

$$U_T = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{pmatrix} \quad \text{or} \quad U'_T = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{pmatrix}. \tag{16}$$

The transition rules for incident beams therefore are

$$|a\rangle \Rightarrow \frac{1}{\sqrt{3}}\{a'\rangle + b'\rangle + c'\rangle\}$$
$$|b\rangle \Rightarrow \frac{1}{\sqrt{3}}\{|a'\rangle + \alpha|b'\rangle + \alpha^2|c'\rangle\}$$
$$|c\rangle \Rightarrow \frac{1}{\sqrt{3}}\{|a'\rangle + \alpha^2|b'\rangle + \alpha|c'\rangle\} \tag{17}$$

for the tritter rule $U_T$. For $U_T'$ the roles of $\alpha$ and $\alpha^2$ are just interchanged. Note also that $U_T^{-1} = U_T'$ and that the two different types of tritter can be converted into each other by an odd number of permutations of rows or columns, e.g.

$$U_T' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} U_T. \tag{18}$$

These results imply that sequential arrangement of tritters does not lead to new nontrivial tritters. In other words, given some tritter one can obtain any tritter by changing external phases and by a permutation of input and/or output ports, which may simply be achieved for example by flipping two output ports. Physically, there are many different possibilities of realising a tritter. A specific type with parallel input beams and parallel output beams is shown in Fig. 5. One can easily see that a tritter has more adjustable parameters than a beam splitter. These are the reflectivities of the partially reflecting mirrors and the nontrivial phase in the internal loop of the tritter.

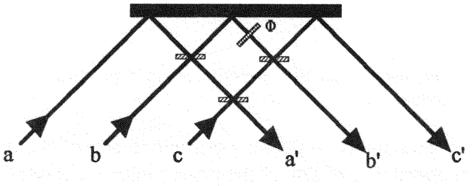


Fig. 5: Possible realization of a tritter using partially reflecting mirrors and a nontrivial internal phase $\phi = 0, \pi$.

Turning to higher multiports the number of experimentally adjustable nontrivial parameters grows quadratically with the number of ports. One of the most interesting results for higher multiports is the existence of distinct classes which cannot be transformed into each other by just changing external phases and by permutation of input and/or output ports. We leave a detailed discussion to a forthcoming paper.

## 5. Two-Particle Three-State Systems

It is evident that with multiports a large number of novel experiments in quantum optics become possible. Because of the availability of down-conversion photon sources, we only discuss here the case where a two-particle source is employed. Assume that such a source emits two particles in the state

$$|\psi\rangle = \frac{1}{\sqrt{3}}\left\{|a\rangle|d\rangle + |b\rangle|e\rangle + |c\rangle|f\rangle\right\}. \tag{19}$$

Again, the first ket in a product state refers to particle 1, and the second to particle 2. The beams $a,b,c,d,e,f$ are subject to the phase shifts $\alpha,\beta,\gamma,\delta,\varepsilon,\zeta$, respectively, and thus the state evolves into

$$|\psi\rangle = \frac{1}{\sqrt{3}}e^{i(\alpha+\delta)}\left\{|a\rangle|d\rangle + e^{i\varphi}|b\rangle|e\rangle + e^{i\chi}|c\rangle|f\rangle\right\} \tag{20}$$

with $\chi = \beta + \varepsilon - \alpha - \delta$ and $\varphi = \gamma + \zeta - \alpha - \delta$.

Suppose now that the three beams excited by particle 1 are fed into a tritter and likewise the three beams excited by particle 2 are fed into another tritter (Fig. 6). Clearly the final state is then obtained by applying the appropriate tritter operator Eq. (16) to state (20). Instead of writing down the final state explicitly, we focus on the count rates and on the correlations to be expected.

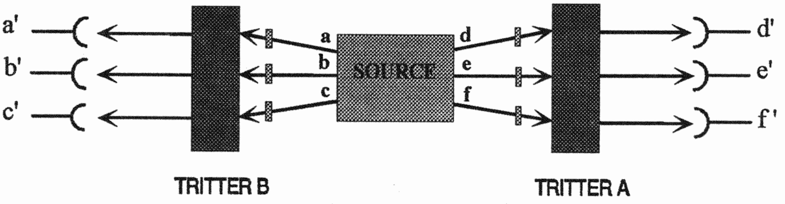


Fig. 6: Principle of a two-tritter, two-photon EPR experiment. In a practical realization the source can be parametric down-conversion.

The unconditional probability to detect a particle in any of the detectors is a constant, e.g. $p(a') = 1/3$. The independence of any of the phases inserted between source and tritters is a consequence of the initial entanglement. Certainly this does not hold anymore for the various joint probabilities of detecting a particle in a given detector on one side together with detecting the other particle on the other side. These joint probabilities are:

$$p(a',d') = p(b',f') = p(c',e') = \frac{1}{27}[3 + 2\cos\chi + 2\cos\varphi + 2\cos(\varphi - \chi)],$$

$$p(a',e') = p(b',d') = p(b',f') = \frac{1}{27}[3 + 2\cos\chi' + 2\cos\varphi' + 2\cos(\varphi' - \chi')]$$

$$\text{with } \chi' = \chi + 2\pi/3,\ \varphi' = \varphi - 2\pi/3$$

$$p(a',f') = p(b',e') = p(c',d') = \frac{1}{27}[3 + 2\cos\chi'' + 2\cos\varphi'' + 2\cos(\varphi'' - \chi'')]$$

$$\text{with } \chi'' = \chi - 2\pi/3,\ \varphi'' = \varphi + 2\pi/3 \tag{21}$$

and where, e.g., $p(a',e')$ is the probability to simultaneously detect a particle in detector $a'$ and a particle in detector $e'$.

The joint probabilities of Eqs. (21) have a number of remarkable features. It is easy to show that all these probabilities are nonnegative and their maximum value is 1/3. This may be understood by analyzing for example the case where the first equation attains its maximum value which occurs when $\chi, \varphi = 2n\pi$. Then $p(a',d') = p(b',f') = p(c',e') = 1/3$ and all other joint probabilities vanish. This implies that if the phases in the two-tritter two-particle interferometer are set to these values then perfect correlations arise, and thus Einstein-Podolsky-Rosen elements of reality may be introduced. Explicitly, if, say, detector $a'$ fires and the phases are set to the parameters just mentioned we can predict with certainty that the other particle will be registered by detector $d'$. Likewise, if particle 1 is registered by detector $b'(c')$, particle 2 will be registered by detector $f'(e')$. Thus, while it is always maximally uncertain which detector will register either of the particles, it is known with certainty which detector will register the second particle once the first particle has been observed, as long as the phases are set according to the above condition.

Another set of similar perfect correlations arises if the phases are set such that $\chi', \varphi' = 2n\pi$. Then the joint probabilities are $p(a',e') = p(b',d') = p(e',f') = 1/3$ with all others being zero. Here again perfect correlations occur but now between different detectors than before. Finally, a third set of perfect correlations arises for $\chi'', \varphi'' = 2n\pi$, then $p(a',f') = p(b',e') = p(c',d') = 1/3$ with all other joint probabilities vanishing. Fig. 7 shows these three possible ways of perfect correlations. Note that of the six possible one-to-one combinations between detectors on either side only three combinations are realized for perfect correlations. Here we should note the fact that these types of perfect correlations arise whenever we use the same tritter on each side (either the one represented by $U_T$ or the one represented by $U_T'$). In case we chose to use different types of tritters on the two sides, the other three types of perfect correlations occur, with the original three now being excluded.

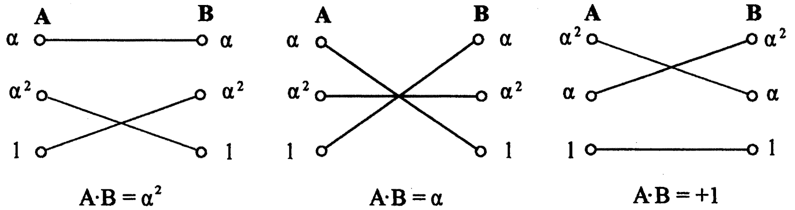


Fig. 7: Perfect correlations occurring in an experiment of the type of Fig. 6. The results on either side are best characterized by assigning them the value $A, B = \alpha, \alpha^2, 1$, where $\alpha = e^{2\pi i/3}$. The three cases of perfect correlations occurring are then signified by $A \cdot B = \alpha, \alpha^2, 1$.

The three types of perfect three-state correlations may be signified in the same way by value assignment as it was done originally by Bell for two-state correlations. One might be tempted to assign the values +1,0,-1 to the three possible outcomes on each side. Such a procedure does not succeed because, when calculating the product $AB$, if $A$ is again the result for one particle and $B$ the result for the other, appearance of a "0"-result always leads to $AB = 0$ independent of which type of correlation occurs and thus information is lost. A rather elegant procedure of value assignment is to choose α, $\alpha^2$, $\alpha^3$ (with $\alpha = e^{2\pi i/3}$) for the three possible outcomes on either side. It then follows that the three cases of perfect correlations are signified by $AB = \alpha, \alpha^2, 1$ (see Fig. 7). These numbers are now the Bell numbers for a three-dimensional Hilbert space.

In general, for the case of correlations between two particles, where each one is defined in an $M$-dimensional Hilbert space, at most $M$ cases of perfect correlations (where EPR-elements of reality may be introduced) occur with a given set of multiports. It is thus natural to generalize the procedure just given by assigning the values $A, B = e^{2\pi i n/M}$ $n = 1, 2, \ldots M$ to the results in order to signify the cases of perfect correlations by $AB = e^{2\pi i n'/M}$. As we will show in a forthcoming paper there is always at least one case of a specific multiport for any $M$ where this procedure succeeds. But, we should point out, for $M \rangle 3$ these are also cases where this procedure fails. Obviously the case $M = 2$ as analyzed originally by Bell is just the most simple nontrivial case. This is the reason why we propose to call these general numbers used in value assignment Bell-numbers.

Concluding this section we note that besides introducing EPR elements of reality the way just given, one can also apply a generalized Bell inequality to the two-tritter correlations[18] thus providing the first feasible test for Bell's theorem for pairs of spins higher than 1/2 via an optical analog.

## 6. Concluding Comments

In general, an experiment using multiports which are fed the two correlated photons created in the process of parametric down-conversion provides a generalization of EPR correlations to Hilbert spaces of higher dimensions. These correlations are fully analogous to those between two particles with higher spin. Thus they are expected to give new interesting results going beyond those realizable in spin correlations between two spin-1/2 particles or two photons. A specific example are those correlations which are necessary to establish the Bell-Kochen-Specker paradox[19]. Using two correlated particles each defined in a higher-dimensional Hilbert space it is possible to establish the results for each individual measurement utilized in the Kochen-Specker argument as Einstein-Podolsky-Rosen elements of reality[20]. It is evident that using multiports together with a down-conversion photon source can provide immediate experimental realization of such correlations.

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## 7. References

1. A. Einstein, B. Podolsky, and N. Rosen, *Phys. Rev.* **47** (1935) 771.
2. J.S.Bell, *Physics* (NY) **1** (1964) 195.
3. D. Bohm, *Quantum Theory*, (Prentice-Hall, Englewood Cliffs, 1951).
4. S.J. Freedman and J.F.Clauser, *Phys. Rev. Lett.* **28** (1972) 938.

5. M.A. Horne and A. Zeilinger, in *Proc. of the Symposium on the Foundations of Modern Physics*, eds. P. Lahti and P. Mittelstaedt (World Scientific, 1985); M. Zukowski and J. Pykacz, *Phys. Lett.* **A127** (1988) 1; M.A. Horne, A. Shimony, and A. Zeilinger, *Phys Rev. Lett.* **62** (1989) 2209.
6. J.G. Rarity and P.R. Tapster, *Phys. Rev. Lett.* **64** (1990) 2495.
7. J.D. Franson, *Phys. Rev. Lett.* **62** (1989) 2205.
8. P.G. Kwiat, W.A. Vareka, C.K. Hong, H. Nathel, and R.Y. Chiao; *Phys. Rev.* **A41** (1990) 2910; Z.Y. Ou, X.Y. Zou, L.J. Wang, and L. Mandel, *Phys. Rev. Lett.* **64** (1990) 321; J.G. Rarity, P.R. Tapster, E. Jakeman, T. Larchuk, R.A. Campos, M.C. Teich, and B.E.A. Saleh, *Phys. Rev. Lett.* **65** (1993) 1348; J. Brendel, E. Mohler, and W. Martiensen, *Phys. Rev. Lett.* **66** (1991) 1142; *Europhys. Lett.* **20** (1992) 575.
9. P.G. Kwiat, A.M. Steinberg, and R.Y. Chiao, *Phys. Rev.* **A47** (1993) R 4272.
10. D.M. Greenberger, M. Horne, and A. Zeilinger, in *Bell's Theorem, Quantum Theory and Conceptions of the Universe*, ed. M. Kafatos (Kluwer Academic, Dordrecht, 1989), p. 73; D.M. Greenberger, M.A. Horne, A. Shimony, and A. Zeilinger, *Am. J. Phys.* **58** (1990) 1131; N.D. Mermin, *Phys. Rev. Lett.* **65** (1990) 1838.
11. L. Hardy (*Phys. Rev. Lett.* **68** (1992) 2981) has recently made a proposal employing two particles, each one defined in more than two states with the attempt at arriving at a GHZ-type contradiction between local realism and quantum mechanics. In this case and in a related proposal (H.J. Bernstein, D.M. Greenberger, M.A. Horne, and A. Zeilinger, *Phys. Rev.* **A47** (1993) 78) that goal is only achieved with some additional assumptions.
12. N.D. Mermin, *Phys. Rev.* **D22** (1980) 356; A. Garg and N.D. Mermin, *Phys. Rev Lett.* **49** (1982) 901; N.D. Mermin and G.M. Schwarz, *Found. Phys.* **12** (1982) 101. M. Ardehali, *Phys. Rev.* **D44** (1991) 3336; G.S. Agarwal, *Phys. Rev.* **A47** (1993) 4608.
13. D.C. Burnham and D.L. Weinberg, *Phys. Rev. Lett.* **25** (1970) 84.
14. A. Zeilinger, H.J. Bernstein, D.M. Greenberger, M.A. Horne, and M. Zukowski, in *Quantum Control and Measurement*, eds. H. Ezawa and Y. Murayama (Elsevier, 1993).
15. A. Zeilinger, *Am. J. Phys.* **49** (1981) 88; S. Prasad, M.O. Scully, and W. Martienssen, *Opt. Comm.* **62** (1987) 139; Z.Y. Ou, C.K. Hong, and L. Mandel, *Opt. Comm.* **63** (1987) 118; H. Fearn and R. Loudon, *Opt. Comm.* **64** (1987) 485.
16. N.G. Walker and J.E. Carroll, *Optical and Quantum Electronics* **18** (1986) 355; N.G. Walker, *J. Mod. Opt.* **34** (1987) 15.
17. H.J. Bernstein, *J. Math. Phys.* **15** (1974) 1677.
18. The method is to be found in M. Zukowski, *Phys. Lett.* **A177** (1993) 290.
19. S. Kochen and E.P. Specker, *J. Math. Mech.* **17** (1967) 59; J.S. Bell, *Rev. Mod. Phys.* **38** (1966) 447.
20. M. Redhead, *Incompleteness, Nonlocality and Realism* (Clarendon, Oxford; 1987).