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Comment

Comment on "Exclusion of time in the theorem of Bell" by K. Hess and W. Philipp

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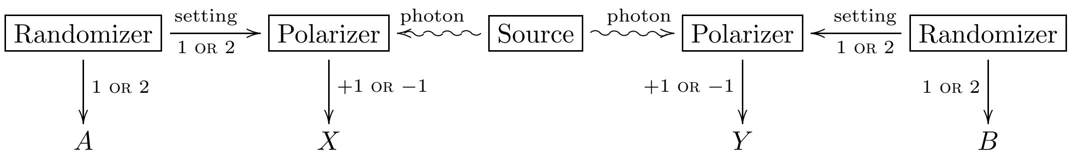
PACS. 03.65.Ud – Entanglement and quantum nonlocality (*e.g.* EPR paradox, Bell's inequalities, GHZ states, etc.).

We point out fatal errors in the recent paper [1] by Hess and Philipp. The most serious one is the lack of recognition of the choice which an experimenter is free to make in the laboratory, and which a theoretician is free to make in a *Gedankenexperiment*. We convert this *freedom* into a statistical independence assumption, and show how it plays a vital role in obtaining Bell's theorem. We will first present our own proof of Bell's theorem and discuss its assumptions, emphasizing aspects of *freedom* and *control*, and then turn to a refutation of the arguments of Hess and Philipp. Our formulation is a summary of attempts of many earlier papers to formulate very precisely the assumptions behind the theorem of Bell.

Here is a schematic view of one trial in a Bell-type delayed choice experiment:



We will use the words "photons", "polarizer", but the picture could be applied to many different realisations. In the two wings, one of two "settings", with labels 1 and 2, is chosen by a random device. We let $A = 1, 2$ and $B = 1, 2$ denote the one actually chosen. The setting is fed into a measurement device, just before a particle arrives. The measurement results in an outcome ± 1. We will denote the outcome left by X and right by Y. The two randomizers and the two polarizers are, all four, well separated from one another. One can consider as complicated local randomization procedures as possible. We assume that the procedure used to generate A and B may be modelled as *independent, fair coin tosses.*

What we mean by *local realism*? *Realism*: any model which allows one to introduce a further *eight* variables, which we denote by X_{ij}, Y_{ij}, where $i, j = 1, 2$, and which are such that $X \equiv X_{AB}$, $Y \equiv Y_{AB}$. In words: one may conceive of "what the measurement outcomes could be, under any of the possible settings". No other hidden variables appear. However, given a stochastic hidden-variables theory, one can define X_{ij} and Y_{ij} as (possibly random) functions of the variables in that theory. *Locality*: the following is supposed to hold for all

i, j: $X_{i1} \equiv X_{i2}$, $Y_{1j} \equiv Y_{2j}$. The outcome which you would see left, under either setting, does not depend on which setting might be chosen, right, and vice versa. We can write $X_i \equiv X_{ij}$, $Y_j \equiv Y_{ij}$. *Freedom*, often tacit in treatments of Bell's theorem: (A, B) *is statistically independent of* (X_1, X_2, Y_1, Y_2). The choice of settings in the two randomizers, summarized in the fair coin tosses A and B, is causally separated from the mechanism which produces the potential outcomes X_1, X_2; Y_1, Y_2. Contained in the above is an assumption of *control*. When Alice and Bob send the chosen settings i, j to their polarizers, they will cause some further unintended disturbance. Any disturbance left, as far as it influences the outcome left, is not related to the coin toss nor to the potential outcomes right, and vice versa.

Now Bell's inequality. The value of X_iY_j encodes the equality or inequality of the variables X_i and Y_j, while $(X_1Y_2) = (X_1Y_1)(X_2Y_1)(X_2Y_2)$. Thus, one can easily show that [2]

$$\Pr\{X_1 = Y_2\} - \Pr\{X_1 = Y_1\} - \Pr\{X_2 = Y_1\} - \Pr\{X_2 = Y_2\} \leq 0. \quad (1)$$

Consider the conditional probability that the outcomes left and right are equal, given any pair of measurement settings, $\Pr\{X = Y \mid AB = ij\}$. By *local realism*, this equals $\Pr\{X_i = Y_j \mid AB = ij\}$. But by *freedom* this probability is the same as the unconditional probability $\Pr\{X_i = Y_j\}$. Therefore we obtain a Bell inequality:

$$\Pr\{X = Y \mid AB = 12\} - \\ - \Pr\{X=Y \mid AB=11\} - \Pr\{X=Y \mid AB=21\} - \Pr\{X=Y \mid AB=22\} \leq 0. \quad (2)$$

But for quantum mechanics the left-hand side can reach $\sqrt{2} - 1 \gg 0$. Hence Bell's theorem.

The only statistical independence we needed was between the chosen settings and the physical system of polarizers and source. Any other kinds of dependencies between hidden variables in any of the locations is allowed. We did not mention time at all because it is irrelevant. Our derivation concerned each time interval, within which one trial of the experiment is carried out. We did not compare *actual* outcomes under different settings at *different* times, but *potential* outcomes under different settings at the *same* time. The argument in [1], formulas (8)-(10), is besides the point.

Do Hess and Philipp provide arguments against our assumptions? If it is freedom that they question, their thesis would have to be that because of long-time periodicities in the physical systems, the outcome of a coin toss and the free will of an experimenter at one location is correlated with the potential outcome of a certain measurement at a distant location. Now locality. When we select a "1" or a "2" on a measurement device, we have supposed that *only* our choice has an impact on the physics at this location. However, at the same time we will be introducing an uncontrolled disturbance. Could it be that this disturbance carries information about the setting being chosen in the far wing? Well, perhaps there is a (spooky) physics in which everything is determined long in advance, so that the setting being generated by Bob is "known" at Alice's location. Then the locality assumption would fail.

In conclusion, Hess and Philipp ignore the freedom of the experimenter to choose either of two settings. The issue of possible time-like dependence and variation is irrelevant. In [2] we show that Hess and Philipp's hidden-variable model is not local.

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REFERENCES

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