HIGH-FIDELITY EXPERIMENTAL QUANTUM TELEPORTATION AND ENTANGLEMENT SWAPPING

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Entanglement swapping in its most fundamental form is the teleportation of a quantum state that is itself entangled to another quantum system. This connection to teleportation provides us with a tool to demonstrate and verify the non-local nature of the teleportation procedure. In this work the non-locality is experimentally confirmed by a violation of Bell's inequality using teleported entanglement.

Within the field of quantum information and quantum communication a very prominent position is taken by the quantum teleportation protocol. It is the only known way in which quantum information could be sent to a different place without actually sending a physical object that carries the quantum information. Not only does this open a completely new way of communication for future quantum computers, but also it can enhance the efficiency of potential linear optics realizations of quantum information processing.

If one were to assess the quality of a certain teleportation[1,2,3,4] device there are two fundamental criteria. One is the efficiency of the apparatus, that is the percentage of cases in which the procedure is completed successfully, after an input has been provided to the machine. The other important parameter is the fidelity, which is defined as the overlap between output and input states.[a] If we restrict ourselves to pure input states the fidelity F can be calculated as follows.

$$F = \langle \Psi_{\text{in}} | \rho_{\text{out}} | \Psi_{\text{in}} \rangle \,, \qquad (1)$$

where $|\Psi_{\text{in}}\rangle$ is the (pure) input and ρ_{out} is the output density operator. A fidelity of 0.5 corresponds to random output, whereas a value 1 indicates perfect operation.

There are two other important thresholds for the teleportation fidelity, the first one being the limit of classically teleporting an unknown quantum state. Its maximum possible fidelity of 2/3 is the state estimation limit for a single copy of an arbitrary pure qubit state.[7] A possible realization, is to measure in an arbitrary basis, communicate the result and prepare the state on the other side corresponding to the measurement result.

Even though this limit cannot be surpassed by classical physics, a local hidden variable theory could in principle do better. Yet, such a theory in turn has to obey the limit of $(1+\sqrt{2})/\sqrt{8} \approx 0.854$ according to Bell's inequality. If hidden variables are considered to be a part of classical physics, then this value is the boundary above which a teleportation device performs provably non-classical.

The fidelity could be measured for a set of possible input states by comparing input and output for a large number of sample transmissions. Another method is to use entanglement swapping[8] to probe the input state space uniformly. Here a quantum state that is itself entangled with an ancilla is teleported and the entanglement of the output with the ancilla is observed. Measuring this "teleported entanglement"[b] we can calculate

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[a] From a particular teleportation point of view this fidelity applies to our experiment only conditioned on the actual detection on the receiver's side due to present technical limitations.[5,6] In the entanglement swapping experiment presented here, however, the particles must be detected in order to test the entanglement. Therefore the fidelity will naturally be conditioned on detections of particles 0 and 3 (see Fig. 1).

[b] Of course one does not actually teleport an *entangled* state but only the relation of the input to the ancilla.

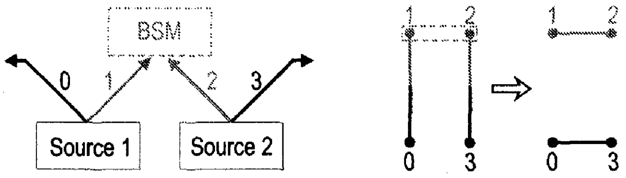

Figure 1. Entanglement swapping or equivalently teleportation of entanglement. The quantum state "1" is teleported to mode "3", (except for the missing application of a unitary transformation) or, by symmetry, state "2" is teleported to mode "0". It turns out, that if the outputs of "0" and "3" are selected according to the result of the Bell-state measurement (BSM), it is not necessary to actually perform the unitary transformation that is part of the teleportation protocol.

the fidelity without varying the input state. Depending on the quality of the original entanglement the whole input state space is probed at once. In the following we will therefore only consider entanglement swapping, which was demonstrated first by Pan et al.[9]

As shown in Fig. 1 (right) we can think of entanglement swapping as a general procedure to redistribute entanglement within a collection of quantum systems. This is completed by entangling a subset of previously separable particles via a projection onto an entangled state basis. In Fig. 1 only the most basic protocol for two two-particle entangled states is shown, which can be generalized to higher numbers of particles.[10]

If we restrict ourselves to start out with two entangled two-particle systems, then entanglement swapping reduces to a Bell-state measurement between two initially separable particles. Consider Fig. 1(left): here we start out with the two entangled systems 0-1 and 2-3. Let their quantum states be $|\Psi^-_{01}\rangle$ and $|\Psi^-_{23}\rangle$, where $|\Psi^\pm\rangle$, $|\Phi^\pm\rangle$ denote the maximally entangled two qubit basis (Bell-basis, Bell-states). The joint state of the four particles is consequently $|\Psi^-_{01}\rangle \otimes |\Psi^-_{23}\rangle$. The Bell-state measurement on particles 1 and 2 will then yield one of the four results $|\Psi^\pm_{12}\rangle$, $|\Phi^\pm_{12}\rangle$ with equal probabilities. The action of the Bell-state measurement is most easily seen in a basis of entangled states between particles 2 and 3. Then the state reads

$$\Psi_{\text{total}} = \frac{1}{2}\Big[|\Psi^+_{03}\rangle\,|\Psi^+_{12}\rangle - |\Psi^-_{03}\rangle\,|\Psi^-_{12}\rangle - \\ - |\Phi^+_{03}\rangle\,|\Phi^+_{12}\rangle + |\Phi^-_{03}\rangle\,|\Phi^-_{12}\rangle\Big]\,. \quad (2)$$

Whenever we measure a specific Bell-state for particles 1 and 2 particles 0 and 3 will end up in exactly the same state.

To subsequently assess the produced entanglement in our experiment we chose to do a Bell-inequality test. Another possibility would be to do a tomography of the two-particle space to recover the full two-particle density matrix.

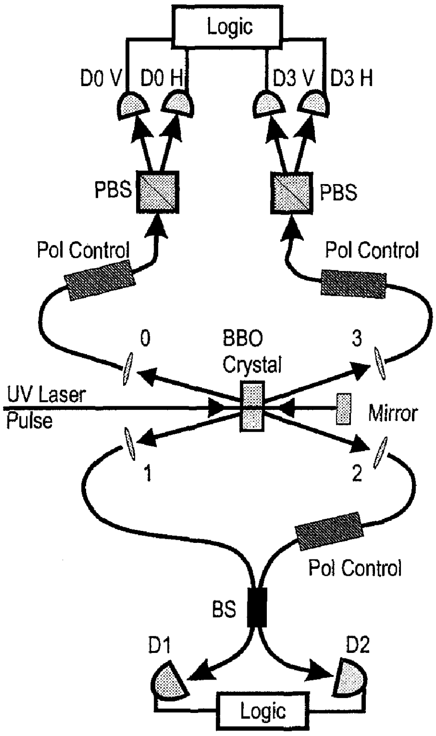

Figure 2. Experimental apparatus to perform teleportation of entanglement created by spontaneous parametric down-conversion from a pulsed laser. Bell-state measurement is performed with the help of a beam-splitter and detectors D1 and D2, while polarization is analyzed on the other side with polarizing beam-splitters followed by detectors D0 H/V and D3 H/V. Due to spurious birefringence in the single-mode optical fibers polarization controllers have to be used to align the various analyzer bases.

Our experimental apparatus is depicted in Fig. 2. We use spontaneous parametric down-conversion from a fs UV laser in a double-pass configuration inside a nonlinear optical crystal to create two $|\Psi^-\rangle$ entangled photon pairs. The photons are coupled to single mode optical fibers. Photons 1 and 2 are overlapped on a fiber beam-splitter. Coincidence detection after the beam-splitter projects destructively on to the state $|\Psi^-_{12}\rangle$. In turn particles 0 and 3 are also projected onto a $|\Psi^-_{12}\rangle$ state whenever the Bell-state measurement was successful.

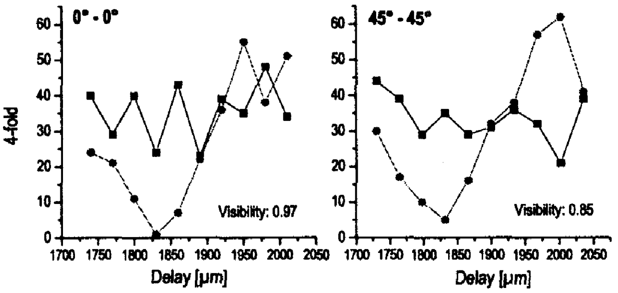

Figure 3. Two photon interferometry of independent photons conditioned on the detection of four photons as used for the Bell-state measurement of $|\Psi^-\rangle$. Polarization anticorrelation shows up as a dip in the coincidence rate for parallel polarizer setting **(circles)** as the relative delay to the beam-splitter is scanned across the point of indistinguishability. For orthogonal polarizers the rate stays constant **(squares)**.

Projection on to the mentioned state of particles 1 and 2 can only be achieved if their indistinguishability is assured. In the experiment we vary the difference between the optical paths of either photon in order to continuously vary the degree of distinguishability. Typical interferograms can be seen in Fig. 3. They are already verifications of entanglement swapping. At matching delay (zero difference) four fold coincidence between all outputs show a clear dip in the curve when we look at identical polarizer outputs for particles 0 and 3. In a $|\Psi^-\rangle$ state we expect the coincidences to go to zero.

The finite experimental visibility V ($V = 2F - 1$) reflects two separate imperfections. On the one hand the initial entanglement between particles 0-1 and 2-3 will never be perfect and also the Bell-state measurement in practice falls short of perfection. The nature of our source makes the quality of the initial entanglement basis dependent. In the vertical-horizontal basis we observe higher contrast than in any other basis. The critical visibility is therefore measured at 45° off horizontal (or at circular polarization alternatively).

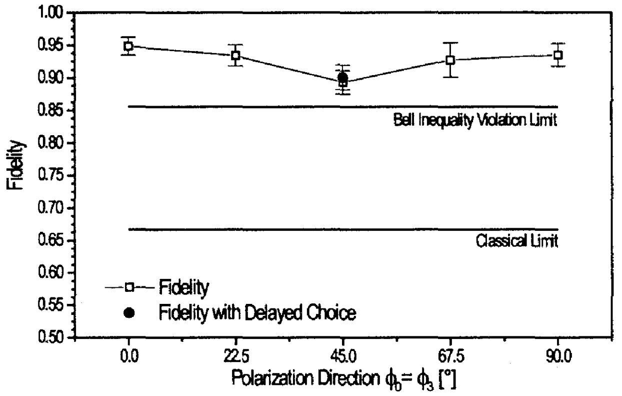

Figure 4. The experimentally achieved fidelity as a function of the absolute analyzer angle. The solid dot shows a measurement for which the Bell-state measurement was delayed until after the other two photons had been detected. Within experimental errors there is no difference in fidelity. The photon pairs are originally produced as HV or VH pairs in the parametric down-conversion crystal. Only by careful compensation are the two components made coherent. The perfection of this compensation is, however, limited by group dispersion and transverse ("walk-off") effects. These imperfections show up in measurements at 45°, where we observe reduced correlation for the individual pairs, which transfer to a reduced correlation for the teleported entanglement if measured at 45°. Comparing the correlation measurements taken for individual pairs and the above data we conclude that the fidelity of our Bell-state measurement must be better than 97%.

In our experiment we achieved a fidelity that was higher than 0.89 for any basis. The results are shown in Fig. 4. This, for the first time made it possible to violate Bell's inequality by a teleportation experiment, i.e. by particles that did not interact locally. For this purpose the correlation is evaluated at the following linear polarization analyzer angles (particles 0, 3): $(0°, 22.5°)$, $(0°, 67.5°)$, $(45°, 22.5°)$, and $(45°, 67.5°)$. The values of the correlation function were determined to $E_{11} = -0.6281 \pm 0.0461$, $E_{12} = +0.6766 \pm$

0.0423, $E_{21} = -0.5407 \pm 0.0454$, and $E_{22} = -0.5748 \pm 0.0472$ respectively. When we evaluate the Bell inequality[11]

$$S = |E_{11} - E12| + |E_{21} + E_{22}| \leq 2 \qquad (3)$$

with this data we get $\mathbf{S = 2.421 \pm 0.091}$, which is 4.6 standard deviations above the limit for any local realistic theory, provided the fair sampling assumption is obeyed by the theory. Also, the values within reasonable experimental error agree with the quantum physical predictions when the imperfections of the apparatus are taken into account.

Because entanglement swapping and teleportation are two aspects of a fundamental quantum communication protocol the appear in various situations. They are part of quantum repeaters, enable basic forms of entanglement purification and lately it has been discovered that they can also enhance quantum operations with linear optics to obtain scalability.[12]

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