17 Bell's Theorem, Information and Quantum Physics

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17.1 Introduction

I had the great fortune to meet John Bell three times in my life. The first encounter was in 1975. I had been working since 1968 at the small Atom Institute in Vienna under Helmut Rauch on applying neutrons to investigate magnetic materials. Then he – together with Wolfgang Treimer and Ulrich Bonse [1] – developed the first crystal-based neutron interferometer as a tool to investigate the foundations of quantum mechanics. Because of my experience with polarized neutrons, I started to think about the role of the spin in such experiments and we were soon able to demonstrate the change of the sign of a spinor wave function upon a complete rotation by 2π [2]. So it was just in time that I discovered the announcement of an Erice workshop entitled "Thinkshops on Physics", which was organized by John Bell together with Bernard d'Espagnat. The topic of this workshop was exactly experiments on the foundations of Quantum Physics. I went there and reported on our newest experimental results in neutron interferometry. I should mention that this was my first real encounter with the international scientific community. There, I heard for the first time about Bell's theorem, about the Einstein–Podolsky–Rosen paradox, about entanglement, and the like. Needless to say that I did not get any real understanding what this was all about, but I got the hunch that something very important was being discussed. This meeting turned out to be very crucial in my life. There, I met a number of colleagues for the first time, some of whom later became personal friends. These include Mike Horne, Abner Shimony and also Val Telegdi. Mike Horne and Val Telegdi then helped me to get to MIT and work with Cliff Shull at the neutron diffraction laboratory there. At that laboratory I also met Danny Greenberger later, and all this resulted in some of my most important collaborations. So it is fair to say that John Bell's organization of that meeting was very important for me in many different ways.

The next time I met John Bell was in Vienna on the occasion of a conference celebrating the 100th anniversary of Erwin Schrödinger. This was in 1987. At that meeting I had the great honor to be on a panel together with John Bell. The other panel members were K. Baumann (Graz), P. Mittelstaedt (Cologne) and H. Pietschmann (Vienna). This panel discussion took place in one of the gilded state halls of the University of Vienna. And I still remember John Bell's insistence on the desideratum that physics should explain why events happen and that something must be missing in the present formulation of quantum mechanics. A position that already at that time I had the pleasure of disagreeing with. John Bell was probably the most pleasant person to disagree with I ever met. The discussions were always very friendly but to the point and succinct.

The third time I met John Bell was in 1990 at the small workshop entitled "The Amherst Workshop on the Foundations of Quantum Mechanics" which, as the name suggests, took place at Amherst College, June 10–15, 1990 and was organized by George Greenstein and Arthur Zajonc. Two years earlier, Dan Greenberger, Mike Horne and I had found what is called today GHZ states [3], the extreme example of what John Bell had demonstrated, namely, disagreement between quantum mechanics and local realistic theories. To our great joy, John Bell expressed his excitement and pleasure about our work. None of us knew that this was the last time we would meet John, as he passed away a few months later completely unexpectedly for everyone.

17.2 Information and Interference

In the famous double-slit experiment [4], which, according to Feynman [5], contains in it the heart of quantum mechanics, a number of deep epistemological questions about quantum mechanics are raised. As is well known, an interference pattern only arises when both slits in the intermediate diaphragm are open (Fig. 17.1). It can be argued that each individual particle has information that both slits are open because all particles avoid the minima in the interference pattern. Yet we still might ask ourselves through which of the two slits does a specific particle pass. Yet, all of the numerous gedanken attempts to demonstrate that one can find out which of the two slits the particle uses, and still obtain the interference pattern, were in vain. Whenever it is possible to determine the path precisely, by whatever means, the interference pattern vanishes. On the other hand, if there is no possibility, not even in principle, to find out which path was taken by a particle registered at the observation screen, the interference pattern arises with perfect visibility. Already here we can note that it is information which plays a crucial role in whether or not interference is observed. Path information and the information contained in the interference pattern exclude the other. It is important to realize that it does not matter whether or not an observer actually takes note of the path the particle took. It is the mere possibility of determining the path which makes it impossible to observe the interference pattern.

The double-slit experiment therefore is a basic example of complementarity in quantum mechanics. Complementarity in general is, as introduced by

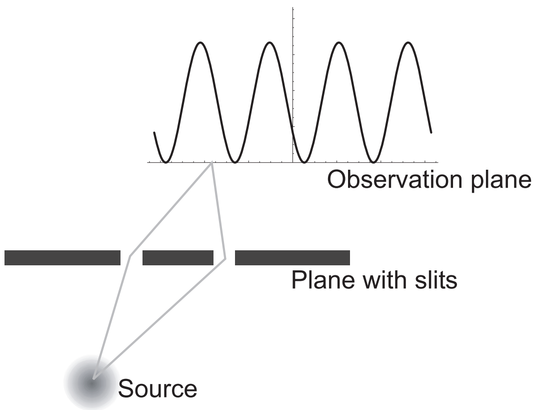

Fig. 17.1. The double-slit interference experiment [4]. The interference fringes in the observation plane are collected one particle at a time, each of them passing individually through the plane with the slits

Niels Bohr, the notion that there exist pairs, or combinations of more than two observables, such that if one is determined perfectly, the others are by necessity completely undetermined. For Niels Bohr [6], this was a consequence of the fact that in order to demonstrate mutually complementary observables, one has to use apparata that exclude each other. This might suggest the impossibility of experimental demonstration, or the simultaneous determination of complementary observables, because of the clumsiness of macroscopic apparata which are unsuitable for determining the fine details of quantum objects. Yet, as signified most clearly by the Kochen–Specker theorem [7], as also found by John Bell [7], it is impossible to assign to a quantum system observable properties per se, i.e. independent of considering the apparatus with which the properties will be measured. Thus, we do not in an experiment reveal a pre-existing feature of a quantum system. Therefore, measurement is constitutive in what can be physical reality in the sense that the experimentalist by deciding which apparatus to use chooses which physical observable can become reality. In the case of the double-slit experiment, by deciding to measure the path the particle takes, the experimentalist decides that "path taken" can become an element of reality. On the other hand by deciding to choose an experimental set-up intrinsically unsuitable to determine the path taken, the observer decides that the interference pattern can become reality. This indicates that the observer, by choosing the experiment, can actually choose between different kinds of information that will manifest themselves in the experiment, although the total amount of information is apparently limited.

This has often led to confusion and to the wrong impression that the observer in a quantum measurement has so much influence that she[1] can define reality. Yet, an important point is suggested by the fact that the observer cannot define which particular value will turn out for the observable chosen out of the class of complementary observables. Specifically, the observer has no influence on which of the two paths the particle will be found in if the path is measured. And likewise she has no influence on where exactly in the interference pattern the particle will be found.

This indicates that the observer has a qualitative, but not a quantitative influence on reality. She can define which quality will show up in the experiment, but not the quantity, the exact value, the latter being completely random, except in the rare case when the quantum system is in an eigenstate of the observed quantity.

This small discussion already indicates that information might be at the root of the interpretation of quantum mechanics. In the double-slit experiment, the observer can decide to obtain information either about the path taken or the information contained in the interference pattern. It also turns out that using a new measure of information the total information in a quantum system is a constant [9]. Complementarity then simply is a consequence of the fact that the total information which is represented by a quantum system is finite [10].

17.3 Information and Entanglement

Bell's theorem [8] states that any local realistic view of the world is incompatible with quantum mechanics. More precisely, John Bell demonstrates that for entangled states, it is not possible to explain all correlations between two particles using a local realistic model. A crucial assumption of such a model is [11] that a measurement result for each of two entangled particles is independent of whatever measurement is performed on the other particle. While this is often interpreted as demonstrating non-locality in quantum mechanics, there are also alternative viewpoints possible, most notably the assumption that the philosophical notion of counterfactuality does not hold or that the existence of a reality independent of observation makes no sense in physics. By now, the conflict between local realism and quantum mechanics has led to numerous experiments, all of which support quantum mechanics [12–14]. It is therefore safe to assume that the world cannot be understood using the rather intuitively reasonable ideas leading to Bell's inequality.

While entanglement apparently seems to be still posing problems as to understanding its nature, the information-theoretical interpretation of quantum mechanics again leads to a very natural point of view, as we will see now for a specific example [15–17].

[1] In some of his papers, John always referred to physicists in the feminine.

Considering just two two-state systems, i.e. two qubits, it is natural to assume that the information carried by the system is one bit of information per qubit. In a classical way of encoding, this would lead to the following factorizable states:

$$
\begin{aligned}
|\Psi_1\rangle &= |0\rangle_1\,|0\rangle_2\\
|\Psi_2\rangle &= |0\rangle_1\,|1\rangle_2\\
|\Psi_3\rangle &= |1\rangle_1\,|0\rangle_2\\
|\Psi_4\rangle &= |1\rangle_1\,|1\rangle_2\,.
\end{aligned}
\tag{17.1}
$$

Here the first (second) ket refers to the first (second) system. In that way of encoding, each system is the representative of one well-defined bit of information. In the system of the states represented by (17.1), we indeed have encoded two bits of information, since each of the two particles is represented by a choice of two orthogonal states which easily can be identified. This is just like in classical coding, where we would have two physical bits, each one carrying either the value "0" or the value "1" with four possible combinations corresponding to the four states (17.1). On the other hand, quantum physics is a holistic theory in the sense that a quantum state intrinsically is not limited by space–time allocation. Therefore, two quantum systems can carry two bits of information in such a way that neither of them carries any well-defined information on its own. An example of such maximally entangled states is given by the so-called "Bell-basis" [18].

$$
\begin{aligned}
\left|\Psi^+\right\rangle &= \frac{1}{\sqrt{2}}(|0\rangle_1\,|1\rangle_2 + |1\rangle_1\,|0\rangle_2)\\
\left|\Psi^-\right\rangle &= \frac{1}{\sqrt{2}}(|0\rangle_1\,|1\rangle_2 - |1\rangle_1\,|0\rangle_2)\\
\left|\Phi^+\right\rangle &= \frac{1}{\sqrt{2}}(|0\rangle_1\,|0\rangle_2 + |1\rangle_1\,|1\rangle_2)\\
\left|\Phi^-\right\rangle &= \frac{1}{\sqrt{2}}(|0\rangle_1\,|0\rangle_2 - |1\rangle_1\,|1\rangle_2)\,.
\end{aligned}
\tag{17.2}
$$

In which way do these four states carry two bits of information in a non-local way? Obviously, one bit of information is encoded in the states $|\Psi^+\rangle$, $|\Psi^-\rangle$ versus $|\Phi^+\rangle$, $|\Phi^-\rangle$. It is the truth-value of the proposition "the two qubits are equal". Apparently, this statement is false for the first two states and correct for the second two states. But where is the other bit of information? This is easily seen if one goes to a conjugate basis by using the transformation

$$
\begin{aligned}
|0\rangle &\to \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\\
|1\rangle &\to \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\,.
\end{aligned}
\tag{17.3}
$$

Then, one will find that in the new basis, the proposition "the two qubits are equal" will now be false for the states $|\Psi^-\rangle$ and $|\Phi^-\rangle$ and correct for the states $|\Psi^+\rangle$ and $|\Phi^+\rangle$. Together, the two statements allow a unique determination of the four non-local states, each one representing a unique combination of the truth values "0", "1" of the two propositions.

The lesson here therefore is that information can be carried by quantum systems in a very non-local way, independent of their spatio–temporal arrangements. One might notice that similar reasoning is possible for higher entangled states, as in GHZ states of the general form [11]

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) \tag{17.4}$$

and W-states [19] of the general form

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|011\rangle + |101\rangle + |110\rangle)\,. \tag{17.5}$$

Here, for example, $|011\rangle = |0\rangle_1|1\rangle_2|1\rangle_3$. In such three-qubit states evidently three propositions define all eight states of the complete basis.

We note that a general criterion exists for quantum non-locality in a two-qubit system, this is that a state violates a Bell inequality if, and only if, more than one bit of information is encoded jointly by the members of the system into the correlations.

These considerations might indicate to the reader that in quantum physics information is a more basic notion than in any classical view of the world.

17.4 Bell's Theorem, Quantum Communication and Quantum Information

Most interestingly, essentially in the last decade a fully unexpected and novel development set in. This is based on the realization that using individual quantum systems one can obtain completely novel ways of encoding, transmitting and processing information. These new ways of communication and of computation include as a fundamental concept quantum entanglement. It is safe to say that this very recent development would not have been possible without John Bell's seminal work. Therefore, it might be appropriate to review now a few of the basic procedures.

17.4.1 Quantum Dense Coding

It was first suggested by *Bennett* and *Wiesner* [20] that if two players have access to entangled states, Alice can actually encode more than one bit of information into one particle. This can easily be seen by referring to the Bell-basis of (17.2). There one sees that starting, for example, with state $|\Psi^+\rangle$,

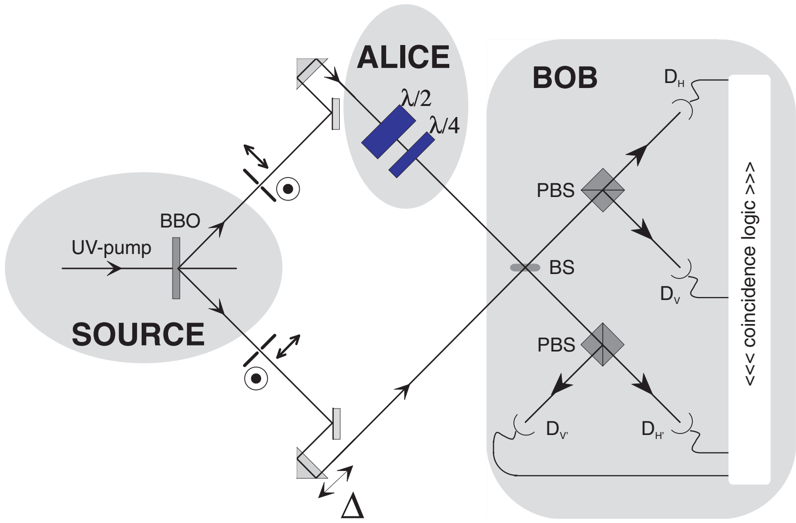

Fig. 17.2. Quantum dense coding setup. Entangled photon pairs are produced by spontaneous parametric down-conversion in a BBO crystal. One photon is sent directly to Bob, and the other is sent to Alice, who encodes the three-valued information on this photon by adjusting the $\frac{\lambda}{2}$ and $\frac{\lambda}{4}$ wave plates, and then passes them on to Bob. Bob combines the two photons in his Bell-state analyzer, which allows him to identify three of the four Bell-states [21]

one can easily obtain any of the other Bell-states by just manipulating one of the qubits, say qubit 2. If qubit 2 is flipped, then one obtains the state $|\Phi^+\rangle$, if the phase of qubit 2 is changed by π, then one obtains $|\Psi^-\rangle$, and if both procedures are applied one obtains $|\Phi^-\rangle$. Thus, Alice can send to Bob more than one bit of information by manipulating just one photon, if Bob has full access to the complete state, i.e. also the other photon, and if he can perform the Bell-state measurement determining which of the four Bell-states characterizes the complete two-qubit system. While at present complete Bell-state analyzers for independent qubits do not exist, the principle of quantum dense coding has been successfully demonstrated in an experiment [21]. In that experiment, see Fig. 17.2, it was possible to determine two Bell-states definitely, and the other two Bell-states gave the same third result. Thus, it is possible to transmit $\mathrm{ld}\,3 = 1.584...$ bits of information for each qubit manipulated, which is clearly larger than the classical limit of 1. Yet, clearly the real limit of 2 bits per pair is not surpassed. Quantum dense coding may be viewed as a direct application of the fact that information is carried in an entangled state in a holistic or non-local way, and that the experimentalist, by manipulating only part of the state, can change the complete two-qubit state such that it becomes another Bell-state, i.e. qualitatively different. We remark that while Alice only has access to one qubit, Bob needs to have access to both qubits in order to extract the information encoded by Alice.

17.4.2 Quantum Teleportation

An extension of these procedures is quantum teleportation, where Alice and Bob initially share an entangled pair of qubits [22, 23]. See Fig. 17.3.

Alice then performs a joint Bell-state measurement on her qubit together with one of the two from the entangled pair. This measurement immediately projects the second qubit from the pair into a specific state directly related to Alice's original. Depending on the result of Alice's Bell-state measurement, Bob then applies a unitary transformation determined by Alice's Bell-state measurement result independent of which state Alice initially had. Thus Bob finally obtains Alice's original state.

It may be worthwhile to analyze this experiment briefly from an information-interpretational point of view [15]. What happens is that initially, Alice and Bob share the non-local information contained in the original entangled quantum state. That is, they know how qubits 2 and 3 relate to each other, should they be measured. By obtaining the Bell-state measurement result, Alice acquires further information on how the original qubit to be teleported and her member of the entangled pair relate to each other. Thus, by a simple logical chain, it is now known how the qubit to be teleported and Bob's entangled member of the pair are related. Thus, one obtains by

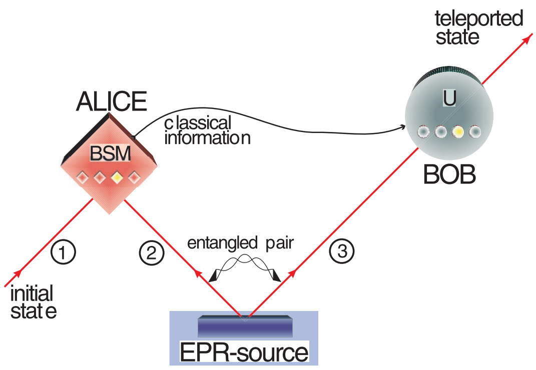

Fig. 17.3. Principle of quantum teleportation: Alice has a quantum system, particle 1, in an initial state which she wants to teleport to Bob. Alice and Bob also share an ancillary entangled pair of particles 2 and 3 emitted by an Einstein–Podolsky–Rosen (EPR) source. Alice then performs a joint Bell-state measurement (BSM) on the initial particle and one of the ancillaries, projecting them also onto an entangled state. After she has sent the result of her measurement as classical information to Bob, he can perform a unitary transformation (U) on the other ancillary particle, resulting in it being in the state of the original particle. In the case of quantum teleportation of a qubit, Alice makes a projection measurement onto four orthogonal entangled states (the Bell states) that form a complete basis. Sending the outcome of her measurement, i.e. two bits of classical information, to Bob will enable him to reconstruct the initial qubit

these measurements a sequence of relational statements, and it is therefore uniquely determined through which unitary operation Bob's qubit is related to the original. This simple information-interpretational point of view supersedes all possible paradoxes, as all that changes by Alice's measurement is the quantum state of the total system, that is, the information which the observers have. No action at a distance or other mysterious processes happen.

It is actually interesting to notice that the information contained in Alice's original state is immediately teleported over to Bob's station as soon as Alice's Bell-state measurement is performed. Bob certainly has four different states at hand, depending on the specific result of Alice's measurement. Yet, each of these states is related to the original through a rotation independent of the properties of the original. Thus, in a sense, the information is already there [24], yet Bob cannot really read it out without knowing Alice's result. Interestingly, this has operational consequences. For example, one can start a quantum computer at a time before the classical information has arrived [25] and thus save computational time.

17.4.3 Teleportation of Entanglement

A most interesting application of these ideas is the teleportation of entanglement [22, 26, 27], also called entanglement swapping. In that experiment, one teleports a photon which does not enjoy its own quantum state but is still entangled with another one. Thus, one cannot even say here that a quantum state is teleported. In a recent experiment [28], it was possible to perform the teleportation of an entangled photon in such a high quality that the original photon and the teleported one were entangled well enough to observe a violation of Bell's inequality between two independently created photons. This, more than any experiment before, underlines that Bell's theorem is not about properties particles carry, but about the information concerning possible measurement results. See Fig. 17.4.

The teleportation of entanglement also can easily be understood on the basis of an information interpretation of quantum mechanics. This is just one step further than standard teleportation. Alice and Bob initially share two relational statements, each one characterizing each of the two entangled pairs. By performing a Bell-state measurement on one photon from each pair, they obtain further relational statements and thus can conclude the chain of logical links from photon 0 to photon 3. About these two photons, they obtain the same joint statement as about the possible states of the Bell basis of (17.2). Again, these are statements about possible experimental results only, should an experiment actually be performed.

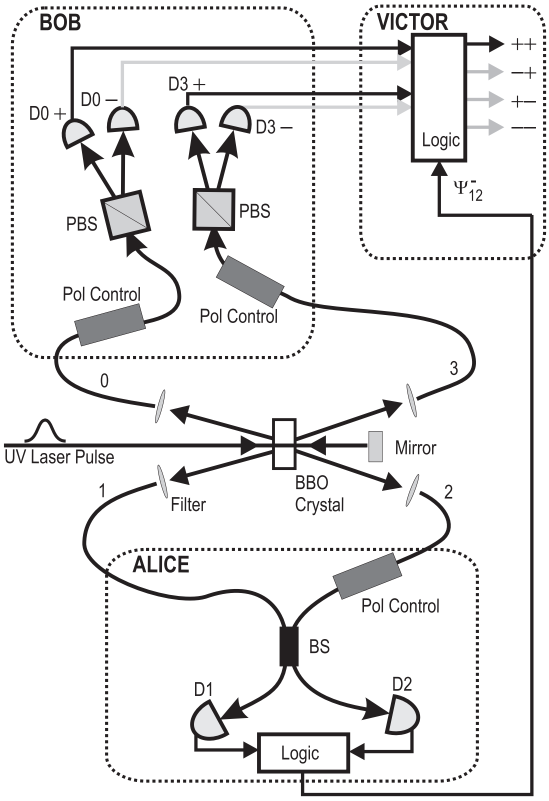

Fig. 17.4. Setup of the experiment teleporting an entangled photon. Two entangled photon pairs are produced by down-conversion in a BBO crystal, pumped by femtosecond UV-laser pulses traveling through the crystal in opposite directions. All photons are collected in single-mode optical fibers for further analysis and detection. For performing the Bell-state analysis, photons 1 and 2 interfere at a fiber beam splitter, where one arm contains a polarization controller for compensating the polarization rotation introduced by the optical fibers. Photons 0 and 3 were sent to Bob's two-channel polarizing beam splitters for analysis, and the required orientation of the analyzers was set with polarization controllers in each arm. All photons were detected with silicon avalanche photodiodes, with a detection efficiency of about 40%. Alice's logic circuit detected coincidences between detectors D1 and D2. It is essential that she passes the result as a classical signal to Victor, who determines whether Bob's detection events violate Bell's inequality [28]

17.4.4 Quantum Cryptography

The technically most advanced application of fundamental quantum concepts in communication is quantum cryptography. The protocols closest related to Bell's ideas are again based on the use of entangled states, as first suggested by Ekert [30]. Without going into details, one uses an entangled state to create the same identical key by independent measurements by two observers, Alice and Bob. The big advantage of Ekert-type quantum cryptography is that the cryptographic key does not need to be transported from A to B, but is really created by measurements in the same basis as a sequence of entangled pairs. Thus, one of the essential security problems of classical cryptography, namely the necessary transportation of a classical key, immediately vanishes. Another advantage of quantum cryptography with entangled states is that any eavesdropper can readily be identified by just observing whether the data measured by Alice and Bob still violate a Bell inequality. If such is the case, then no essential information could have leaked out to an eavesdropper, and Alice and Bob can readily use the key obtained. See Figs. 17.5 and 17.6.

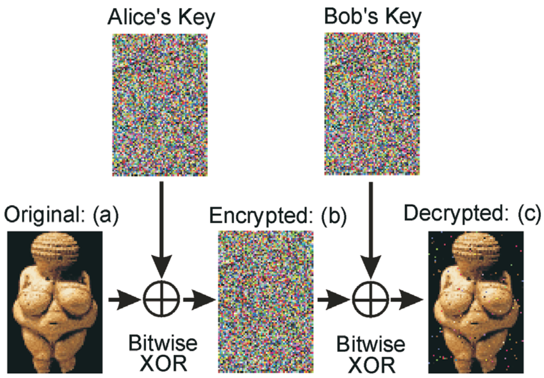

Fig. 17.5. In an experimental realization using polarization entangled pairs of photons, a key was created and it was used to actually transmit visual information, in this case a picture of the famous Venus von Willendorf sculpture [29]

There is a very fundamental information-theoretic interpretation of the security of entangled-state quantum teleportation. The basic observation is that a two-qubit system can carry only two bits of information. For a maximally entangled state, three bits are defined non-locally, i.e. the two bits are all used up to define the entanglement. Now, if the eavesdropper extracts any information, this can only be at the expense of the two non-local bits, thus reducing the entanglement.

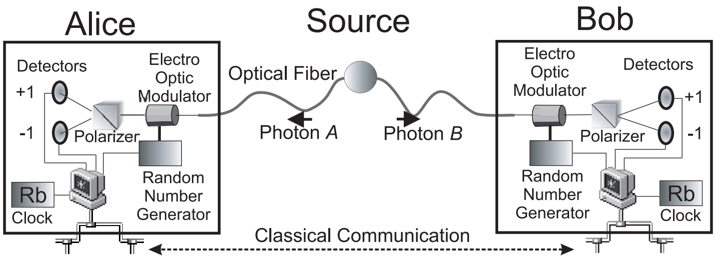

Fig. 17.6. Polarization-entangled quantum cryptography [29]. The most salient feature of this experiment is, besides the creation of the entangled pair, that each photon is sent to experimental stations which finally are separated by more than 300 m. In each station, one has an independent Rubidium clock, which simply registers the photon arrival times and two-channel polarizers, where one channel is identified as "0" and the other is identified as "1". If the two polarizers are parallel, the correlations are perfect, and Alice and Bob obtain the same random sequence after eliminating those events where only one photon was registered. In order to identify a possible eavesdropper, both Alice and Bob randomly switch between two bases independent of each other. They then identify those situations where they happen to have the same basis, and they know that in these situations they obtain the key. The situations where they happen to have different bases can be used to check for a possible eavesdropper

17.5 John Bell's Desiderata and the Interpretation of Quantum Mechanics

In his famous paper "Against measurement" [31], John Bell suggests:

"*Here are some words which, however legitimate and necessary in application, have no place in a formulation with any pretension to physical precision:* ***system, apparatus, environment, microscopic, macroscopic, reversible, irreversible, observable, information, measurement.***"

By now, the reader might have gathered that the present author does not agree with John Bell's statement. In contrast it is suggested that *information* is the most basic notion of quantum mechanics, and it is *information* about possible *measurement* results that is represented in the quantum states. *Measurement* results are nothing more than states of the classical *apparatus* used by the experimentalist. The quantum *system* then is nothing other than the consistently constructed referent of the *information* represented in the quantum state. In a *measurement*, one of the possible *measurement* results becomes reality with a relative frequency as indicated by the quantum state. For a quantum *system*, the *environment* therefore is just an external bath *information* can leak into, and the situation is *reversible* if this *information* can be recovered somehow from the environment, and *irreversible* if that is not the case.

The point where I agree with John Bell is that *microscopic* and *macroscopic* should not command any fundamental place in any physical theory. Experimental progress will certainly make it possible to push the regime where quantum phenomena have been demonstrated very far into what we would consider *macroscopic*. Thus, while the dichotomy *microscopic–macroscopic* should not have any place in a physical theory, the dichotomy *quantum–classical* is a most fundamental one.

While it is clear that I respectfully disagree with the general philosophical position of John Bell concerning the foundations of quantum mechanics, one cannot but show deep respect for his high intellectual integrity. His way of thinking led him to advocate the quest for more complete theories than quantum mechanics, which, as he hoped, would finally explain why individual events happen. Even if the interpretation of quantum mechanics based on information – which runs completely against his expectation - will turn out to be the correct one, then John Bell will turn out to have raised the correct challenges.

Acknowledgment

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