Three-photon GHZ entanglement and quantum information

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Abstract

We report the first experimental test of local realism versus quantum mechanics that utilizes three-particle entanglement. This test, put forward by Greenberger, Horne and Zeilinger (GHZ), addresses a strong conflict with local realism for those cases where quantum theory makes definite, i.e. nonstatistical, predictions. This is in contrast to the case of Einstein-Podolsky-Rosen experiments with two entangled particles, where according to Bell's theorem the conflict with local realism only arises for the statistical predictions. Our experimental results are in agreement with the quantum prediction and in distinct conflict with a local realistic interpretation.

1 Introduction

Ever since its introduction by Schrödinger [1] entanglement has commanded a central position in the discussions of the interpretation of quantum mechanics. Originally that discussion has focused on the proposal by Einstein, Podolsky and Rosen (EPR) of measurements performed on two spatially separated entangled particles [2]. Most significantly John Bell then showed that there is a conflict between any attempt to explain the correlations observed in such systems by a local realistic model and the predictions made by quantum mechanics [3].

An increasing number of experiments on entangled particle pairs having confirmed the statistical predictions of quantum mechanics [4],[5],[6] have thus provided increasing evidence against local realistic theories. Yet, one might find some comfort in the fact that such a realistic and thus classical picture can explain perfect correlations and is only in conflict with statistical predictions of the theory. After all, quantum mechanics is statistical in its core structure. In other words, for entangled particle pairs the cases where the result of a measurement on one particle can definitely be predicted on the basis of a measurement result on the other particle can be explained by a local realistic model. It is only that subset of statistical correlations where the measurement results on one particle can only be predicted with a certain probability which cannot be explained by such a model.

Yet in 1989 it was shown by Greenberger, Horne and Zeilinger (GHZ) that for certain three- and four-particle states [7], [8] a conflict with local realism arises even for perfect correlations. That is, even for those cases where, based on measurement on $N-1$ of the particles, the result of the measurement on particle N can be predicted with certainty. Local realism and quantum mechanics here both make definite but completely opposite predictions.

Utilizing a recently developed source for three-photon GHZ-entanglements in the present paper we report on the first realization of such a three-particle test against local realism [9].

2 The conflict with local realism

How are the quantum predictions of a three-photon GHZ-state in stronger conflict with local realism than the conflict for two-photon states as implied by Bell's inequalities? To answer this, consider the state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|H\rangle_1|H\rangle_2|H\rangle_3 + |V\rangle_1|V\rangle_2|V\rangle_3), \quad (1)$$

where H and V denote horizontal and vertical linear polarizations respectively. This state indicates that the three photons are in a quantum superposition of the state $|H\rangle_1|H\rangle_2|H\rangle_3$ (all three photons are horizontally polarized) and the state $|V\rangle_1|V\rangle_2|V\rangle_3$ (all three photons are vertically polarized).

Consider measurements of linear polarization along directions H'/V' rotated by 45° with respect to the original H-V directions, or of circular polarization L/R (left-handed, right-handed). These new polarizations can be expressed in terms of the original ones as

$$|H'\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle), \quad |V'\rangle = \tfrac{1}{\sqrt{2}}(|H\rangle - |V\rangle), (2)$$

$$|R\rangle = \frac{1}{\sqrt{2}}(|H\rangle + i|V\rangle), \quad |L\rangle = \tfrac{1}{\sqrt{2}}(|H\rangle - i|V\rangle) (3)$$

Let us denote $|H\rangle$ by $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|V\rangle$ by $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$, they are thus the two eigenstates of Pauli operator σ_z, correspondingly with the eigenvalues $+1$ and -1. We can also easily verify that $|H'\rangle$ and $|V'\rangle$ or $|R\rangle$ and $|L\rangle$ are two eigenstates for Pauli operator σ_x or σ_y with the values +1 and −1, respectively. For convenience we will refer to a measurement of H'/V' linear polarization as a y measurement and of L/R circular polarization as a y measurement.

Representing state (1) in the new states using Eqs. (2) and (3) one obtains the quantum predictions for measurements of these new polarizations. For example, for the case of measurement of circular polarization on, say, both photon 1 and 2, and measurement of linear polarization H'/V' on photon 3, denoted as a yyx experiment, the state may be expressed as

$$\begin{aligned} |\Psi\rangle = {} & \tfrac{1}{2}(|R\rangle_1|L\rangle_2|H'\rangle_3 + |L\rangle_1|R\rangle_2|H'\rangle_3 \\ & + |R\rangle_1|R\rangle_2|V'\rangle_3 + |L\rangle_1|L\rangle_2|V'\rangle_3) \end{aligned} \tag{4}$$

This expression has a number of significant implications. Firstly, we note that any specific result obtained in any individual or in any two-photon joint measurement is maximally random. For example, photon 1 will exhibit polarization R or L with the same probability of 50%, or photons 1 and 2 will exhibit polarizations RL, LR, RR, or LL with the same probability of 25%.

Secondly, because only those terms yielding a −1 product for a yyx measurement appear in the expression, we realize that, given any two results of measurements on any two photons, we can predict with certainty what the result of the corresponding measurement performed on the third photon will be. For example, suppose photons 1 and 2 both exhibit right-handed (R) circular polarization (i.e., both having the value +1). By the third term in Eq.(4), photon 3 will definitely be V' polarized (i.e., having the value -1).

By cyclic permutation, we can obtain analogous expressions for any experiment measuring circular polarization on two photons and H'/V' linear polarization on the remaining one. Thus, in every one of the three yyx, yxy and xyy experiments any individual measurement result both for circular polarization and for linear H'/V' polarization can be predicted with certainty for any one of the three photons given the corresponding measurement results of the other two.

Now we will analyze the implications of these predictions from the point of view of local realism. First note that the predictions are independent of the spatial separation of the photons and independent of the relative time order of the measurements. Let us thus consider the experiment to be performed such that the three measurements are performed simultaneously in a given reference frame, say, for conceptual simplicity, in the reference frame of the source. Thus we can employ the notion of Einstein locality which implies that no information can travel faster than the speed of light. Hence the specific measurement result obtained for any photon must not depend on which specific measurement is performed simultaneously on the other two nor on the outcome of these measurements. The only way then to explain from a local realist point of view the perfect correlations discussed above is to assume that each photon carries elements of reality for both x and y measurements considered and that these elements of reality determine the specific individual measurement result [7],[8],[10].

Calling these elements of reality of photon i X_i with values +1(-1) for $H'(V')$ polarizations and Y_i with values +1(-1) for polarizations $R(L)$ we obtain the relations $Y_1Y_2X_3 = -1$, $Y_1X_2Y_3 = -1$ and $X_1Y_2Y_3$ in order to be able to reproduce the quantum predictions of Eq. (4).

We now consider a fourth experiment measuring linear H'/V' polarization on all three photons, i.e. a xxx experiment. What possible outcomes will a local realist predict here based on the elements of reality introduced to explain the earlier yyx, yxy and xyy experiments? Due to Einstein locality any specific measurement for x must be independent of whether on the other photon a x or y measurement is performed. As $Y_iY_i = +1$, we can write $X_1X_2X_3 = (X_1Y_2Y_3)\cdot(Y_1X_2Y_3)\cdot(Y_1Y_2X_3)$ and obtain $X_1X_2X_3 = -1$. Thus from a local realist point of view theonly possible results for a xxx experiment are $V_1'V_2'V_3'$ $H_1'H_2'V_3'$, $H_1'V_2'H_3'$, and $V_1'H_2'H_3'$.

How do these predictions of local realism for a xxx experiment compare with those of quantum physics? If we express the state given in Eq. (1) in terms of H'/V' polarization using Eq.(2) we obtain

$$\begin{aligned} |\Psi\rangle = {} & \tfrac{1}{2}(|H'\rangle_1|H'\rangle_2|H'\rangle_3 + |H'\rangle_1|V'\rangle_2|V'\rangle_3 \\ & + |V'\rangle_1|H'\rangle_2|V'\rangle_3 + |V'\rangle_1|V'\rangle_2|H'\rangle_3)\,. \end{aligned} \tag{5}$$

Here the local realistic model predicts none of the terms occurring in the quantum prediction. This implies that whenever local realism predicts a specific result definitely to occur for a measurement on one of the photons based on the results for the other two, quantum physics definitely predicts the opposite result. For example, if two photons are both found to be H' polarized, local realism predicts the third photon to carry polarization V' while the quantum state predicts H'.

Thus, while in the case of Bell's inequalities for two photons the conflict between local realism and quantum physics arises for statistical predictions of the theory, for three entangled particles the difference occurs already for the definite predictions, statistics is now only due to inevitable measurement errors occurring in any and every experiment, even in classical physics.

3 Generating three-photon GHZ entanglement

The experiment reported here is based on the observation of three-photon GHZ entanglement that was achieved recently in our laboratory [11]. The method to produce GHZ entanglement for three spatially separated photons is a further development of the techniques that have been used in our previous experiments on quantum teleportation [12] and entanglement swapping [13]. Here we only present a brief summary of our method of the three particle entanglement generation and refer to our original paper [11] for further elaboration.

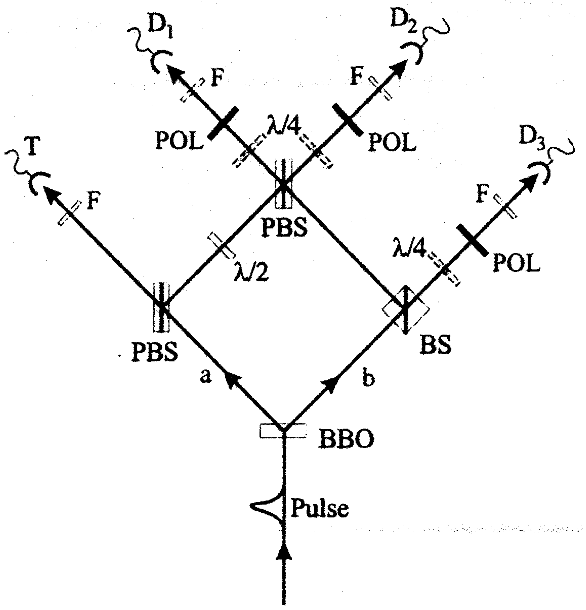

Figure 1: Sketch of the experimental setup for observing GHZ entanglement. Pairs of polarization-entangled photons are generated by a short pulse of UV-light ($\approx$ 200fs, λ = 394nm from a frequency-doubled, mode-locked Ti-sapphire laser) in a nonlinear crystal (Beta-Barium-Borate, BBO). The setup uses a beam splitter BS, polarizing beam splitters PBS and narrow-bandwidth filters F ($\approx$ 4nm). In between the two PBSs, vertical polarization is rotated to 45° polarization using a $\lambda/2$ plate. Conditioned on the registration of one photon at the trigger detector T, the three photons registered at D$_1$, D$_2$ and D$_3$ exhibit the desired GHZ correlations. Polarizers and $\lambda/4$ plates have been used to perform polarization analysis. More specifically, we insert a polarizer oriented at 45° or -45° in front of a certain detector to perform a H' or V' polarization measurement respectively, and further insert a $\lambda/4$ plate in front of the polarizer to perform a R or L circular polarization measurement.

A schematic drawing of our experimental setup is shown in Figure 1. We use two pairs of polarization-entangled photons both in the state [14]

$$\frac{1}{\sqrt{2}}(|H\rangle_a|V\rangle_b + e^{i\chi}|V\rangle_a|H\rangle_b). \qquad (6)$$

This is a superposition of the possibility that the photon in arm a is horizontally polarized and the one in arm b is vertically polarized $|H\rangle_a|V\rangle_b$, with the opposite possibility $|V\rangle_a|H\rangle_b$. As proposed in Ref.[15], the main idea is to detect one photon from two pairs of polarization-entangled photons such that any information as to which pair it belongs is erased. This photon is observed by detector T (see Fig. 1). Observation of the remaining three photons, one in each of the outgoing beams, then exhibits GHZ-entanglement in the state

$$\frac{1}{\sqrt{2}}(|H\rangle_1|H\rangle_2|H\rangle_3 + |V\rangle_1|V\rangle_2|V\rangle_3). \qquad (7)$$

For simplicity of argumentation we have assumed here that for photon 3 H and V are defined at right angles compared to photons 1 and 2. We adjust the path lengths a and b and use the same narrow band-width filters as in ref.[11] to stretch the coherence time of the photons to approximately 500 femtoseconds, which is substantially longer than the pulse length (200fs) that created the photon pairs. This effectively erases temporal pair-correlation information [16].

4 Experimental test of local realism versus quantum mechanics

As explained in section 2 demonstration of the conflict between local realism and quantum mechanics for GHZ entanglement consists of four experiments each with three spatially separated polarization measurements. First, one performs yyx, yxy, and xyy experiments. If the results obtained are in agreement with the predictions for a GHZ state then the predictions for an xxx experiment are exactly opposite for a local realist theory as to that of quantum mechanics.

For each experiment we have 8 possible outcomes of which ideally 4 should never occur. Obviously, no experiment neither in classical physics nor in quantum mechanics can ever be perfect and therefore, due to principally unavoidable experimental errors, even the outcomes which should not occur will occur with some small probability in any realistic experiment.

All individual fractions which were obtained in our yyx, yxy and xyy experiments are shown in Figs. 2(a), 2(b) and 2(c), respectively. From the data we conclude that we observe the GHZ terms of Eq.(4) predicted by quantum mechanics in 85% of all cases and in 15% we observe spurious events.

If we assume the spurious events are just due to experimental errors and thus conclude within the experimental accuracy that for each photon 1, 2 and 3, quantities corresponding to both c and l measurements are elements of reality. Consequently a local realist if he accepts that reasoning would thus predict that for a xxx experiment, the combinations $V'V'V', H'H'V', H'V'H'$, and $V'H'H'$ will only be observable (Fig. 3(b)). However referring back to our original discussion we see that quantum mechanics predicts the exact opposite terms should be observed (Fig. 3(a)). To settle this conflict we then perform the actual xxx experiment. Our results, shown in Fig. 3(c), disagree with the local realism predictions and are consistent with the quantum mechanical predictions. The individual fractions in Fig. 4(c) clearly show within our experimental uncertainty that only those triple coincidences predicted by quantum mechanics occur and not those predicted by local realism. In this sense, we claim that we experimentally realized the first three-particle test of local realism following the GHZ argument.

We have already seen that the observed results for a xxx experiment confirm the quantum mechanical predictions when we assume that deviations from perfect correlations in our experiment, and in any experiment for that matter, are just due to unavoidable

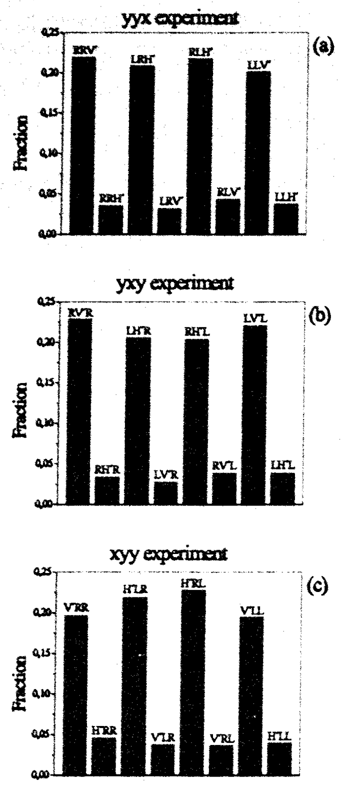

Figure 2: Fractions of the various outcomes observed in the yyx, yxy, and xyy experiments. The experimental data show that we observe the GHZ terms predicted by quantum physics in 85% of all cases and in 15% the spurious events.

experimental errors. However, a local realist might argue against that approach and suggest that the non-perfect detection events indicate that the original GHZ argumentation cannot succeed.

To face this blame, a number of inequalities for N-particle GHZ states have been derived [19], [20], [21]. For instance, Mermin's inequality for a three-particle GHZ state reads as follows[19]

$$|\langle\sigma_x\sigma_y\sigma_y\rangle+\langle\sigma_y\sigma_x\sigma_y\rangle+\langle\sigma_y\sigma_y\sigma_x\rangle-\langle\sigma_x\sigma_x\sigma_x\rangle| \leq 2, \tag{8}$$

where symbol $\langle\cdots\rangle$ denotes the expectation value of a specific physical quantity. The necessary visibility to violate this inequality is 50%. The visibility observed in our GHZ experiment is 71% ± 4% and obviously surpasses the 50% limitation. Substituting our results measured in the yyx, yxy and xyy experiments into the left-hand of Eq. 8 we obtain the following constraint

$$\langle\sigma_x\sigma_x\sigma_x\rangle \leq -0.1, \tag{9}$$

by which a local realist can thus predict that in a xxx experiment the probability fraction for the outcomes yielding a +1 product, denoted by $P(xxx = +1)$, should be no larger than 0.45 ± 0.03 (also refer to the first bar in Fig. 4).

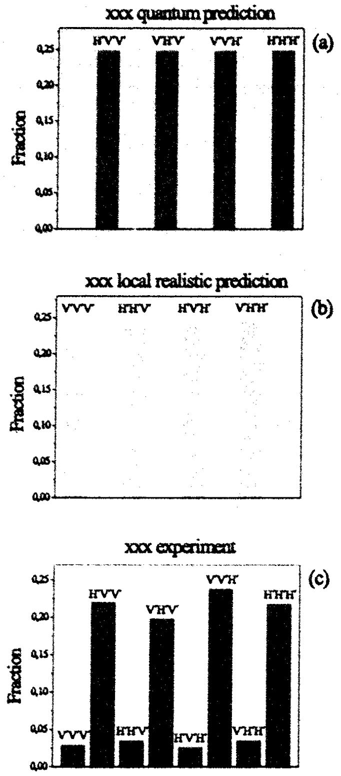

Figure 3: The conflicting predictions of quantum physics (a) and local realism (b) of the fractions of the various outcomes in a xxx experiment for perfect correlations. The experimental results (c) are in agreement with quantum physics within experimental errors and in disagreement with local realism.

What is the quantum prediction for a xxx experiment following from the yyx, yxy and xyy experiment results? Because our experimental visibility is due mainly to the finite width of the interference filters, the finite pulse duration, quantum mechanically it is expected that the same visibility should be observed in a xxx experiment, hence we obtain the quantum prediction as shown in the second bar of Fig. 4.

The visibility observed in our xxx experiment is 74% ± 4%, which consequently gives $P(xxx = +1) = 0.87 \pm 0.04$ (shown in the third bar of Fig. 4). Comparing the results in Fig. 4 we therefore conclude that our experimental results well verify the quantum prediction while contradicting the local realistic prediction by over 8 standard deviations, and there is no local hidden variable model which is capable to describe our experimental results.

5 Discussion and Prospects

Since the first tests of quantum mechanics versus local realism there have been strong debates as to what extent these experiments fully refute the notion of local realism. In this paper we presented the first experimental test of quantum nonlocality in three-particle

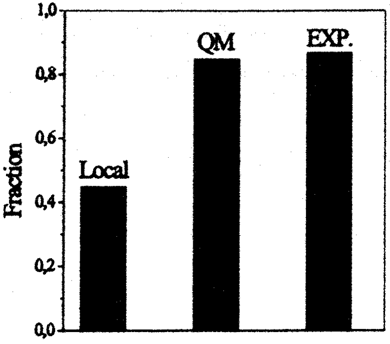

Figure 4: Predictions of local realism(Local)and quantum physics(QM) for the probability fraction of the outcomes yielding a +1 product in a xxx experiment based on the real experimental data measured in the yyx, yxy and xyy experiments. The experimental results (EXP.) are well in agreement with quantum physics and in distinct conflict with local realism.

entanglement where the theories make definite but opposite predictions. Our experiment fully confirms the predictions of quantum mechanics and is in conflict with local hidden variable theories.

However, we have by no means the illusion that our new test will once and for all convince the disbelievers of quantum mechanics. Our experiment shares with all existing two-particle tests of local realism the property that the detection efficiencies are rather low. Therefore we had to invoke the fair sampling hypothesis [17],[18] where it is assumed that the registered events are a faithful representative of the whole ensemble.

It will be interesting to further study GHZ correlations over large distances with space-like separated randomly switched measurements [6], to extend the techniques used here to the observation of multi-photon entanglement [22], to observe GHZ-correlations in massive objects like atoms [23], and to investigate possible applications in quantum computation and quantum communication protocols [24],[25].

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