The Equivalence Principle in Quantum Mechanics and Neutrons that Fall Upwards

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We report the deflection of neutrons by Earth's gravitational field, when propagating along the Brillouin-zone boundary inside a silicon crystal. The deflection is enhanced by more than 5 orders of magnitude compared to the deflection in free space due to the minute effective mass of a neutron inside the crystal. Neutrons in the negative effective mass state fall upwards. In connection with an earlier observation of the deflection in a crystal due to a Coriolis force this experiment enables a verification of the equivalence principle in quantum mechanics while the strong interaction is present simultaneously with inertial or gravitational forces.

The principle of equivalence of the inertial and the gravitational mass is the foundation of the general theory of relativity. It has been tested with an accuracy of better than 10^{-12} with macroscopic test masses[1), 2) 3)]. This is different in quantum mechanics, where in the COW experiments the phase shift due to the gravitational field of the Earth was measured with a neutron interferometer[4)]. That experiment requires a wave mechanical description and cannot be understood by using classical mechanics alone. The recent experiments by Littrell et.al.[5)] and Werner et. al.[6)] showed a deviation from theory[7)] in the range of 1%. This is significantly larger than the measurement accuracy of 0.1%.

Here we report a new experiment observing gravitational effects in the quantum limit: The effective mass enhanced deflection of neutrons in a gravitational field and we compare the results with an earlier inertial experiment. The experiments are based on the fact that the effective mass of a neutron inside a silicon crystal can be reduced by a factor of more than 10^5 compared to the mass of the free particle. This ratio can be explained by solving the Schrödinger equation. The solution is a wave function consisting of a coherent superposition of plane waves. Thus the experiments can only be understood by wave mechanics.

Another remarkable feature of these experiments is the existence of two effective mass states inside the crystal, one with a positive and one with a negative effective mass. Neutrons in the negative effective mass state are accelerated opposite to the appropriate component of the external force and thus can fall upwards in Earth's gravitational field. The trajectories in the experiment are not analogous to those of a free particle because they appear bent even in a frame of reference which is freely falling in Earth´s gravitational field.

With the effective mass m* one obtains an equation of motion which is a generalization of Newton's second law[8] (μ, ν = x, y, z):

$$\left(\frac{1}{m^*}\right)_{\mu\nu} F_\nu = a_\mu . \tag{1}$$

The effective mass is calculated from the dispersion relation $\omega(\mathbf{K})$ when $\mathbf{K}$ is the wave vector inside the crystal

$$\left(\frac{1}{m^*}\right)_{\mu\nu} = \frac{1}{h}\frac{\partial^2}{\partial K_\mu \partial K_\nu}\omega(\mathbf{K}). \tag{2}$$

The direction and speed of the neutron propagation is given by the group velocity $\mathbf{v} = \nabla_K \omega(\mathbf{K})$.

The dispersion relation in a crystal can be derived by solving the Schrödinger equation within the periodic crystal potential. Assuming neutrons traveling close to the Bragg angle Θ_B - equivalent to the Brillouin-zone boundary for one single set of parallel lattice planes characterized by the reciprocal lattice vector $\mathbf{G}$ - it is sufficient to use an ansatz for the wave function that is a superposition of two plane waves, one propagating approximately parallel to the incident monochromatic neutron beam and one in the Bragg reflected direction ("two beam approximation")[9].

$$\Psi = A_\pm e^{i\mathbf{K}_\pm \mathbf{r}} + B_\pm e^{i(\mathbf{K}_\pm + \mathbf{G})\mathbf{r}} \tag{3}$$

The calculation shows that two solutions of this form exist (index $\pm$) in the crystal for one given wavevector $\mathbf{k}$ outside the crystal. These two solutions correspond to the two effective mass states mentioned. For a given neutron energy E there are only nonzero solutions for the amplitudes in Eq. 3, when the following dispersion relation is fulfilled by the wavevector $\mathbf{K}$.

$$E = h\omega(\mathbf{K}_{\pm}) = \frac{h^2 K_{\pm}^2}{2m} + V_0 + V_G(y \pm \sqrt{1+y^2}) \tag{4}$$

Here $y = h^2(G^2 - 2\mathbf{K}_{\pm}\mathbf{G})/(4mV_G)$ is a dimensionless parameter describing the deviation from the Bragg angle inside the crystal, V_0 is the mean crystal potential and V_G is the Fourier component of the crystal potential corresponding to **G**. In the symmetric Laue case (**G** is parallel to the entrance surface, see Fig. 1) y is related to the deviation from the Bragg angle $\delta\Theta_B$ by $y = (E/V_G)\,sin\,(2\Theta_B)\,\delta\Theta_B$, that follows from the boundary conditions at the crystal surface.

From Eq. 4 it follows that the group velocity inside the crystal is given by[10]:

$$\mathbf{v}_{\pm} = \frac{h}{m}\left[\mathbf{K}_{\pm} - \frac{\mathbf{G}}{2}(1 \pm \Gamma)\right] \tag{5}$$

The parameter Γ of Eq. 5 which is given by $\Gamma = y_{\pm}/(\sqrt{1+y_{\pm}^2}) = tan\Omega/tan\Theta_B$ characterizes the slope of the neutron trajectory with respect to the lattice planes. Here Ω is the angle between the lattice planes and the group velocity. As the typical neutron kinetic energy E is about five orders of magnitude larger than the crystal potential, and Γ depends on the ratio E/V_G, a small change in the deviation from the Bragg angle results in a large change in the direction of **v** (angular amplification).

From the effective mass tensor of the equation of motion Eq. 1 combined with Eq. 2 and Eq. 4 the acceleration of a neutron due to an external force is obtained:

$$\mathbf{a} = \frac{\mathbf{F}}{m} \pm (1-\Gamma^2)^{3/2}\mathbf{F}_{\perp}\frac{E_G}{2mV_G} \tag{6}$$

Here $\mathbf{F}_{\perp} = \mathbf{F}\cdot\hat{\mathbf{x}}$ is a vector parallel to **G** with a length equal to the component of **F** parallel to **G**. The energy E_G is the characteristic kinetic energy $h^2G^2/2m$ of a neutron with a wavelength equal to the lattice spacing $2\pi/G$. For the Si(220) reflex used in the experiments the ratio $E_G/(2V_G) = 2.1\times10^5$. The two possible signs correspond to the two effective mass states mentioned above. In our experiments the deflection and the collimation chosen were such that $|\Gamma| \ll 1$. In this case the first term in Eq. 6 corresponding to the acceleration of a free particle can be neglected and the acceleration is approximately independent of Γ. For that reason the trajectories appear bent even in a frame of reference that is freely falling in a gravitational field. The component of the acceleration perpendicular to the lattice planes is then given by:

$$\mathbf{a}_{\perp} = \pm(E_G/(2mV_G))\,\mathbf{F}_{\perp} \tag{7}$$

The minus sign in Eq. 7 results in an acceleration opposite to $\mathbf{F}_{\perp}$.

The external force in our earlier experiment[11)] was an inertial force in an accelerated frame namely the Coriolis force in a rotating crystal[12)]. In our present experiment, which was carried out at the T13A test facility of the ILL, the force on the neutron in the gravitational field of the Earth was used. For our geometry $\mathbf{F}_{\perp}$ is given by $m_g g\, cos(\Theta_B)\, sin(\phi)$ when ϕ is the crystal tilt angle of **G** in Earths gravitational field. It is defined as zero if **G** lies in the horizontal plane. The setup and the resulting neutron trajectories are shown in Fig. 2.

Due to the angular amplification effect a high collimation ($\approx 10^{-7}$ rad) is necessary to perform the experiments because the divergence in the crystal has to be small compared to the deflection. This is achieved by using a setup consisting of two silicon single crystals (L=52mm) joined by a common base and cut from one crystal to provide parallel lattice planes in both parts. The first crystal together with two Cadmium slits at the entrance and the exit acts as a collimator by selecting only symmetric trajectories in the first crystal. Neutrons following the symmetric trajectories in the first crystal show a well defined deviation from the Bragg angle. In the second crystal the trajectories of the two states are well separated (see Fig. 2). The spatial separation d of the two effective mass states after passing through both crystals (length L) due to an acceleration $a_{\perp}$ follows from Eq. 5 and Eq. 7 as

$$d = 2a_{\perp}(L/v_z)^2 cos(\Theta_B)\,(1 + s/L)\,. \tag{8}$$

Here s is the width of the gap between the two crystals (see Fig. 2) and v_z is the component of the group velocity parallel to the lattice planes. The factor $(1 + s/L)$ is due to the angular deflection in the gap because the deflection there changes the deviation from the Bragg angle and therefore v_x at the entrance of the second crystal is increased.

The experiment is very sensitive to a bending of the crystal under its own weight[4)]. Such a bending was avoided by placing the crystal in a fluid of equal density as silicon (approx. 274 g $ZnBr_2$ in 100 ml D_2O). The appropriate concentration of $ZnBr_2$ was determined by placing a small piece of silicon into a solution with roughly calculated concentration and adding the appropriate component until the small test piece floated freely in the fluid.

The container for the crystal and the fluid had to meet several requirements: To decrease neutron losses inside the fluid due to absorption and scattering the distance between the walls of the container and the crystal had to be kept as small as possible everywhere the beam passed through. Another requirement is that the material of the container should not react with the solution. Finally the wall material in the gap between the two silicon crystals had to be plane enough to avoid defocussing effects due to refraction. To meet the last two requirements mentioned the container was made of quartz glass.

The experiment employed a monochromatic neutron beam with a mean wavelength of 2.35 Å provided by a graphite monochromator. The assembly containing the silicon crystals and the detector could be tilted around the axis of the incoming neutron beam. After passing through the crystal the neutrons were detected in a position sensitive detector (ND&M Handmonitor)[13)].

The axis of rotation of the tilt mechanism was adjusted to coincide with the axis of the incident neutron beam and the mean wavelength was determined by replacing the silicon crystal by a pyrolytic graphite crystal serving as analyser for the wavelength. Behind this crystal a ^{3}He counter detected the transmitted neutrons. By turning the graphite monochromator around an axis perpendicular to the axis of rotation of the tilt turntable two dips in the transmitted intensity occurred at the two Bragg angles. By comparing the angles of minimal transmission for different tilt angles any misalignement could be determined and corrected. After this procedure the maximum difference of the angles for the Bragg position for different tilt angles was 0.02° which is negligible in comparison to the mosaic spread of the monochromator crystal of 0.4° and the resulting divergence of the beam.

After replacing the graphite by the container and the Si crystal, the distance of the two effective mass peaks (Fig. 3) was measured at 12 different tilt angles. The measurement time for each position varied between 3.5 and 17.3 hours.

The measured separation of the two effective mass states of $(d/\sin\phi)_{exp} = (4.740 \pm 0.05)$ mm agreed with the predicted value of $d/\sin\phi = (4.737 \pm 0.011)$ mm calculated using known crystal parameters. The error in the theoretical value arises from uncertainties in the wavelength and the crystal dimensions. The experimental errors are mainly due to counting statistics.

The experimental results (see Fig. 4) showed a constant offset of 1.3 mm in the separation of the two effective mass peaks. This can be explained by a small intrinsic bending of the crystal and can be interpreted for example as an angle of 0.06 μrad between the lattice planes of the first and second crystal. An offset of the same order of magnitude was observed in all former experiments with the same crystal[11, 12].

From the results of the present experiment and the earlier one using Coriolis force deflection an experimental value for the ratio between the gravitational mass m_g and the inertial mass m_i of the neutron can directly be obtained by realizing that m in Eq. 7 is in effect m_i and $\mathbf{F}_\perp$ is the x component of the gravitational Force $m_g\mathbf{g}$. From the results of our previous experiment with the rotating crystal all factors except $\mathbf{a}_\perp$ in Eq. 8 can be determined experimentally. By comparing both experiments we obtain the ratio:

$$m_g/m_i = 1.011 \pm 0.015 \tag{9}$$

directly without having to report to any crystal properties.

The accuracy of this result is limited by counting statistics and by the uncertainty of the measurement of the angular velocity of the crystal in the Coriolis experiment. An improved version of these experiments could certainly reduce the errors to the 0.1% level.

In comparison to the COW experiments an essential difference is that in our experiments the effects inside the crystal are dominant, while in the interferometric experiments the nonzero thickness of the crystals results only in small, but important corrections. Thus our experiments allowed a verification of the equivalence principle in quantum mechanics and in the presence of strong interaction.

We wish to thank Prof. W. Gläser for the opportunity to work at the FRM, W. Drexel for his support at the ILL and C. Rausch for helping us with the detector. We appreciate the expert fabrication of the crystal by Cliff Holmes at the machine shop of the physics department of the University of Missouri. This work was supported by the Austrian Science Foundation FWF project number S6503, by the US NSF grant number PHY 97-22614 and the TMR program of the EU.

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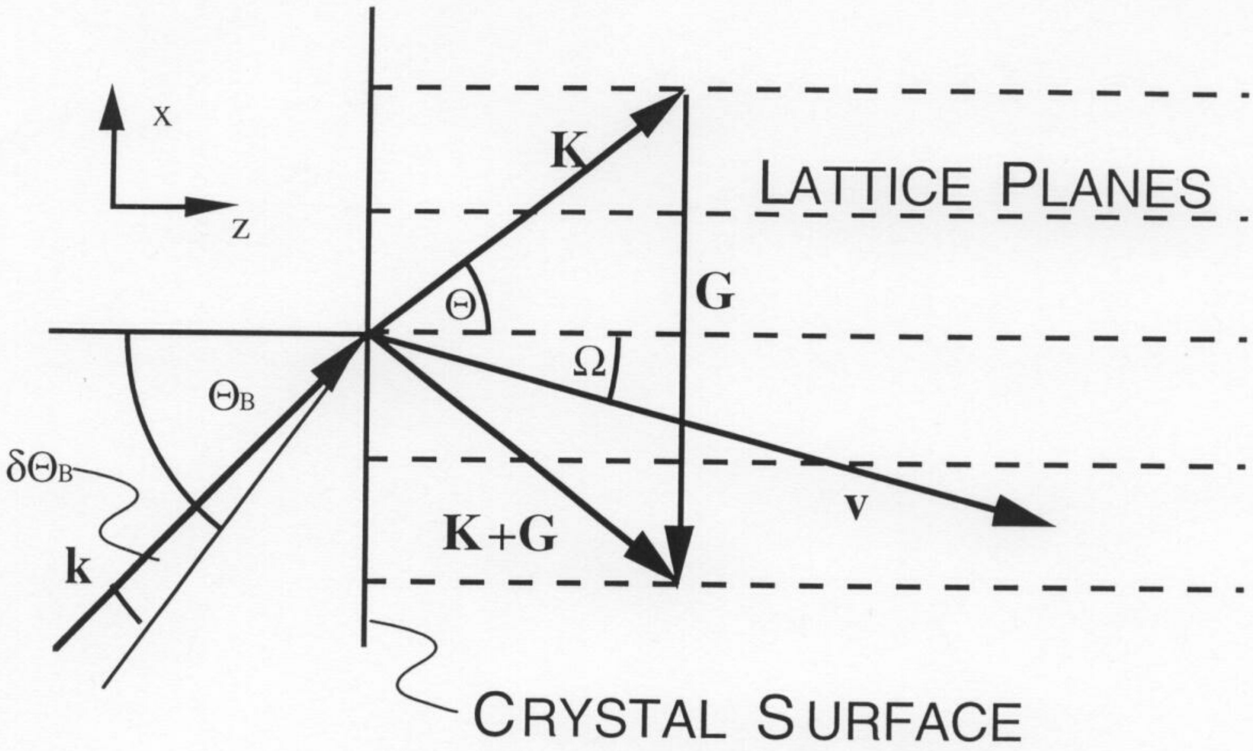
Figure captions

FIG. 1. The group velocity **v** in the symmetric Laue case. The direction of the group velocity is limited to the region between **K** and **K**-**G** and thus $-\Theta_B < \Omega < \Theta_B$ (Borrmann fan).

FIG. 2. Principle of the setup showing the neutron trajectories. The fine dashed line represents the neutron trajectories without external force, the fat lines with external force. In the first crystal only the symmetric neutron trajectories that pass through both slits are shown. The rest of the Borrman fan is absorbed in the second slit. The distances were L=52.3 mm and s=9.6 mm. The slits were 1.5 mm wide.

FIG. 3. Count rate as a function of the position on the detector for four different tilt angles of the crystal and detector assembly. The positive effective mass peak is marked with a + sign and the negative one with a - sign.

FIG. 4. Separation of the two effective mass peaks as a function of the tilt angle. The offset at zero tilt angle is due to imperfection of the silicon crystal. The solid line represents the theoretical prediction shifted by the offset.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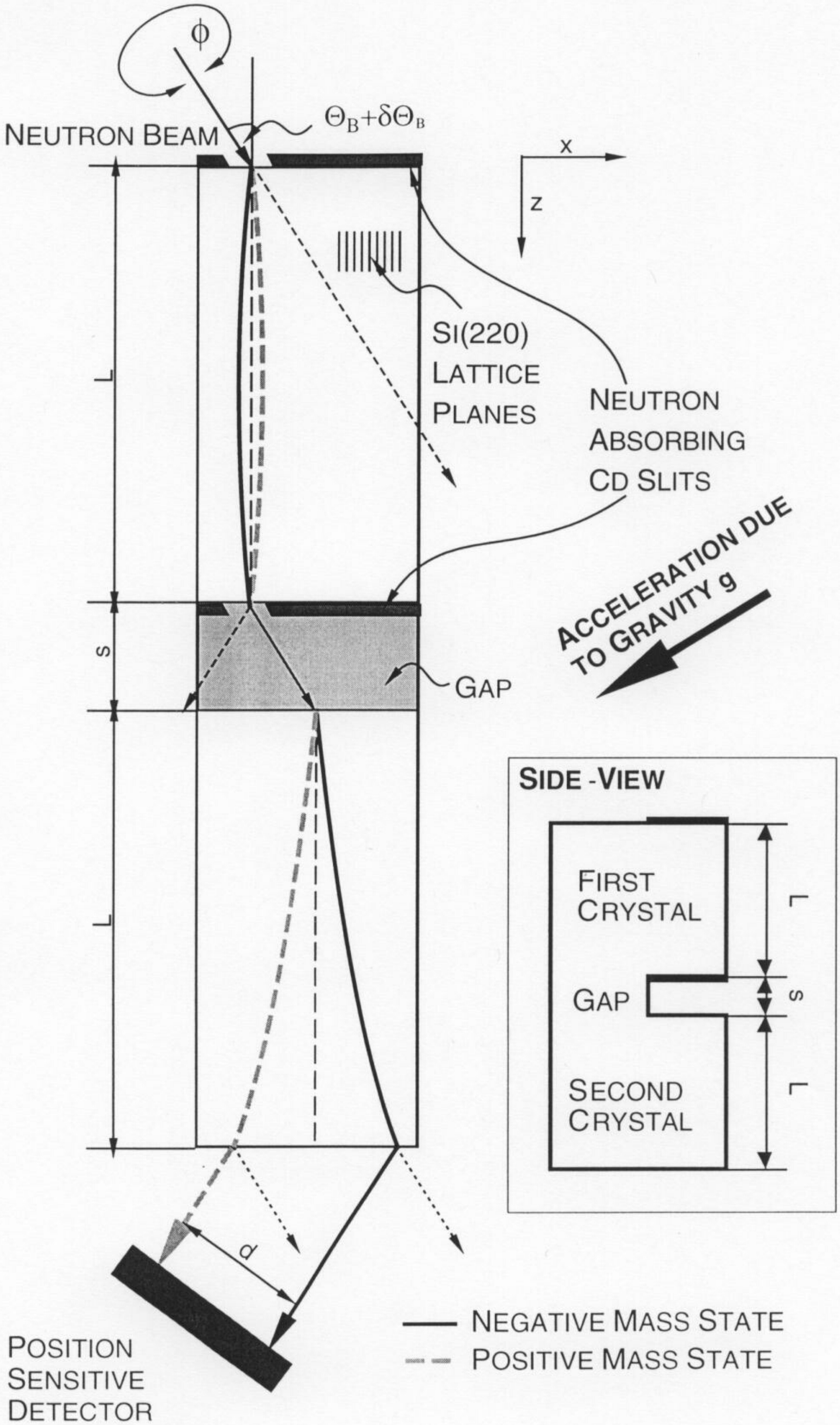
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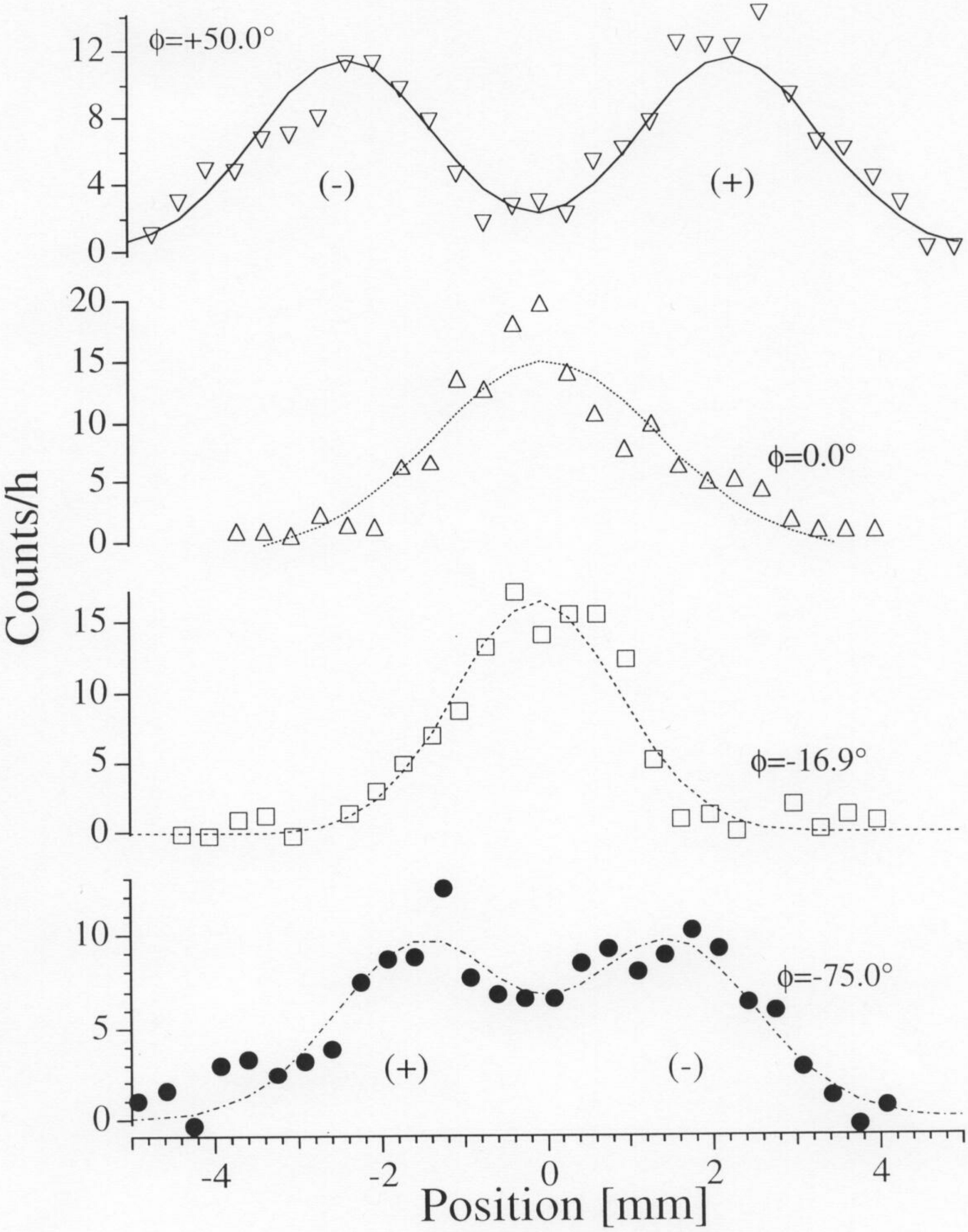

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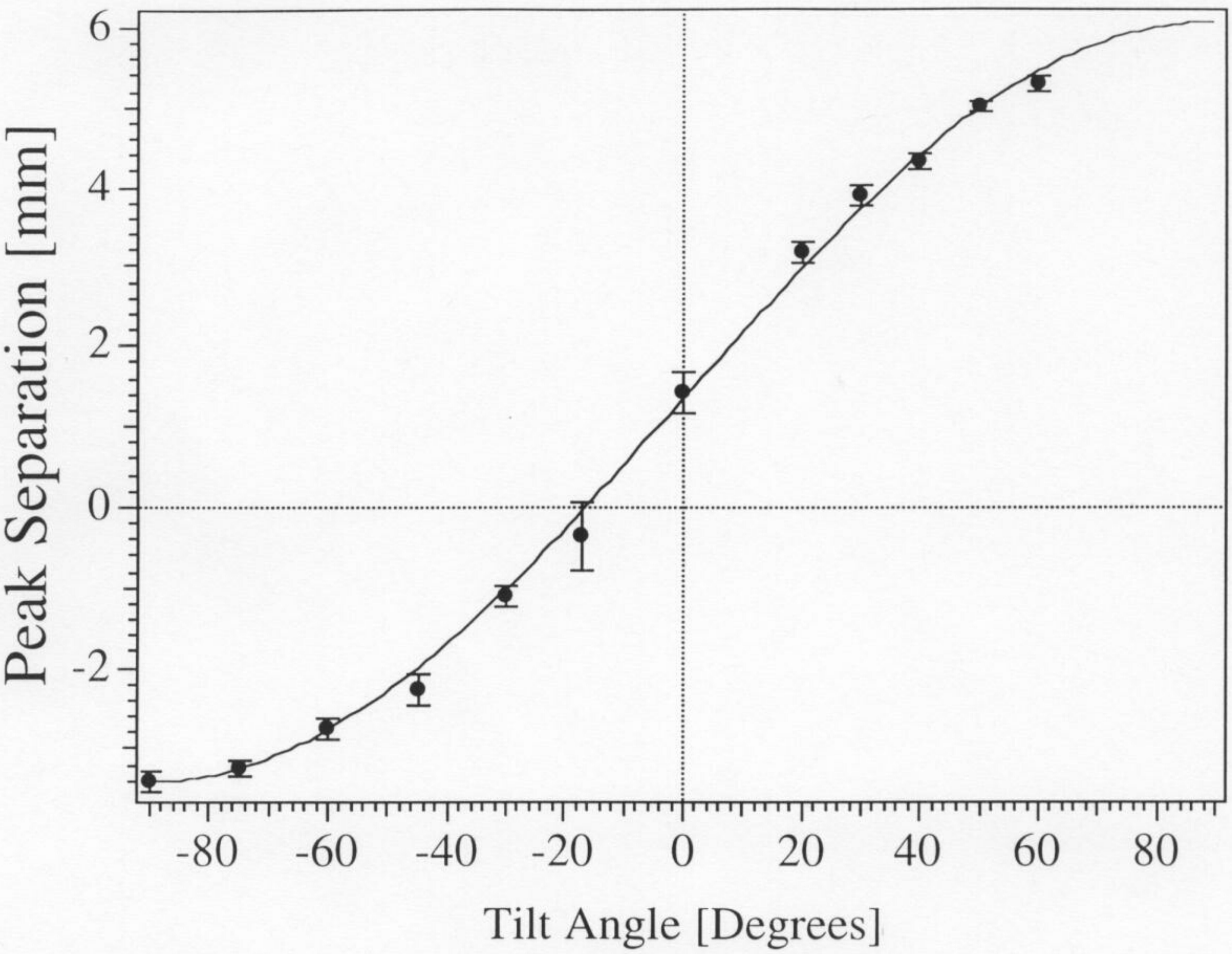
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