Diffractive matter wave optics in time

Stefan Bernet, Roland Abfalterer, Claudia Keller, Jörg Schmiedmayer, and Anton Zeilinger

Institut für Experimentalphysik, Universität Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria

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We demonstrate a programmable sideband modulator for atoms. A controlled phase modulation is engraved on an atomic matter wave by interaction with a frequency-modulated light potential. The principle allows for tailoring atomic optical elements, e.g., temporal lenses, adjustable beam splitters, and holograms, in the time domain. As an illustration we present the time analog of a blazed grating that is generating asymmetric sidebands of atomic matter waves. © 1998 Optical Society of America [S0740-3224(98)01012-1]

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1. PHASE MODULATION OF MATTER WAVES

Methods that provide coherent control over matter waves are subjects of increasing interest, both in fundamental research and in applications such as atom lithography.[1] Their importance may even grow with advances in the development of an atom laser.[2] Although static atom optics elements such as atom mirrors, lenses, and beam splitters already exist, to date no programmable modulation methods for atomic matter waves have been developed, although the feasibility of sideband generation was shown recently.[3–5]

Here we demonstrate two elementary advances in this field: First we show how to imprint a programmable phase modulation onto an atomic matter wave and then we explain how to use this technique to transfer the principles of spatial diffractive light optics to the time domain for matter waves. This process promotes the existence of atomic optical elements that are generalizations of traditional optics to the time domain, such as temporal lenses, gratings, and time-domain holograms. As an illustration we present the temporal analog of a nontrivial optical element, which is a blazed grating. The method extends the possibilities of light modulation by exploiting the long interaction times achievable for atoms in an extended light potential. By temporal modulation of the potential an atom can be transformed into a tailored superposition of different energy and momentum states. We achieve this result by using a frequency-modulated diode laser without any mechanical device. Our method provides full control over the spectral composition of matter waves and can be used for coherently shaping an atomic wave packet.

2. DIFFRACTIVE OPTICS IN SPACE AND TIME

We start by recapitulating the idea of diffractive optics. The basic principle is that many optical elements (e.g., lenses) operate by just spatially modulating the phase of an incident wave front by their transmission function. However, a phase shift of multiples of 2π cannot be distinguished from a zero phase shift. Therefore one can remove excessive material (in steps corresponding to phase delays of 2π) just until the wave-front modulation is less than 2π at each position. The resultant phase object has imaging properties that are similar to those of the original optical element. In optical holography these phase objects are called kinoforms.[6] Thus imaging properties, which are commonly attributed to refraction, can be also achieved by diffraction in a quite natural way.

As an example, we consider a prism that has the well-known property of refracting incident waves in only one direction. This deflection is caused by the wedged refractive material's producing a phase shift that varies linearly transversely to the beam direction. The corresponding kinoform has the shape of a sawtooth function and is known as a blazed grating. It deflects light in a single direction only, by means of diffraction instead of by refraction. Blazed gratings are used for achieving high diffraction efficiencies (as much as 100%) in many technical applications, e.g., in optical spectrometers. Blazed gratings for atomic matter waves in space have already been demonstrated experimentally by use of superposed standing light waves.[7]

How can this blazing principle now be transferred to the time domain? Similar to the way in which a prism changes the momentum of an incident wave, by applying a phase shift that varies linearly in space, the corresponding time-domain element changes the frequency of the wave by applying a phase shift that varies linearly in time. Thus the time analog to the lopsided momentum transfer produced by a prism is a frequency shifter that shifts the frequency of an incident wave in one direction only. One can achieve such a frequency shift by transferring a time-dependent linear phase shift to the original wave. According to the blazing principle such a linear phase shift can be approximated by a sawtooth function, i.e., an originally linear function that is reset to zero each time a total phase shift of 2π is reached (see Fig. 1). Applying such a phase shift to a matter wave should produce the same result as applying a linear phase shift: i.e., a pure frequency shift in one direction. Note that the device is a full temporal analog of a blazed grating in space because it acts on the frequency spectrum of the matter waves in a way analogous to that in which a real blazed

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grating acts on the transverse momentum distribution of the scattered atoms.

Now the practical question arises of how to program such a time-dependent phase shift of an atomic matter wave. It has been already demonstrated that atoms can be diffracted from material gratings[8] and even from standing light waves.[9] The reason that this can be done is that the complex interaction potential[10] between a light field and an atom gives rise to a complex index of refraction. The spectral dependencies of the real and the imaginary parts of the potential are given by a dispersion curve and by a Lorentzian, respectively, both centered at an atomic transition line (inset A of Fig. 2). These curves describe refraction and absorption (diffuse scattering), respectively, of the incident matter waves. In our experiment the absorption is realized[11] by resonant excitation of the atoms, which leads to relaxation to an undetected ground state (see inset B of Fig. 2).

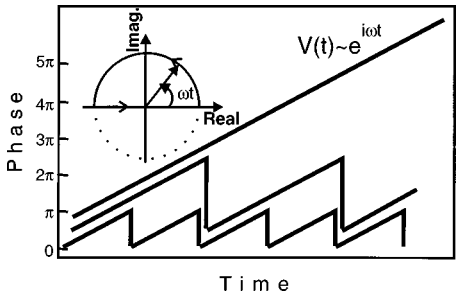

Fig. 1. Principle of a blazed grating in time: An ideal frequency shifter is characterized by a linear time-dependent phase (linear function). An ideal blazed grating approximates this shape by a sawtoothed function with the same slope and a (peak-to-peak) amplitude of 2π (larger jagged curve). In our experiment we realized a sawtoothed function with an amplitude of π (lower curve). Inset, a complex pointer representation of the phase shifts. Our realization corresponds to a pointer, which suddenly takes a shortcut to its starting point after each semicircle.

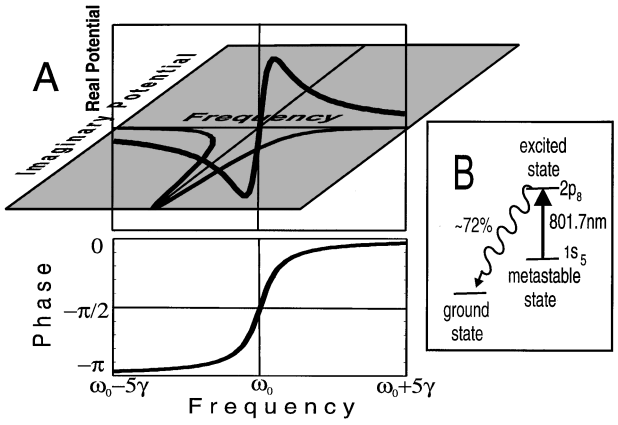

Fig. 2. A, Real part (real potential), imaginary part (absorption), and phase of the complex interaction potential between light and atoms near an open electronic transition. The curves are plotted as a function of the excitation frequency in units of the linewidth γ. The center of the plots corresponds to the resonance frequency. The corresponding complex phase is an arctangent function with an offset of $-\pi/2$ extending over a range from $-\pi$ to 0. Atoms diffracted at such a potential pick up this complex phase angle. B, In our experiment we use the $1s_5 \rightarrow 2p_8$ (801.7-nm) transition in metastable argon. After resonant excitation the atoms relax to their ground state with high probability (72%). This corresponds to absorption because our channeltron detector does not register ground-state atoms.

From light optics it is well known that the scattering phase of a photon at an atom depends on the detuning of the light frequency from an atomic transition; i.e., the scattered photon acquires the phase of the complex refractive index that corresponds to the absorption line. For example, this is the reason that the phase of light scattered at purely absorptive gratings differs by $\pi/2$ from that scattered at purely refractive gratings. However, it is less obvious that in the course of the scattering process the matter-wave phase of the atom also has to change, because the resultant phase of the total system of photon and atom has to stay constant. Therefore the phase of a diffracted matter wave also jumps by $\pi/2$ if an originally refractive light grating is interchanged with an otherwise identical absorption grating.[11] However, to date it has not been demonstrated that such a phase shift can be performed smoothly by continuous shifting of the light frequency across the atomic transition. Nevertheless, such a shift is exactly the mechanism that we use to obtain control of the matter-wave phase; i.e., we sweep the frequency of a light grating, which diffracts the atoms. This process changes the complex phase of the potential continuously as an arctangent function of the light frequency in a range from $-\pi$ to 0 (inset A of Fig. 2). In what follows, we apply this phase control to transfer the principle of diffractive optics to the time domain. Thus our experiment can be seen as a confirmation both of this relation and of the fact that the scattered atoms continuously pick up the spectral phase angle of the complex interaction potential.

3. EXPERIMENTAL DEMONSTRATION: A BLAZED GRATING IN TIME

Our experiment is sketched in Fig. 3.[12] A collimated (<6 μrad) beam of metastable argon atoms (v = 700 m s^{-1}; 60% velocity spread) is Bragg scattered at a thick standing light wave (length: 4 cm), which forms an extended periodic potential for the atoms that corresponds to a one-dimensional light crystal (grating constant, 401 nm). This light crystal is set up by retroreflection of a collimated expanded diode laser beam at a mirror in vacuum. The laser frequency can be modulated in a wide range (120 MHz ≈ 20 linewidths) across the atomic absorption line by direct modulation of the laser diode current with frequencies up to 100 kHz by use of a programmable signal generator. Thus the complex potential phase can be changed a few times (>5) during the passage of each atom through the crystal. The metastable atoms with their high internal energy (12 eV) can be registered by a channeltron detector. A thin slit in front of the detector guarantees that only first-order diffracted atoms are counted. By tilting the mirror with a piezo actuator one can measure the diffraction efficiency of the atoms as a function of their incidence angle at the light crystal. Because of the angular selectivity of Bragg diffraction, atoms are scattered only if their incidence angle lies within a narrow range around the Bragg angle

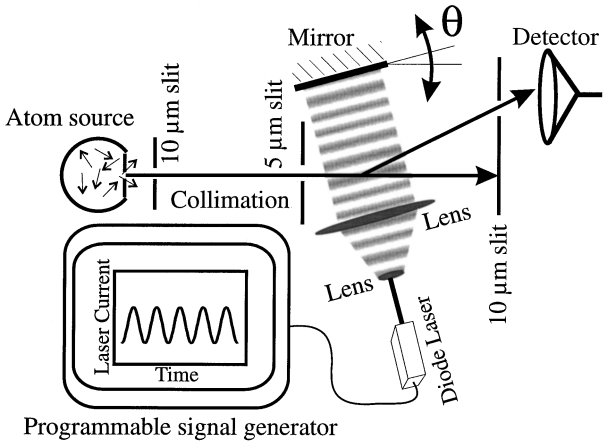

Fig. 3. Experimental setup (not to scale): A collimated thermal beam of metastable argon atoms is Bragg diffracted at a thick standing light wave created by retroreflection of an expanded laser beam at a mirror. The laser frequency can be modulated across the atomic resonance line (at 801.7 nm) by direct modulation of the laser diode current by use of a programmable signal generator. We can vary the incidence angle Θ by tilting the mirror. A channeltron detector located 1.4 m downstream, masked by a 10-μm slit, counts first-order diffracted atoms as a function of their incidence angle at the light crystal.

(18 μrad), and therefore the result of such a measurement consists of a single peak at the Bragg angle in static experiments.

The situation changes when atoms are scattered at a periodically time-modulated light crystal. It was shown earlier[5] that in this case coherent matter-wave sidebands can be created that are symmetrically offset to higher and lower energies by the modulation frequency. However, because the frequency shift of a matter wave is always related to a momentum change, there is only one specific incidence angle under which a particular frequency shift can be obtained. Therefore the frequency-shifted atoms appear at new discrete Bragg angles, and a rigid relation connects the new Bragg angles with the corresponding frequency shift of the diffracted atoms.[13,14] In our case of small incidence angles, the angular separation $\Delta\Theta$ of the new Bragg angles from the static Bragg angle Θ_B is proportional to the modulation frequency ω_M, which is simultaneously the frequency shift experienced by the scattered atoms[15]:

$$\frac{\Delta\Theta}{\Theta_B} = \pm\frac{\omega_M}{\omega_{\mathrm{rec}}}, \tag{1}$$

where ω_{rec} is a geometric constant of the experiment, namely, the two-photon recoil frequency that an atom acquires when it is absorbing a photon from one light-propagation direction within the standing light field and then reemitting the photon in the other direction. In our case $\omega_{\mathrm{rec}} = 2\pi \times 30$ kHz. The derivation of Eq. (1) is a straightforward generalization of the Ewald construction to a Bragg diffraction process in space and time. Because of the wide transverse extension of the monochromatic standing light wave in the Bragg case, both the length and the direction of the light grating vector are sharply defined. Then the energy of a scattered atom after addition of such a grating vector is strictly determined by momentum conservation. In the Ewald picture the Bragg condition for first-order elastic scattering is expressed by the requirement that incident and diffracted waves connected by such a grating vector form a symmetric triangle. At all other incidence angles there is an energy difference between incident and diffracted atoms, suppressing any elastic scattering. However, diffraction can be enabled if the corresponding energy difference is resonantly supplied by a periodic time-dependent perturbation with the appropriate frequency realized, e.g., by temporal modulation of the light crystal. The relation of Eq. (1) merely connects the deviation of any incidence angle from the static Bragg angle to the corresponding frequency change of the atoms after diffraction. The traditional static Bragg condition results in a special case, assuming equal energies of incident and diffracted atoms. In particular, the new Bragg angles are related to the sign of the frequency shift; i.e., frequency upshifted atoms are diffracted closer to perpendicular incidence and vice versa. Consequently the angular deviation from the static Bragg angle is a direct measure of the frequency offset of the diffracted atoms, and our light crystal represents both a frequency shifter and a spectrum analyzer at the same time.

In our present experiment we exploit this specific feature of light crystals together with the temporal blazing principle to demonstrate, for the first time to our knowledge, an asymmetric sideband creation (or suppression) of matter waves. We proceed in three steps to approach our closest implementation of the blazing principle.

First we demonstrate the effect of a time-dependent phase on the complex potential by sinusoidally modulating the frequency of the light grating symmetrically from the red side to the blue side of the atomic transition line. The laser frequency is changed sinusoidally at a rate of $\omega_M = 100$ kHz in a range of ± 30 MHz (corresponding to ± 5 atomic linewidths) across the atomic resonance line. The resultant diffraction efficiency is plotted in Fig. 4A as a function of the incidence angle. It shows one central peak and two symmetrically located side peaks. The center of the plot corresponds to the static Bragg angle. The atoms in the central peak are not frequency shifted, whereas the atoms in the side peaks are shifted by the modulation frequency of ± 100 kHz, as explained above. The sign of the frequency shift differs for the two side peaks. The same experiment with an unmodulated Bragg crystal results only in the central peak of elastically scattered atoms.

To realize our blazed grating in time we next applied a temporally asymmetric phase shift. We chose a sawtoothed frequency modulation symmetrically across the atomic resonance line with a frequency of 100 kHz and a modulation amplitude of ± 5 linewidths, as in the previous experiment. The result is displayed in Fig. 4B. Now, aside from the central peak of unshifted atoms, only one side peak of frequency-shifted atoms appears. This result demonstrates that an asymmetric modulation in the time domain can produce a corresponding asymmetry in the frequency domain.

However, some reflection shows that our pure sawtoothed modulation of the light frequency is not the best realization of the blazing principle, because it does not result in the desired sawtoothed modulation of the scatter-

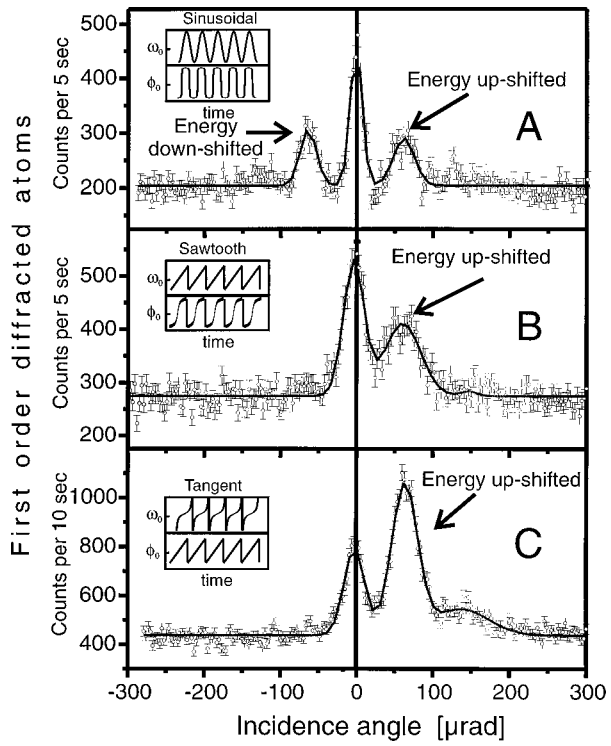

Fig. 4. Diffractive optics in time: The intensity of first-order diffracted atoms is plotted as a function of the atomic incidence angle at the frequency-modulated light crystal. Solid curves, Gaussian fits used as guides for the eye. Insets show the frequency modulation ω applied to the laser and the corresponding phase modulation of the atom ϕ. A, A temporal symmetric (sinusoidal) frequency modulation (100 kHz) about the atomic resonance frequency results in symmetric frequency sidebands. B, A sawtoothed frequency modulation suppresses the lower-energy sideband. C, A tangent frequency modulation compensates (inset) for the arctangent spectral variation of the complex potential phase (Fig. 2), yielding the closest approximation to a sawtoothed phase grating (blazed grating) in time (note that the integration time per data point is doubled relative to those of the previous figures).

ing phase (Fig. 4, insets). For optimum realization, the frequency modulation has to compensate for the nonlinear (arctangent) spectral variation of the complex potential phase (see Fig. 2) when one is scanning across the atomic transition. Therefore the laser frequency has to sweep as the corresponding inverse function, i.e., a tangent function with appropriate slope. The result of such modulation is displayed in Fig. 4C, for which we chose a tangent frequency modulation with a 100-kHz modulation rate symmetrically across the atomic absorption line. As expected, we obtained a more pronounced sideband asymmetry than in the previous results. Additional experiments with a temporally inverted (negative) tangent modulation showed that the sideband peak now appeared at the other side of the static Bragg peak, corresponding to an opposite sign for the frequency shift. Thus the energy shift depends only on the temporal orientation of our frequency modulation, in accordance with our blazed-grating model.

4. DISCUSSION

It should be mentioned that in our experiment the absolute and relative efficiencies of sideband generation, i.e., the ratios of frequency-shifted atoms to incoming and transmitted atoms, were of the orders of ≈5% and ≈50%, respectively. Thus the absolute efficiency was less than that of a previous experiment (≈20%) in which an intensity-modulated, purely refractive light crystal was used.[5] This result is due to the additional absorption in the present experiment. However, the demonstrated phase-modulation scheme allows for arbitrarily shaping the sideband structure, whereas in the previous experiment the sideband composition was fixed and always symmetric. One application of this arbitrary access to the sideband composition appears if the modulation scheme is applied in a thin-grating regime. There, the angular selectivity of sideband production will be lost, and all allowed sidebands will arise simultaneously in the diffracted beams. Thus a temporally modulated diffracted matter wave can be produced, with a tailored sideband structure. Because this kind of sideband manipulation is a coherent process,[16] wave packets with programmed features can be be generated or coherently manipulated. Detection of such a tailored wave packet would require a new interferometric detection scheme because in the thin-grating regime our setup loses its ability to act as a frequency analyzer; i.e. the frequency-shifted atoms can no longer easily be identified as new Bragg peaks.

It is interesting to investigate how the feasibility of our phase-modulation scheme is affected by the simultaneous modulation of the potential strength connected with the laser frequency modulation. In fact, the scheme explains the deviation from the behavior of an ideal blazed grating in time, which should provide only one single-frequency sideband, without any unshifted atoms, in a way similar to that in which a spatially blazed grating deflects an incident beam only in a single direction. The potential modulation arises because the potential strength depends on the frequency detuning from resonance during a frequency sweep with constant light intensity. We can analyze this behavior by regarding the spectral dependence of the complex potential drawn in Fig. 2:

$$V(\omega) = \frac{\hbar \Omega_{\mathrm{Rabi}}{}^2}{4} \frac{1}{\omega - \omega_0 + i\gamma/2}. \tag{2}$$

Here $\Omega_{\mathrm{Rabi}} = \mathrm{d}E/\hbar$ is the Rabi frequency of an atom in light field E at resonant excitation, γ is the width of the absorption line, and $\omega - \omega_0$ is the detuning of the light frequency from the resonance frequency ω_0. This complex function can also be expressed in a representation that uses absolute value and complex phase:

$$V(\omega) = \frac{-i\hbar \Omega_{\mathrm{Rabi}}{}^2}{4} \frac{\exp\left[i \arctan\left(\dfrac{\omega - \omega_0}{\gamma/2}\right)\right]}{[(\omega - \omega_0)^2 + (\gamma/2)^2]^{1/2}}. \tag{3}$$

This representation already suggests that a periodic time-dependent frequency modulation with rate ω_M of the shape

$$\omega(t) = \omega_0 + (\gamma/2)\tan(\omega_M t) \tag{4}$$

compensates for the complicated spectral dependence of the phase in the exponential factor. Inserting Eq. (4) into Eq. (3) yields, after some trigonometric manipulations,

$$V(t) = \frac{-i\hbar\Omega_{\mathrm{Rabi}}{}^2}{4\gamma}\cos(\omega_M t)\exp(i\omega_M t)$$

$$= \frac{-i\hbar\Omega_{\mathrm{Rabi}}{}^2}{4\gamma}[1 + \exp(2i\omega_M t)]. \qquad (5)$$

This result corresponds to the Fourier representation of the potential modulation, consisting of a static component and a second component at frequency $2\omega_M$. Note that the Fourier spectrum is not symmetric (non-Hermitian); i.e., there is no conjugate frequency component at $-2\omega_M$. In a simplified model, which is valid for small modulation efficiencies, the sideband intensity of a frequency shift should be proportional to the square of the corresponding Fourier component. Accordingly, the modulation of Eq. (5) results in two equal fractions of elastically scattered atoms and frequency-shifted atoms with a frequency offset of $2\omega_M$. These fractions correspond to the two peaks in the data of Fig. 4C.[17] Further analysis of the Fourier spectrum of the potential modulation shows that the intensity of the frequency-shifted sideband grows if the amplitude of the potential modulation in Eq. (4) exceeds the value $\gamma/2$. Actually, such an overcompensation of the slope of the complex phase is the reason that the peak of frequency-shifted atoms is larger than the peak of elastically scattered atoms in the data of Fig. 4C.

The above analysis shows that the deviation from the behavior of an ideal blazed grating, which should suppress the peak of unshifted atoms, is due to the intensity modulation connected with our phase modulation. However, it is not trivially possible to compensate for the effect of this intensity modulation [the cosine-function in Eq. (5)] by simultaneously controlling the light potential strength and the reciprocal function, i.e., $\propto 1/\cos(\omega_M t)$. The reason is that such simultaneous control would require a periodic change of the sign of the potential, which one can attain only by flipping the sign of the frequency detuning. Unfortunately, this sign flip would be accompanied by an undesired phase jump of π, canceling the required effect. This problem could be circumvented in an inverted system that also provided a negative absorption process, i.e., gain, because there the corresponding complex potential switches sign compared with the complex potential drawn in Fig. 2. Practically, this solution might be accomplished in a three-level system by use of a repumping transition from the absorbed state. In such a system the efficiency of sideband generation would no longer be limited by the absorptive contributions, because they were compensated for by repumping. Scanning the laser frequency over two adjacent absorption and emission lines provides the jump of the potential sign in the middle between the two lines (assuming equal line strengths) that is required for compensation of the cosine function in Eq. (5). Simultaneously, this scanning step corresponds to the step of performing full circles, instead of our actual semicircles, of the potential in the complex plane (see Fig. 1). This procedure would yield a perfect analog of a blazed grating in time. More generally, such a system allows one to adjust any scattering phase in the whole interval from 0 to 2π by controlling the light frequency and to adjust the interaction strength independently by modulating the light intensity. Thus it provides all features to yield arbitrary access to the sideband composition of atomic matter waves by use of the principles of diffractive light optics.

Summarizing, we have demonstrated that spatial concepts of traditional diffractive optics with light can be transferred to matter wave optics in the time domain, yielding a mechanism, similar to a frequency synthesizer, to shape the distribution of matter wave sidebands. As an example, we realized a lopsided frequency shifter for matter waves that acts as a time analog of a blazed grating. The generalization of the principle to other optical elements in the time domain is straightforward. For example, the realization of a refractive-index Fresnel lens in time requires transferring the well-known spatial phase profile of a Fresnel lens as a temporal phase profile to the diffracted atoms, which is achieved by a tailored frequency modulation of the light grating. As in our experiment, the tangent variation of the spectral phase of the light potential needs to be compensated for by appropriate programming of the frequency-modulation function. Diffraction of a monochromatic atomic beam at such a modulated light grating should result in a frequency-chirped matter wave that focuses periodically in time.

Such experiments, which exploit the principle of diffractive optics in the time domain, can give fundamental new insights into the quantum mechanics of wave packets. Additionally, we expect numerous practical applications, e.g., in active atomic interferometry, atomic lithography, and coherent matter wave manipulations, especially in combination with progressing developments in coherent sources of atomic matter waves.

ACKNOWLEDGMENTS

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15. For generalization of Eq. (1) to arbitrary incidence, the angles have to be replaced by their sines; i.e., $\Delta\Theta \rightarrow \sin(\Theta) - \sin(\Theta_B)$ and $\Theta_B \rightarrow \sin(\Theta_B)$.
16. The coherence of atomic matter-wave sidebands created with a modulated refractive light crystal has been already demonstrated.[5] This coherence is not destroyed in the present modulation scheme because diffraction at absorptive light crystals is also a coherent process, as was shown previously.[11]
17. It might be striking that a frequency shift of $2\omega_M$ results from a modulation $\propto \tan(\omega_M t)$, i.e., with modulation rate ω_M. However, the tangent function is periodic in an interval of π instead of 2π; i.e., the actual periodicity of the applied modulation is $2\omega_M$. Note that a Fourier component at a positive ω value in the spectrum of the modulated potential creates a frequency sideband with negative frequency offset. In the Schrödinger equation this negative frequency is expressed by the sign in front of the time derivative of a wave function; e.g., the representation of a plane de Broglie wave with positive kinetic energy corresponds to a negative sign of the time-dependent exponent $\psi \propto \exp(-i\omega t + ikx)$.