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Direct verification of the quantum spin-state superposition law†‡

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Abstract. Coherent superposition of two neutron waves, with mutually opposite polarisation direction, is used to demonstrate the quantum-mechanical principles of spin-state superposition for fermions.

1. Introduction

Quantum theory predicts that the superposition of two coherent beams of spin-$\frac{1}{2}$ particles with opposite spin states leads not, as one might visualise classically, to a mere mixture of these states but results in a final polarisation state which neither of the constituent beams had. In a famous article on the problem of measurement in quantum physics (Wigner 1963) a Gedanken experiment was suggested to demonstrate this phenomenon, which is a consequence of the principle of linear superposition of probability amplitudes, one of the outstanding foundations of quantum mechanics. In recent years it was proposed theoretically (Eder and Zeillinger 1976, Zeilinger 1976, 1979) to realise such an experiment by means of neutron interferometry.

Today neutron interferometry has developed from the very first beginnings (Rauch *et al* 1974, Bauspiess *et al* 1974) to a broad and challenging field of research (Bonse and Rauch 1979). In particular it initiated a series of innovative experiments of basic physics, as for example the first experimental verification of the 4π symmetry of spinors (Rauch *et al* 1975, 1978, Werner *et al* 1975) and the observation of gravitationally induced quantum interference (Overhauser and Collela 1974). Recently a neutron experiment has been reported (Summhammer *et al* 1982a, b) where coherent separation and subsequent recombination of polarised neutron beams were used to verify the above mentioned spin-state superposition principle, for the first time actually in an explicit way. It is the aim of the present paper both to summarise the essential aspects of this basic physics experiment and to discuss respectively its possible future continuations and modifications.

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2. Theoretical background

Let the spin states of two completely polarised neutron beams be denoted by the kets $|\mathrm{I}\rangle$ and $|\mathrm{II}\rangle$, respectively. Suppose that both states are identical except for an arbitrary real phase difference χ between them, that is

$$|\mathrm{II}\rangle = \mathrm{e}^{\mathrm{i}\chi}|\mathrm{I}\rangle. \tag{1}$$

Following the usual notation, we choose the coordinates in such a way that each state is represented by a normalised spinor of the form

$$|\theta, \phi\rangle = \cos\tfrac{1}{2}\theta|\uparrow_z\rangle + \mathrm{e}^{\mathrm{i}\phi}\sin\tfrac{1}{2}\theta|\downarrow_z\rangle \tag{2}$$

where θ and ϕ are the polar angles of the neutron polarisation vector with respect to the z axis and $|\uparrow_z\rangle$, $|\downarrow_z\rangle$ are the eigenstates of σ_z with $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ the familiar Pauli spin operator. The spatial part of the neutron wavefunction is not considered here since it is not necessary for the interpretation of the particular experiment we will describe.

Now let the unitary operator

$$U_{\mathrm{R}}(\alpha) = \exp[\tfrac{1}{2}(-\mathrm{i}\boldsymbol{\sigma}\cdot\boldsymbol{\alpha})] = \cos\tfrac{1}{2}\alpha - \mathrm{i}\boldsymbol{\sigma}\cdot\hat{\alpha}\sin\tfrac{1}{2}\alpha \tag{3}$$

act solely on state $|\mathrm{II}\rangle$, leaving the other one unaffected. U_{R} describes a rotation of the spinor around an axis $\hat{\alpha} = \boldsymbol{\alpha}/\alpha$ by an angle α. According to the definition of exponential operators the validity of the right-hand side of equation (3) can be easily proved by series expansion. Due to the magnetic moment $\boldsymbol{\mu} = \mu\boldsymbol{\sigma}$ ($\mu = -9.663 \times 10^{-27}\,\mathrm{J\,T^{-1}}$) of the neutron, such spin rotations can be accomplished by means of a homogeneous magnetic field $\boldsymbol{B} = B\hat{\alpha}$. The angle of rotation is given in that case by

$$\alpha = -\frac{2\mu}{\hbar}\int B\,\mathrm{d}t \tag{4}$$

where the integration has to be performed over the total time of interaction. The generalisation to spatially inhomogeneous magnetic fields would have to take into account the spin-dependent momentum changes which occur in that case due to the gradients of the field. For example, the purely longitudinal momentum changes which are caused by field gradients parallel to the neutron beam trajectory lead to different interaction time intervals for both spin states and thus make exact spin rotations impossible (Bernstein 1967). We will not, however, consider further such effects in the present context.

It is particularly illustrative if the size and orientation of the homogeneous field are chosen to produce a rotation of the spin, which is assumed to be aligned initially parallel to the $+z$ direction, around the y axis by an angle of exactly 180°. In this case the unitary operator reads $U_{\mathrm{R}}(\pi, \hat{e}_y) = -\mathrm{i}\sigma_y$ and state $|\mathrm{II}\rangle = \mathrm{e}^{\mathrm{i}\chi}|\downarrow_z\rangle$ is transformed into

$$|\mathrm{II}\rangle_{\mathrm{R}} = U_{\mathrm{R}}(\pi, \hat{e}_y)|\mathrm{II}\rangle = -\mathrm{i}\sigma_y\,\mathrm{e}^{\mathrm{i}\chi}|\uparrow_z\rangle = \mathrm{e}^{\mathrm{i}\chi}|\downarrow_z\rangle. \tag{5}$$

If beam I and beam II are now superposed coherently with equal weights to give a state $|\mathrm{O}\rangle = \frac{1}{2}|\mathrm{I}\rangle + \frac{1}{2}|\mathrm{II}\rangle_{\mathrm{R}}$ we obtain immediately the remarkable result

$$|\mathrm{O}\rangle = \tfrac{1}{2}|\uparrow_z\rangle + \tfrac{1}{2}\mathrm{e}^{\mathrm{i}\chi}|\downarrow_z\rangle = \mathrm{e}^{\mathrm{i}\chi/2}(\cos\tfrac{1}{2}\chi|\uparrow_x\rangle - \mathrm{i}\sin\tfrac{1}{2}\chi|\downarrow_x\rangle) = 2^{-1/2}|\tfrac{1}{2}\pi, \chi\rangle. \tag{6}$$

This means that the polarisation of the final state lies in the x–y plane, where it can be rotated by variation of the scalar phase shift between the two interfering states. In contrast to such a coherent superposition of states, an incoherent mixture of two oppositely polarised beams could evidently never produce a beam with polarisation perpendicular to its constituents. To recombine beam I and beam II their trajectories cannot be parallel but cross each other somewhere in space. The simplest interferometer device that can be placed there would be a semi-transparent mirror. Because of the phase change of $\frac{1}{2}\pi$ associated with each reflection the two emerging beams behind the interferometer have a net phase difference of π and hence are polarised in opposite directions. In view of this fact it is no longer paradoxical that a scalar phase shift can produce spin rotations as described by equation (6), since the net effect on the spin of the complete wavefunction always remains zero. Furthermore from equation (6) it follows that $\langle O|O\rangle = \frac{1}{2}$. This means that the intensity of the recombined beams is independent of the phase shift between the two oppositely polarised substates. This compares with the case when the flipper is not activated and both partial beams have the same polarisation. Here the final intensity is a periodic function of χ,

$$\langle O|O\rangle = \tfrac{1}{4}\langle I|(1-e^{-i\chi})(1+e^{i\chi})|I\rangle = \tfrac{1}{2}(1+\cos\chi). \tag{7}$$

3. Experimental realisation

The experimental demonstration of the spin-state superposition principle according to the ideas presented in § 2 was performed using the neutron interferometer set-up (instrument D18) at the high-flux reactor of the Institute Laue–Langevin, Grenoble. A schematic sketch of the experimental arrangement is shown in figure 1. Exploiting

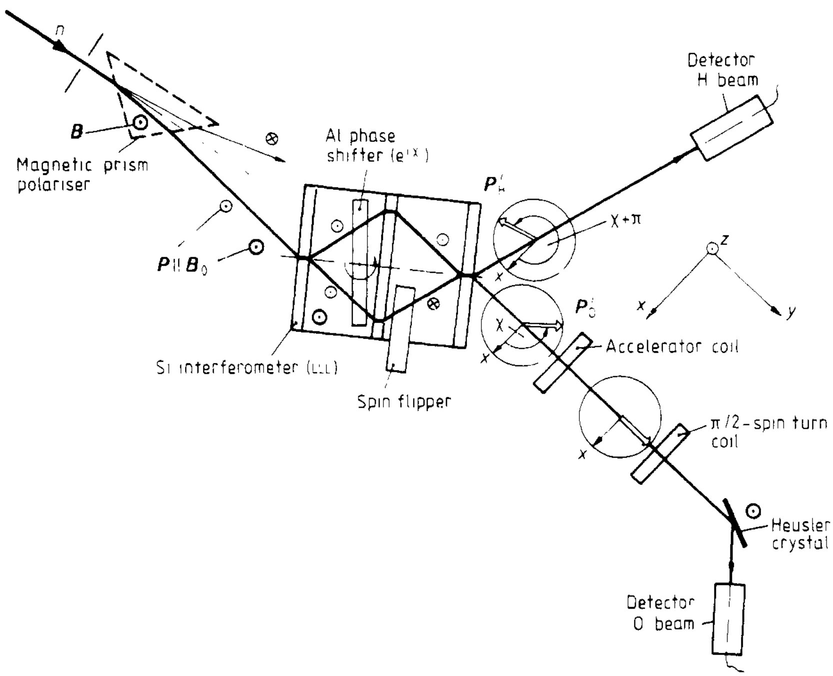

Figure 1. Schematic arrangement of the spin-state superposition experiment.

the extremely narrow reflection width (approximately two seconds of arc) of the Si-crystal slabs of the triple-Laue-case (LLL) interferometer (Bauspiess *et al* 1978) the two partial beams propagating in the interferometer could be polarised by means of spin-dependent double refraction at a magnetic field prism (Badurek *et al* 1979). By use of two subsequently arranged prisms with 120° apex angle and 0.8 T field strength, a total angular separation of the two spin states of about 3.9 seconds of arc could be achieved, corresponding to a polarisation degree of about 95%. To keep depolarisation effects small, the interferometer was placed in the centre of a Helmholtz coil pair of 62 cm diameter, which produced an adiabatic magnetic guide field along the beam trajectories. At the site of the crystal the field strength was about 4 mT. By means of a miniaturised DC spin-flip coil the polarisation of one partial beam could be inverted with a measured efficiency $e = 0.87 \pm 0.05$. To avoid an inhomogeneous phase shift over the beam cross section, its windings were made of a $Nb_{5,4}V_{94,6}$ wire whose refracting power for neutrons is practically negligible. Particular care had also been taken to avoid both the occurrence of magnetic stray fields at the path of the other partial beam in the interferometer and of temperature gradients between the three plates of the interferometer crystal. A variable phase difference $\chi = -N\lambda b_c \Delta D$ between the two wavetrains could be produced in the usual way by insertion of a plane-parallel, non-magnetic sample. Here N is the number of nuclei per unit volume of the phase shifter material, b_c their coherent scattering length and ΔD the geometric path difference caused by rotation of the sample.

The polarisation of the beam diffracted into a forward direction (O-beam) was analysed by means of Heusler crystal which reflects only neutrons that are polarised antiparallel to its $(+z)$ magnetisation direction. A $\frac{1}{2}\pi$-spin turn coil mounted in front of the analyser allows the y component of the polarisation vector to turn into the z direction and *vice versa*. In order to be able to compensate for the spatial precession of the polarisation vector around the magnetic guide field, a so-called accelerator coil was installed along the neutron flight path between the last crystal plate of the interferometer and the $\frac{1}{2}\pi$-spin turn device. It allowed for a controlled shift of the interference pattern by causing an additional variable Larmor angle.

4. Results and discussion

The objective of the measurements was to verify that the polarisation of the emerging beams behind the interferometer has no z component but rotates entirely within the x–y plane although their constituent waves are polarised into the $+z$ and $-z$ directions, respectively. This could be achieved as follows.

First of all a usual interferometer 'scan' was performed by recording the intensities of the forward (O) and the deviated (H) beam behind the interferometer as a function of the path difference or respective phase shift between the two interfering partial beams. Since the spin flipper is not in action during this scan, one observes according to equation (7) the typical coherent intensity oscillations, which because of particle number conservation are mutually complementary for the two beams, as shown in figure 2(*a*). Note the different scales due to the presence of the analyser crystal in the forward beam and that the $\frac{1}{2}\pi$-spin turn was activated during this and the following scan. If the flipper is turned on and the two partial beams have opposite spin states the situation changes totally, as also indicated in the figure. As expected, the intensity oscillations disappear completely. However, since the intensity reflected from the

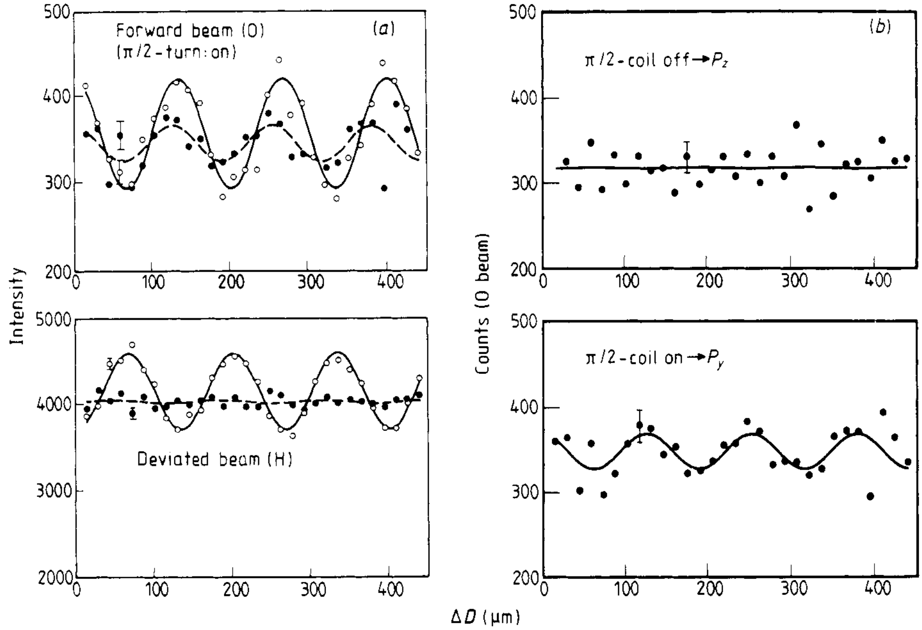

Figure 2. (*a*) Results of spin superposition from both emerging beams behind the interferometer. The coherent intensity oscillations of the deviated beam (and, because of particle conservation, of the forward beam also) which are caused by a variation of the path difference ΔD of the interfering waves within the Al-phase shifter vanish if the spin state of one of these waves is inverted. Since the $\frac{1}{2}\pi$-spin turn coil is in operation during these scans the intensity oscillations observed behind the analyser crystal indicate the existence of a polarisation component that is perpendicular to the polarisation of the interfering states. ○ indicates flipper off and ● indicates flipper on. (*b*) Verification that the polarisation of the emerging beams has no component in the z direction but rotates entirely within the x–y plane if the spin flipper is switched on. The curves correspond to least-squares fits of sinusoidal functions to the measured data points.

analyser is proportional to that component of the polarisation vector of the O beam that is parallel to the z direction, we observe a periodic variation of the measured intensity, which must be due to a polarisation component rotating within the x–y plane in front of the $\frac{1}{2}\pi$-spin turn coil. By variation of the accelerator coil current these oscillations could be shifted in an arbitrary direction. In the case shown in figure 2(*a*) they are nearly in phase with the intensity oscillations measured without spin-flip. As a final proof of the spin superposition principle according to equation (6), it was verified that the beams emerging behind the interferometer have, in fact, no component of polarisation parallel to that of the interfering constituents. As shown in figure 2(*b*) the phase-shift-dependent coherent intensity oscillations behind the analyser vanish if the z component of the polarisation vector is measured instead of its y component. Note that both the full and the broken curves in figure 2 correspond to least-squares fits of a sinusoidal function to the measured data points.

We will continue the experiments in the future to study in particular the influence of partial absorption of one wave inside the interferometer. Another motivation is to use a radiofrequency (RF) spin flipper whose action is basically different from that of a static one (Badurek *et al* 1980a, b, Alefeld *et al* 1981). If the spin state of a neutron propagating within a magnetic field ***B*** is reversed, by means of an interaction that depends not explicitly on time, the total energy of the neutron is conserved. Because of the different Zeeman energies $E_\pm = \pm|\mu|B$ of the two spin eigenstates, this implies a change of the kinetic energy by an amount $\Delta E = 2|\mu|B$. In that case, the wavelengths of the two interfering beams differ by $\Delta\lambda = 2m\mu B\lambda^3/h^2$. In the field of 4mT that the interferometer crystal was exposed to, this wavelength difference is extremely small ($\sim 10^{-8}$ Å). According to dynamical diffraction theory (Rauch and Petraschek 1978), it nevertheless influences the reflectivity of the third crystal lamella and can therefore lead to a measurable reduction of the interference contrast. If necessary one could eliminate this effect by reducing the guide field to zero in the vicinity of the interferometer. However, careful shielding of magnetic stray fields would be necessary in that case to avoid an increase of beam depolarisation.

The time-dependent action of a RF spin flipper, on the other hand, leaves the kinetic energy of the neutrons, and hence their wavelength, completely unchanged. There the total energy of the neutrons is not a constant of the motion due to an exchange of photons of energy $\hbar\omega_s$ between the neutrons and the RF field. This interaction has a resonant maximum if the photon energy equals the Zeeman energy difference of the two neutron spin states within the applied static field $\boldsymbol{B}$, that is if $\hbar\omega_s = 2|\mu|B$. After passage through that flipper, neutrons which were initially polarised in the $+z$ direction and had an energy $E = \hbar^2k_+^2/2m + |\mu|B$ are flipped into the $|\downarrow_z\rangle$ state and have lost the amount of energy $\Delta E = 2|\mu|B$, whereas they maintain their initial momentum $\boldsymbol{k}_+$. The two interfering beams within the interferometer are therefore completely described by the spinors

$$\begin{aligned} |\mathrm{I}\rangle &= \exp(\mathrm{i}\boldsymbol{k}_+\boldsymbol{r}_\mathrm{I})\exp[-(\mathrm{i}/\hbar)Et]|\uparrow_z\rangle \\ |\mathrm{II}\rangle_\mathrm{R} &= \exp(\mathrm{i}\boldsymbol{k}_+\boldsymbol{r}_\mathrm{II})\exp[-(\mathrm{i}/\hbar)(E-\Delta E)t]\,\mathrm{e}^{\mathrm{i}\chi}|\downarrow_z\rangle. \end{aligned} \tag{8}$$

Neglecting again the fact that the beams propagate into different directions $\boldsymbol{r}_\mathrm{I}$ and $\boldsymbol{r}_\mathrm{II}$ and eliminating all common phase factors the combined state $|\mathrm{O}\rangle = \frac{1}{2}(|\mathrm{I}\rangle + |\mathrm{II}\rangle_\mathrm{R})$ is described by the spinor

$$|\mathrm{O}\rangle = 2^{-1/2}\cos[\tfrac{1}{2}(\chi+\omega_s t)]|\uparrow_x\rangle - 2^{-1/2}\mathrm{i}\sin[\tfrac{1}{2}(\chi+\omega_s t)]|\downarrow_x\rangle. \tag{9}$$

Thus the polarisation vector behind the interferometer is not independent of time,

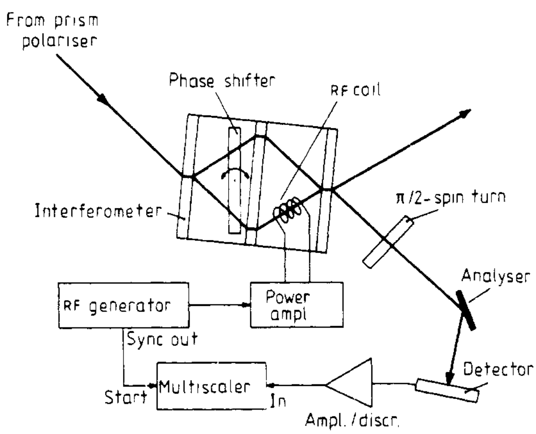

Figure 3. Proposed experimental arrangement if a RF spin flipper is to be used instead of a static one.

but points for any given position at each moment into a different direction according to

$$\boldsymbol{P}(t)=\frac{\langle O|\boldsymbol{\sigma}|O\rangle}{\langle O|O\rangle}=\begin{pmatrix}\cos(\omega_s t+\chi)\\ \sin(\omega_s t+\chi)\\ 0\end{pmatrix}. \tag{10}$$

To avoid an averaging with time when a RF flip system is used, on measuring the spin-state superposition the neutron detection has to be synchronised with the RF field. In figure 3 a possible experimental arrangement is sketched.

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