HIGHER-ORDER QUANTUM ENTANGLEMENT

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ABSTRACT

In quantum mechanics the general state describing two or more particles is a linear superposition of product states. Such a superposition is called entangled if it cannot be factored into just one product. When only two particles are entangled the stage is set for Einstein-Podolsky-Rosen discussions and Bell's proof that the EPR viewpoint contradicts quantum mechanics. If more than two particles are involved new possibilities and phenomena arise. For example the GHZ disproof of EPR applies. Furthermore, as we point out in this paper, with three or more particles even entanglement itself can be an entangled property.

INTRODUCTION

Of the many conceptual innovations of quantum mechanics the notion of *entanglement* is gaining increasing attention over the last few years. This is because entanglement implies the quantum nonlocality as discussed by Einstein, Podolsky, and Rosen (Ref. 1) and Bell (Ref. 2). The increased attention has also led to a somewhat loose discussion of these topics, quite often lacking care with respect to the fundamental issues involved. It is therefore one of the purposes of the present paper to give a detailed discussion of the notion of entanglement.

The term "Entanglement" (in Schrödinger's original German "Verschränkung") in quantum mechanics goes back to Schrödinger's famous 1935 paper (Ref. 3) where he gives a general confession, as he calls it, of his understanding of the situation of quantum mechanics at that time. Most of the analysis of the

measurement problem presented in Schrödinger's paper rests on the properties of nonfactorizable states of two-particle systems as first discussed in the same year by Einstein, Podolsky and Rosen (EPR). Here we shall first briefly review entanglement for the case of two particles. Then, focussing on situations where three particles are entangled, we will see that entanglement itself can be an entangled property.

TWO PARTICLES

For two-particle systems the best known entangled states are those which exhibit entanglement of spin variables, e.g. in the case of two spin-1/2 particles with total spin zero this is the singlet state (Ref. 4)

$$|\psi\rangle = \sqrt{1/2} \; [\; |+\rangle_1 |-\rangle_2 - |-\rangle_1 |+\rangle_2]. \tag{1}$$

Here, $|+\rangle_1$ describes particle one with spin up etc. Because of the rotational symmetry of the singlet state the direction along which the spins are defined in Eq. (1) need not be specified.

As can easily be seen, the state of Eq. (1) does not make any specific predictions for spin measurement results on either particle, but it makes the definite prediction that, as soon as the spin of one particle is found to be oriented along one direction, the spin of the other one will be found to be oriented along the opposite direction should it be measured along that direction. Schrödinger calls this property "entanglement of predictions" or "entanglement of our knowledge of the two bodies". As in classical physics, one might draw up before the measurement an expectation catalog which gives the possible measurement results together with the probabilities of these various results. In quantum mechanics, the expectation catalog has to be calculated from the quantum state of the system.

While in classical mechanics the combined expectation catalog for two objects (bodies, particles, pieces...) can always be written as the logical sum of the expectation catalogs of the individual systems, this is not possible anymore in quantum mechanics for the case of entangled states. Or, in other words, while in classical physics (and certainly in the case of factorizable quantum states) disjoint catalogs for two bodies that once did interact exist, in quantum mechanics this is generally not true anymore. Or, in Schrödinger's original words: *Maximal knowledge of a total system does not necessarily include total knowledge of all its parts, not even when these are fully separated from each other and at the moment are not influencing each other at all*. This results in the interesting nonlocality questions in quantum mechanics.

Consider a measurement on particle 1 along some direction. The experimenter is certainly free to choose this direction at will, call it direction **n**. The experimental result for particle 1 along that

direction can either be + ("up") or – ("down"). The entangled state (Eq. 1) implies that particle 2 is then either in the state $|-n>_2$ if particle 1 was found to be up along n, or particle 2 is in the state $|+n>_2$ if particle 1 was found to be down along n. This is simply a consequence of the von Neumann reduction of the state vector which is equivalent to the property that, upon measurement of particle 1, the expectation catalog for particle 2 changes to be in agreement with the result for particle 1 and the predictions obtained from the entangled state (Eq. 1).

In the case of two particle-entanglement just discussed it is the experimenter's choice of the direction along which she measures the spin of particle 1 which determines that particle 2 will be in an eigenstate along that direction. Which specific eigenstate of the two possible ones it will be is completely random and outside the influence of the experimenter, it is "Nature's choice". We might also express this as the property that after measurement of particle 1, the expectation catalog for particle 2 gives a definite prediction for measurement along the same direction, the specific result being objectively undefined until the measurement on particle 1 is actually performed.

To summarize, for two particles, entanglement implies that no disjoint catalogs for all observable properties of the two particles exist and that the specific result of the measurement of an entangled quantity instantly permits prediction with certainty of the result of a measurement of the related quantity on the other particle.

THREE PARTICLES, TWO TERMS

All the discussion on entanglement in quantum mechanics until recently exclusively focused on two-particle states only. Yet it is evident that correlations between three and more particles provide a richer abundance of new quantum phenomena. For example, while any entangled state of two particles can always be written as a sum of just two terms (see e.g. Eq. 1) this in general is not true anymore for three particles. Specifically, there may be experimental situations where the state of the three-particle system consists of two terms or maybe even one term only, while in other situations three or more product terms are necessary for a complete description. Here we shall first analyze a specific case of a three-particle experiment where the state contains two terms.

The introduction of three-particle correlations into discussions of the EPR-paradox (Refs. 5 – 7) and related questions not only did lead to more stringent contradictions between local realistic models and quantum mechanics than in two-particle situations, it also provides qualitatively new entanglement phenomena. Let us consider a three-particle interferometer experiment of the type recently proposed (Ref. 7). A suitable source, say a nonlinear crystal exploiting a second-order nonlinearity of the

electric polarizability, emits three photons in the entangled state

$$|\psi\rangle = \sqrt{1/2}\ [|A\rangle_1 |B\rangle_2 |C\rangle_3 + |A'\rangle_1 |B'\rangle_2 |C'\rangle_3].\tag{2}$$

Here, e.g., $|A\rangle_1$ describes photon one in beam A etc. (see Figure 1). We now consider two possible choices the experimenter has:

(a) She might determine which path photon 1 takes by placing detectors into beam path A and beam path A'. As soon as one of these detectors fires, the state of the system collapses due to von Neumann wave packet reduction. This implies that depending on whether the detector in beam path A fires or the one in beam path A' the state of the remaining two particles is different. If detector A registers photon 1 (we assume, as is customary in photon experiments, that the photon is absorbed by a detector registering it) that state is

$$|\psi\rangle' = |B\rangle_2 |C\rangle_3.\tag{3}$$

But, if detector A' registers photon 1 the state of the remaining two photons is

$$|\psi\rangle' = |B'\rangle_2 |C'\rangle_3.\tag{4}$$

In either case, after registration of photon 1 the remaining two photons are left in a product state, i.e. they are not entangled. In other words, registering photon 1 in either beam A or A' did untangle the other two photons. From a complementarity point of view this might readily be understood on basis of the fact that registering any of the photons in a beam path before it encounters the recombining beam splitter instantaneously provides information not only in this photon's path but, because of the momentum correlations implied in state (2), also on the paths taken by the other two photons.

(b) The experimenter might alternatively decide not to insert detectors into any of the beam paths before the recombining beam splitter but measure the interference fringes instead. In order to simplify the analysis we assume that the phase shifter phases (Fig. 1) are all chosen to obey the condition

$$\phi_1 + \phi_2 + \phi_3 = \pi/2.\tag{5}$$

Let us now call D (E) the beam path of particle 1 leading to detector R_1 (L_1) and likewise F (G) and H (J) the beam paths of particles 2 and 3 leading to their detectors. Without loss of generality we assume that photon 1 is registered in detector D: The state of the remaining 2 photons is then (see Ref. 7)

$$|\psi\rangle = \sqrt{1/2}\ [|E\rangle_2 |F\rangle_3 - |E'\rangle_2 |F'\rangle_3].\tag{6}$$

Thus photon 2 and photon 3 are clearly in an entangled state now. This holds always for the remaining two photons if a photon was registered after any of the other beam splitters. In other words, registering a photon in a detector behind its recombining beam splitter does not untangle the other two photons.

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Clearly, the experiment can be set up in such a way that the detection events for the three photons occur at spacelike separation. Nevertheless, the experimenter's decision as to which measurement is to be performed at one of the photons determines - upon registration of the measurement result - whether or not the other two photons are left in an entangled state.

In Schrödinger's terms this means that the experimenter, simply by deciding whether to measure photon 1 before or after its recombining beam splitter, also decides whether or not the other two photons each enjoy their own disjoint expectation catalog, no matter how far these other photons might be away at the time that decision is made or at the instant of registration of photon 1. For completeness we simply remark here that, as in two-particle entanglement, the detection events of the three photons might be arranged in any time order.

A THREE-TERM STATE

For three or more particles with each particle enjoying its own two-dimensional Hilbert space there are evidently a number of different three-term states. Clearly the details of the experimental situation determine which state is present. To be specific, let us analyze the experiment represented in Figure 2. There, the incident beam A bearing radiation with wave number $k = 2\pi/\lambda$ is split by a series of successive partially reflecting mirrors into the beams B, C, and D and a through going beam. This latter one is assumed to enter a nonlinear crystal where it is upconverted into radiation with wavenumber 3k and then passes a filter set at the wavelength $\lambda' = 2\pi/3k$. Afterwards it is downconverted again. We shall call such a device consisting of an upconverter, a monochromatic filter and a downconverter a "number filter" because it lets only pass states with a certain number of photons, in our case three. This beam subsequently encounters a partially reflecting mirror again where some of it is deflected into a beam which we call A' towards a detector set to count a particle, call it particle 1. Registration of particle 1 in that detector acts as a trigger signal to indicate that a three-particle state has passed the number filter.

The transmitted beam again encounters some mirrors such that the beams B', C', and D' result as indicated in Figure 2. These latter beams are then superposed at a set of three semireflecting mirrors with the corresponding beams B, C, and D and we assume that detectors are placed into the outgoing beams of these semireflecting mirrors. We arrange the experiment such that the amplitudes incident on either port of any recombining mirror are equal and we look only for such events where in one and only one of the outgoing beams of each semireflecting mirror a photon is found (assuming 100% efficient detectors). Let us call these photons by their numbers 2, 3, and 4 respectively and let us agree to look only at the detectors if particle 1 has been registered in its detector.

The state of these photon quadruples is

$$|\psi\rangle = \sqrt{1/3} \, |A'\rangle_1 \left[|B'\rangle_2 |C'\rangle_3 |D\rangle_4 + |B'\rangle_2 |C\rangle_3 |D'\rangle_4 + |B\rangle_2 |C'\rangle_3 |D'\rangle_4 \right] \tag{7}$$

because this describes exactly the situation where photon 1 triggers the detector in beam path A' thus indicating that a three-photon has passed the number filter and one of these is photon 1. The other two photons coming through the filter might either be photon 2 and 3, or photon 2 and 4, or photon 3 and 4 as indicated by the primes in the terms above. For each term one other photon must have used the unprimed beam completing the quadruple.

We immediately note that in the state (7) photons 2, 3, and 4 are entangled with each other, while photon 1 is not entangled with any of the others, it enjoys its own disjoint catalog of predictions. More importantly, let us now consider what happens if we place a detector into any of the primed beam paths of photon 2, 3, or 4. Suppose explicitly and without loss of generality that a detector is placed into path B' of photon 2. Still we assume that we only look at such cases where the detector in beam A' has registered photon 1. Then two different possibilities arise: Either the detector in beam path B' fires or it does not fire. If it fires, the state of the remaining two photons, the photons 3 and 4, is

$$|\psi'\rangle = \sqrt{1/2} \, [|C'\rangle_3 |D\rangle_4 + |C\rangle_3 |D'\rangle_4] \tag{8}$$

which is clearly an entangled state for these two photons. On the other hand detector B' might not fire. In that case the predictions for the quadruple of photons can be described by the last term of Eq. (8)

$$|\psi'\rangle = |A'\rangle_1 |B\rangle_2 |C'\rangle_3 |D'\rangle_4 \tag{9}$$

which now implies that none of the photons is entangled with any of the other photons. Most remarkably, if the detector inserted into the beam path B' of photon 2 fires, photon 3 and 4 are entangled. If that detector does not fire, photon 3 and 4 are not entangled anymore! The absence of a registration event of the detector in beam B' untangles the other two photons. Whether or not such a registration event happens is completely and objectively random, at least within the standard interpretation of quantum probabilities. In this situation it is therefore a totally random event happening to photon 2 which determines whether or not photons 3 and 4 are entangled.

We point out here too that the apparatus might easily be arranged in such a way that the detection events on particles 2, 3 and 4 are spacelike separated from each other. Again we might call the specific random event "Nature's choice" and we find it quite remarkable that a spacelike separated random event happening to particle 2 decides whether or not particle 3 and 4 are entangled with each other.

CONCLUDING COMMENTS

Several comments might be in order. Firstly we remark that the analysis given above is not restricted to multiparticle interferometry. In fact, it is rather straightforward to give an example in terms of spin correlations. We might also point out that such experiments, though they have not been performed yet, should be feasible given further development in our knowledge of the nonlinear conversion processes in quantum optics. Furthermore, it was implicitly assumed that the incident radiation is rich enough to contain the multiparticle states exploited in the various experiments. Finally, entangled entanglement also is consistent with special relativity in the sense that it does not permit information to be transmitted with a speed larger than that of light.

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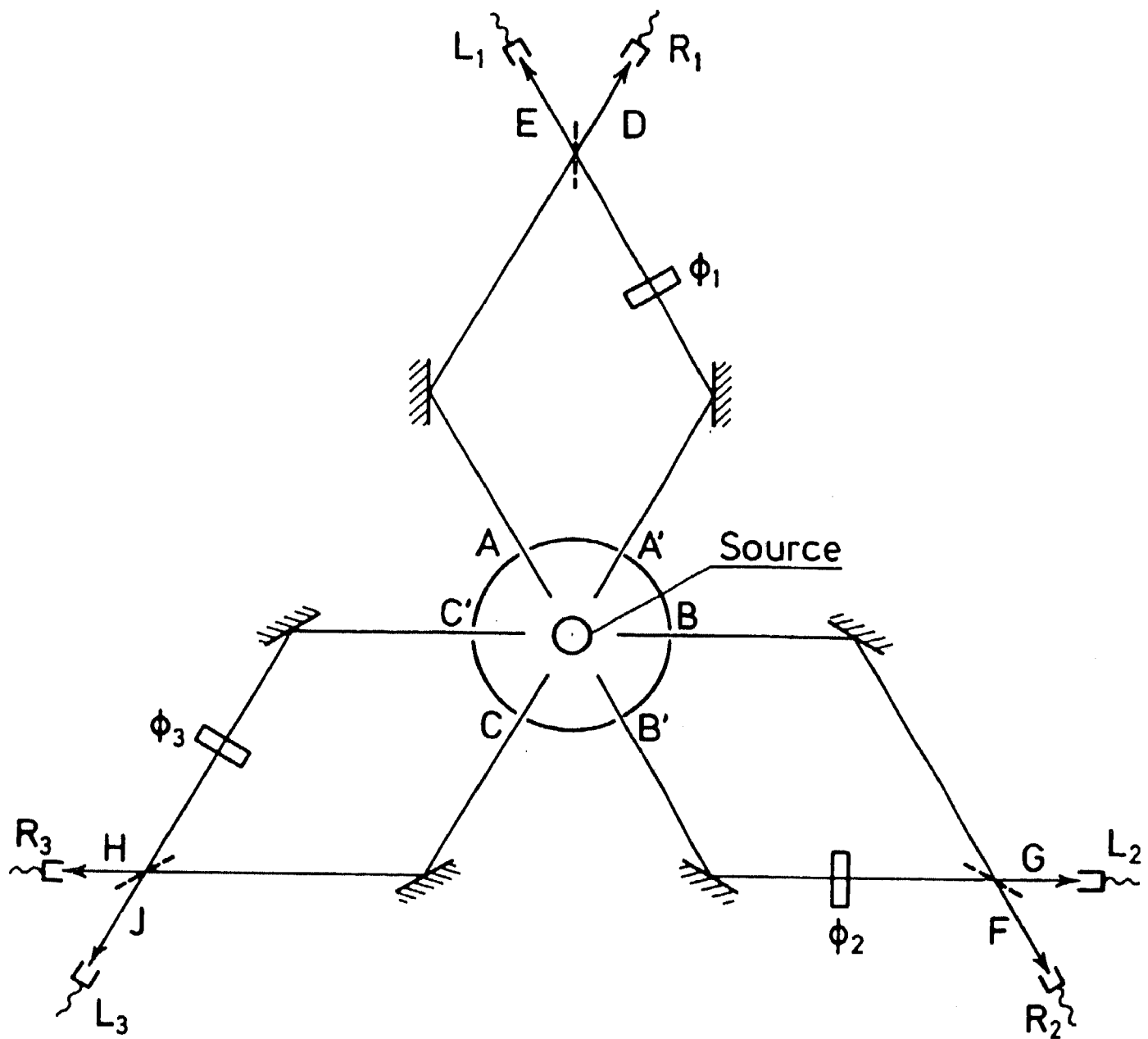
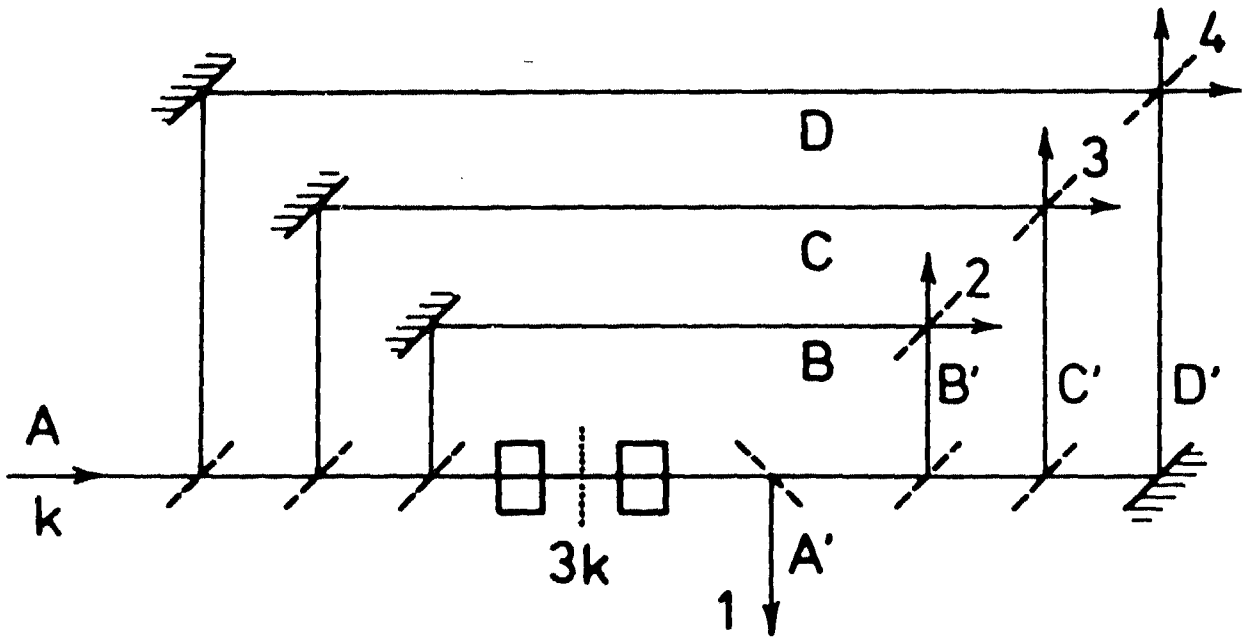Fig. 1: Principle of a 3-particle interferometer experiment.

Fig. 2: Principle of an interferometer experiment exhibiting entanglement of more than two terms.

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