# BELL'S THEOREM WITHOUT INEQUALITIES AND BEYOND

Anton Zeilinger[1,2], Daniel M. Greenberger[3], and Michael A. Horne[4]

1. Atominstitut der Österreichischen Universitäten Schüttelstraße 115, A-1020 Wien (Austria)
2. Institut für Experimentalphysik Universität Innsbruck, A-6020 Innsbruck (Austria)
3. City College of the City University of New York New York, NY 10013 (USA)
4. Stonehill College North Easton, MA 02357 (USA)

## 1. INTRODUCTION

The investigation of the foundations of quantum mechanics has received momentous stimulation when, in 1964, John Bell established his famous theorem[1]. It is specifically remarkable that Bell's work not only triggered an enormous amount of theoretical work, it also gave rise to an extensive series of experiments. Both the discussion and the experiments focussed on the quantum mechanical correlations in two-particle systems. Recently, an extension to systems with more than two particles was found with rather interesting novel properties[2]. In the present paper we shall firstly point out the similarities and the differences between the two situations and secondly we shall briefly discuss some new aspects concerning entangled states consisting of more than two particles partly by further generalizing the quantum states considered.

We remark in the beginning that in particular the more philosophical aspects of our paper might not be of immediate importance for the practicing quantum mechanic, be she an experimentalist or a theorist.

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Therefore anybody just interested in the application of quantum mechanics may safely ignore these aspects of our paper. That some of the more technical aspects of multiparticle correlations might have even practical applications is signified by the recent discussions in quantum cryptology[3].

## 2. TWO PARTICLES VERSUS THREE PARTICLES

Here we would like to give a brief analysis of the two-particle situations where Bell's inequality applies and compare it to the new three- and four-particle situation. The purpose of this chapter is to give an operational introduction for the newcomer to the field. For technical details we refer the reader to the papers in ref. 2.

An important point we wish to stress once more is that it was not the motive of John Bell to introduce some hidden variables into physics in order to go beyond quantum mechanics. All he did was to accept the original arguments of Einstein-Podolsky-Rosen (EPR)[4] at face value and - continuing their line of reasoning - he arrived at a conflict with quantum mechanics. The EPR line of argumentation starts from the observation that in two-particle systems exhibiting *perfect correlation* for some properties one can predict from the results of measurements on one particle exactly the results of certain measurements on the other particle. Thus EPR did explicitly accept predictions of quantum mechanics. EPR then employ their idea of *locality* that the measurement performed on one particle can cause no real change in the other particle. This because the measurement events on the two particles can very well be spacelike separated and this would not change the correlations. A keystone for the EPR argument is then the famous *reality* principle which states that an element of reality must correspond to a physical quantity which can be predicted with certainty. Thus the measured quantity on particle 2 must correspond to an element of reality. But because of the locality condition this must hold for any of the observables for which the quantum state prescribes perfect correlation. The important point now is that states exist which predict perfect correlations for noncommuting observables. Thus, according to EPR, elements of reality must exist corresponding to each of these noncommuting observables. Hence, finally, quantum mechanics cannot be a complete physical theory because there is no quantum state which predicts definite values for noncommuting observables.

Thus EPR arrived at the conclusion that quantum mechanics is an incomplete description of physical reality. They explicitly left open whether or not a complete theory is possible. Yet they expressed their believe in the possibility of constructing such a theory.

Bell, continuing along the lines of reasoning of EPR, explicitly assumed that the elements of reality as demanded by EPR on the basis of the existence of perfect correlations really do exist. Bell then demonstrated that this assumption leads to contradiction with quantum mechanics in cases where measurements on not perfectly correlated observables are performed. The formal expression of this statement are Bell's inequalities.

Operationally, one needs a source emitting particle pairs in correlated states, either one or two detectors for either particle and some variable parameter on both particles. The observables have to be dichotomic. These dichotomic variables usually are spin variables, either the two spin states of a spin-1/2 particle or the polarization states of a photon. Yet, properly chosen momentum-position variables might also lead to violations of Bell's inequalities. That explicit proposal[5] has recently led to the first non-spin verification[6] of violations of Bell's inequalities.

It is rather interesting to note that more than 20 years had to pass after Bell's seminal work until it was found that the application of the EPR program to systems consisting of more than 2 particles can result in even more striking disagreements with quantum mechanics[7]. The spirit of reasoning is the same as the one leading to the Bell inequalities. Here too one starts from the prediction of quantum mechanics of *perfect correlations* between some observables of the three or more particles. Through the application of *locality* and *reality* one arrives again at new theoretical predictions. In contrast to the two-particle case these predictions now are not statistical bounds anymore but they are predictions of perfect correlations for other combinations of variables. Quantum mechanics, on the other hand, also makes definite predictions for the combination of variables but with completely opposite eigenvalues.

Operationally, the three-particle situation is very similar to the two-particle one. One also has dichotomic variables for each particle and a variable parameter. Here too, experiments exploiting spin correlations and experiments exploiting momentum correlations have been proposed.

The most interesting aspect of the new situation is that reasoning using EPR *locality* and *reality* leads to wrong predictions even for perfect correlations. This is an intrinsic inconsistency within the EPR line of argument not present in the two-particle correlations. It shall be interesting to exploit this intrinsic inconsistency further in particular with the aim at identifying the possible sets of assumptions about the physical world consistent with quantum mechanics.

## 3. ENTANGLED ENTANGLEMENT

The central feature of all EPR-type situations is that the properties of the particles described by the quantum state considered are *more strongly correlated* than any correlations in classical physics. Schrödinger[8] has called this property *entanglement* (in the German original "Verschränkung"). Entanglement means that the properties of a particle in a correlated quantum state cannot be described only upon information obtained from within the particles's backward light cone but that the *results* of measurements on one or more of the other particles of the system are also required. It seems that the great majority of physicists ascribes this to some breakdown of locality or separability (evidently these two terms have to be specified very carefully) in quantum mechanics. Here we do not wish to enter that segment of the discussion but refer the reader to a very thorough presentation of the issues involved[9].

In the present paragraph we rather want to analyze the property of entanglement itself. Consider for example the three-particle correlated state proposed by Mermin[7]

$$|\psi\rangle = \sqrt{1/2}\ [\,|z+\rangle_1|z+\rangle_2|z+\rangle_3 + |z-\rangle_1|z-\rangle_2|z-\rangle_3]\,. \qquad (1)$$

where $|z+\rangle_i$ and $|z-\rangle_i$ mean states of particle i with spin up or down respectively along the z-direction. The coordinate system defined for each particle need not be the same. For example, in the case of photons propagating in different directions the spin is assumed to be defined along the direction of momentum of each photon. We now consider measurements of a given particle, say particle 1, along its z-direction. According to the reduction of the state vector the state of the remaining particles 2 and 3 after the measurement of particle 1 is either

$$|\psi>' = |z+>_2|z+>_3 \qquad (2)$$

or

$$|\psi>' = |z->_2|z->_3 \qquad (3)$$

depending on whether the result of the measurement on the first particle is + or -. In any case, after measurement of one particle along its z-direction the other two particles are in a factorizable state. In other words, each of the two remaining particles enjoys a well-defined state on its own after a z-measurement of the first particle, the two particles are not entangled anymore.

On the other hand, suppose we decide to measure particle 1 along some direction within the x-y-plane, say, along the x-axis. Then the state of the other two particles is not factorizable, independent of the specific result of the measurement on particle 1. In other words, the decision of experimenter 1 to measure the spin of particle i along a direction within the x-y-plane or along the z-direction decides also whether the other two particles are left in an entangled state or not after the first measurement. Therefore, in this case, entanglement is an entangled property dependent on the experimenters choice.

Another rather intriguing situation arises if we generalize the EPR discussion even further. After the GHZ generalization to 3 and 4 particles we now consider states consisting of more than two nonfactorizable terms. This, we note, is not possible within the standard Bell-inequality discussions since it is easy to see that any sum of product states of two two-state particles can always be written as a sum of just two terms. For more than two two-state particles this can only be done in limited cases. For our discussion we just consider the rather innocent looking state

$$|\psi> = \sqrt{1/3}\ [\,|z+>_1|z+>_2|z->_3 + |z+>_1|z->_2|z+>_3 + |z->_1|z+>_2|z+>_3 \qquad (4)$$

For this state we again contemplate measurements along the z-direction of, say, particle 1. If the result of that measurement is + then the remaining two particles are found in the state

$$|\psi\rangle' = \sqrt{1/2}\ [\,|z+\rangle_2 |z-\rangle_3 + |z-\rangle_2 |z+\rangle_3\,]. \tag{5}$$

and if the result of the measurement on particle 1 is - the remaining two particles are found in the state

$$|\psi\rangle' = |z+\rangle_2 |z+\rangle_3. \tag{6}$$

Therefore, if the result of the measurement on particle 1 was +, the other two particles are left in an entangled state and if the result of the first measurement was - the two remaining particles are left in a factorizable state, they each enjoy their own pure state. In this example now it is not the experimenters choice but Nature's choice which decides whether or not the other two particles are in a pure state after the first measurement. We point out that before the first measurement all 3 particles are entangled with each other and there is nothing in state (4) which would decide whether or not after the first measurement the other two particles will be entangled or not.

We therefore have shown in the present paragraph that entanglement itself can be an entangled property and that whether or not the remaining particles are left in an entangled state after a measurement on one particle might depend either on the experimenters choice or on the specific statistical outcome of the first experiment.

## 4. CONCLUDING COMMENTS

Entanglement is becoming a more and more common phenomenon in experiments. As a specific example of possible future work we might note the possible entanglement in atomic interferometry. There an atom enjoying both beam paths could be entangled with a spontaneously emitted photon. Evidently. our considerations also apply if the base states are not spin states and if other particles than photons are involved.

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