On the topological nature of the Aharonov–Casher effect

Anton Zeilinger
Atominstitut der Österreichischen Universitäten, Schüttelstrasse 115, A-1020 Vienna, Austria

Roland Gähler
Physik-Department E21, Technische Universität München, D-8046 Garching, Germany

and

Michael A. Horne
Stonehill College, North Easton, MA 02356, USA

Received 14 January 1991; accepted for publication 4 February 1991
Communicated by J.P. Vigier

In order to manifest the topological nature of the Aharonov–Casher effect more strongly, we propose to demonstrate experimentally the non-dispersiveness of the Aharonov–Casher phase shift and to perform experiments with geometries varying the position of the line charge relative to the neutron interferometer beams, with and without changing the topology of the arrangement.

Recently, Cimmino et al. [1] reported the observation of the Aharonov–Casher (AC) effect [2] in a very beautiful experiment. In the idealized experiment, the two beams in a neutron interferometer enclose a region containing an electric line charge with linear charge density Λ. This charge causes a phase shift

$$\Delta\Phi_{AC} = \pm \frac{4\pi\mu\Lambda}{\hbar c}, \qquad (1)$$

where μ is the neutron magnetic moment. The two signs hold for the two spin states of the neutron. Due to the smallness of this phase shift even for sizeable laboratory electric fields ($\Delta\Phi_{AC} = 1.50$ milliradians for the parameters of the experiment) a large number of neutrons (of the order of 10^9) had to be counted. Nevertheless, Cimmino et al. could clearly demonstrate that the effect exists and that it is of the magnitude predicted.

The Aharonov–Casher effect is a topological analogue of the magnetic Aharonov–Bohm [3] effect. As a consequence of the topological nature of the AC effect the phase shift depends only on the total amount of charge enclosed between the interferometer beams and no classical force acts on the neutron traversing the electric field produced by the line charge. In the present Letter we propose additional experiments to further clarify the topological nature of the AC effect and we raise various related points we consider interesting.

The AC effect shares with the AB effect a controversy on its quantum mechanical nature. A central question in that controversy is whether there is indeed no classical force acting on the neutron. Associated with such a controversial force there would be an equally controversial lag [4] of the wave packet passing the line charge on one side compared to that on the other side. Though it has been argued [5] on theoretical grounds that this lag occurs only in a naive approach there is clearly a need for detailed experimental resolution. Unfortunately, if it existed, such a lag would be extremely small. The experimentally observed AC phase shift of 2 milliradians, for neutrons with wavelength $\lambda = 1.477$ Å, would correspond to a spatial lag of only 0.5×10^{-3} Å or, with the neutron velocity of 2700 m/s, to a temporal lag of only 1.8×10^{-17} s between the two wave packets travelling the two routes through the interferometer. Clearly, this is much too small for a direct experimental search. Yet, we point out, the question whether or not there is a lag can be subject to an indirect experimental test by measuring the AC phase shift with neutrons of different wavelengths thereby confirming that the observed phase shift is non-dispersive as predicted.

It is easy to see that confirmation of the wavelength independence proves that there can be no lag. If, with no charge present, the wave function ψ_0 of the neutrons leaving the interferometer is a superposition of the two wave functions ψ_1 and ψ_2 of the two routes through the interferometer, $\psi_0=\psi_1+\psi_2$, then, with the charge present, this wave function is changed into $\exp(i\phi)\,[\psi_1+\exp(i\Delta\Phi_{AC})\psi_2]$, where ϕ is an irrelevant common phase factor. Clearly, ψ_2 and $\exp(i\Delta\Phi_{AC})\psi_2$ have identical space–time distributions if the phase shift $\Delta\Phi_{AC}$ is independent of wavelength. Identical space–time distribution implies that there can be no lag. This may also be seen from the property that the position $x=x_0$ of the maximum of the wave packet as given by the constant-phase condition

$$\frac{d}{dk}\psi(k)\bigg|_{x=x_0}=0 \tag{2}$$

does not change due to a wavelength-independent phase factor $\exp(i\Delta\Phi_{AC})$, i.e.

$$\frac{d}{dk}\left[\exp(i\Delta\Phi_{AC})\psi(k)\right]\bigg|_{x=x_0}=0\,. \tag{3}$$

For this result the non-dispersiveness of the phase shift is crucial. Consider, in contrast, that an arbitrary wave packet

$$\psi(\boldsymbol{r},t)=\int A(\boldsymbol{k})\mathrm{e}^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}\,d\boldsymbol{k}$$

if subject to a dispersive phase shift $\varphi(\boldsymbol{k})$ is changed into

$$\psi'(\boldsymbol{r},t)=\int A(\boldsymbol{k})\mathrm{e}^{i\varphi(\boldsymbol{k})}\mathrm{e}^{i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)}\,d\boldsymbol{k}\,.$$

Then the space–time distributions of $\psi(\boldsymbol{r},t)$ and $\psi'(\boldsymbol{r},t)$ are not identical anymore. In particular, the positions of constant phase will not coincide anymore, there will be a lag of the wave packet. In general, there can be a lag only if the phase shift is dispersive. We would like to stress here that our analysis as to whether or not there is a lag in the AC experiment was done purely within quantum mechanics and did not at all rely on classical correspondence considerations.

It is therefore desirable to experimentally demonstrate that the AC phase shift is indeed non-dispersive. Of the many possibilities for testing with neutrons the wavelength independence of the AC phase shift the most forthcoming one would be to use the apparatus of Cimmino et al. without modification if the interferometer is illuminated by neutrons incident at the same angle but with half the wavelength. These half-wavelength neutrons will undergo second order diffraction in the interferometer crystal and therefore follow the same paths through the apparatus. One might also be interested in approximating more closely the line charge ideal to demonstrate directly the topological nature of the AC phase shift, i.e., that the phase shift (a) is independent of where the line charge penetrates the area enclosed by the beams and (b) is zero if the line charge is located anywhere outside that area no matter how close to a beam. We suggest that such future experiments may be facilitated by utilizing the large-area interferometer recently tested for $\lambda=100$ Å neutrons [6].

We would like to make three final points. Firstly, the AC phase shift may also be viewed as a manifestation of the spin–orbit interaction [7] [#1] of the neutron magnetic moment with the electric field $\boldsymbol{E}$. The non-dispersiveness of the AC phase shift is then a direct consequence of the linear dependence of the spin–orbit interaction Hamiltonian $H=-\boldsymbol{\mu}\cdot(\boldsymbol{v}\times\boldsymbol{E})$ on the neutron velocity $\boldsymbol{v}$. One might therefore expect AC type effects to arise also in other situations where a Hamiltonian linearly dependent on particle velocity applies. Secondly, the usual description that the moving neutron "sees" an effective magnetic field is not conforming to a description in the laboratory frame. There, a more satisfactory view is that the moving neutron can be considered to carry the electric dipole moment $\boldsymbol{\mu}_{el}=\boldsymbol{\mu}_{mag}\times\boldsymbol{v}$ which interacts directly with the electric field. Thirdly, the phase effect

[#1] The resulting spin–orbit scattering of neutrons at nuclei has been observed by Shull [8].

of this interaction can be radians instead of milliradians if one uses the electric field of atomic nuclei instead of the electric field of macroscopic electrodes and exploits the spatially periodic arrangement of the nuclei in a perfect crystal [9].

We believe that the experiments proposed here would add significantly to the evidence on the topological nature of the AC effect.

This work was supported by the Austrian Fonds zur Förderung der wissenschaftlichen Forschung project No. P6635 and by the US National Science Foundation (grants No. DMR 87-13559 and INT 87-13341).

References

[1] A. Cimmino, G.I. Opat, A.G. Klein, H. Kaiser, S.A. Werner, M. Arif and R. Clothier, Phys. Rev. Lett. 63 (1989) 380.
[2] Y. Aharonov and A. Casher, Phys. Rev. Lett. 53 (1984) 319.
[3] Y. Aharonov and D. Bohm, Phys. Rev. 115 (1959) 485; 123 (1961) 1521.
[4] T.H. Boyer, Phys. Rev. A 36 (1987) 5083.
[5] Y. Aharonov, P. Pearle and L. Vaidman, Phys. Rev. A 37 (1988) 4052;
A.S. Goldhaber, Phys. Rev. Lett. 62 (1989) 482.
[6] M. Gruber, K. Eder, A. Zeilinger, R. Gähler and W. Mampe, Phys. Lett. A 140 (1989) 363.
[7] J. Schwinger, Phys. Rev. 73 (1948) 407;
L.L. Foldy, Rev. Mod. Phys. 30 (1958) 471.
[8] C.G. Shull, Phys. Rev. Lett. 10 (1963) 297.
[9] M.A. Horne, K.D. Finkelstein, C.G. Shull, A. Zeilinger and H.J. Bernstein, Physica B 151 (1988) 189;
K.D. Finkelstein, Ph.D. thesis, M.I.T. (1988).