INTRODUCTION TO TWO-PARTICLE INTERFEROMETRY

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Ordinary interferometry employs beams of particles -- photons, electrons, neutrons, and possible other particles -- but the phenomena which it studies arise when two amplitudes associated with a single particle combine at a locus. When the single particle is characterized by a quantum state, the two amplitudes have a definite phase relation. The variation of the relative phase as one or more parameters vary gives rise to the familiar interferometric "fringe" pattern, which characteristically is sinusoidal.

The phenomena of two-particle interferometry also arise from the combination of two amplitudes with a definite phase relation. The radical innovation is the employment of beams of two-particle systems, with each pair in an "entangled" state, that is, a state which cannot be expressed as a simple product of quantum states of the two particles separately. That quantum mechanics permits in principle the existence of pairs of spatially separated particles in entangled states has been known at least since the classical paper of Einstein, Podolsky, and Rosen (1935), and the actual existence of such pairs has been known since the analysis by Bohm and Aharonov (1957) of the experiment of Wu and Shaknov (1950). It is only in the last five years, however, that beams of entangled two-particle systems have been subjected to the traditional interferometric techniques of splitting, directing, and combination.[1]

In this lecture we shall analyze a schematic arrangement (Fig. 1) to show that when the particle pairs are appropriately prepared, then quantum mechanics predicts two-particle interference fringes and predicts at the same time the non-occurrence of single-particle fringes. We shall then illustrate the experimental potentialities of two-particle interferometry by showing how this arrangement makes possible a test of Bell's Inequality without polarization analysis.

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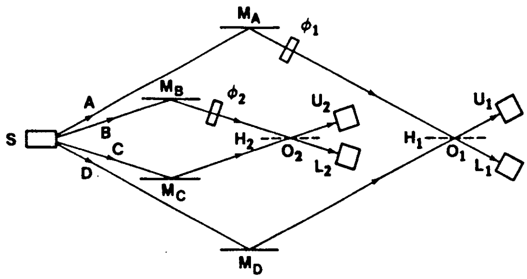

Fig. 1. An arrangement for two-particle interferometry with variable phase shifters.

In the arrangement of Fig. 1 an ensemble of particle pairs is emitted from the source S into the beams A,B,C,D, each pair in the ensemble being in the entangled quantum state

$$|\psi\rangle = 2^{-\frac{1}{2}}(|A\rangle_1|C\rangle_2 + |D\rangle_1|B\rangle_2). \tag{1}$$

This state describes a coherent superposition of two distinct pairs of correlated paths for particles 1 and 2. In one of these, particle 1 enters beam A and is reflected from mirror M_A to phase shifter ϕ_1 en route to beam splitter H_1, from which it proceeds either into the upper channel U_1 or the lower channel L_1; while particle 2 enters beam C and is reflected from mirror M_C to beam splitter H_2, from which it proceeds either into the upper channel U_2 or the lower channel L_2. In the other pair of correlated paths particle 1 enters beam D and proceeds to U_1 or L_1 via mirror M_D and H_1, while particle 2 enters beam B and proceeds to U_2 or L_2 via mirror M_B, phase shifter ϕ_2, and H_2. The beams A,B,C,D are assumed to be in a single plane, and their directions ensure momentum conservation (i.e., the sum of the momenta of particles 1 and 2 in A and C respectively equals the sum of the momenta of particles 1 and 2 in D and B respectively). We wish to calculate the probabilities that the two particles will jointly enter each of the four possible pairs of exit channels: (U_1,U_2), (U_1,L_2), (L_1,U_2), and (L_1,L_2). Quantum mechanically each of these probabilities is expressed as the absolute square of a total probability amplitude, for instance,

$$P_\psi(U_1,U_2,|\phi_1,\phi_2) = |A_\psi(U_1,U_2|\phi_1,\phi_2)|^2, \tag{2}$$

where the dependence of this probability upon the initial quantum state and upon the variable phase shifters ϕ_1 and ϕ_2 has been indicated explicitly. There are two contributions to the probability amplitude A_ψ: one comes from particle 1 entering beam A and eventually being reflected from H_1, while particle 2 enters beam C and eventually is transmitted through H_2; whereas the other comes from particle 1 entering beam D and eventually being transmitted through H_1, while particle 2 enters beam B and is reflected from H_2. In the first contribution particle 1 encounters the phase shifter ϕ_1, and in the second particle 2 encounters the phase shifter ϕ_2. We need to calculate the relative phase of these two contributions.

A necessary preliminary to this calculation is the derivation of an equation governing the phase relations of reflected and transmitted rays from a lossless beam-splitter, when two rays are incident symmetrically upon its two faces, as indicated in Fig. 2. The rays correspond to quantum states of definite linear momentum and are denoted by $|I\rangle$ and $|J\rangle$ respectively. If the beam-splitter is symmetric, the moduli of the reflected and the transmitted output from each incident ray are equal. Let $|I'\rangle$ denote the total

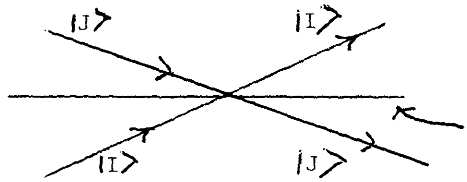

Fig. 2. Incident rays |I>, and |J'> reflected and transmitted from a symmetric, lossless beam-splitter.

output state from incident |I>, and |J'> denote the total output state from incident |J>. Following Zeilinger (1981) we use losslessness to connect output to input by a unitary operator U and use the symmetries to write

$$|I'\rangle = 2^{-\frac{1}{2}}(e^{it}|I\rangle + e^{ir}|J\rangle) = U|I\rangle, \tag{3a}$$

$$|J'\rangle = 2^{-\frac{1}{2}}(e^{ir'}|I\rangle + e^{it'}|J\rangle) = U|J\rangle, \tag{3b}$$

where the real numbers r and r' are the phase shifts due to reflection, and the real numbers t and t' are the phase shifts due to transmission through the beam-splitter. Because of the orthogonality of |I> and |J> and unitarity, |I'> and |J'> are orthogonal, and hence

$$0 = \langle I'|J'\rangle = \tfrac{1}{2}[e^{i(r'-t)} + e^{i(t'-r)}] \tag{4}$$

so that

$$r' - t = t' - r + \pi (\mathrm{mod}\ 2\pi). \tag{5}$$

We now return to Fig. 1 in order to calculate the probability amplitude $A_\psi(U_1,U_2|\emptyset_1,\emptyset_2)$. Let r_1 and t_1 be the phase shifts of Eq. (3a) associated with reflection and transmission of the ray incident upon beam-splitter H_1 from below, and r_1' and t_1' be the phase shifts of Eq. (3b) associated with reflection and transmission of the ray incident upon H_1 from above. Let r_2,t_2,r_2',t_2' have analogous meanings for beam-splitter H_2. Let s_1 be the phase change associated with the upper path of particle 1 from S to H_1, omitting $\emptyset_1$, and s_1' be the phase change associated with the lower path, via beam D; likewise, let s_2 be the phase change associated with the upper path of particle 2 from S to H_2, omitting $\emptyset_2$, and s_2' the phase change associated with the lower path, via beam C. Finally we use the letters $\emptyset_1$ and $\emptyset_2$ not only to designate the apparatus used for variable phase shifting, but also for the amounts of these phase shifts -- an ambiguity of notation which will cause no confusion. Using Eqs. (3a) and (3b) and collecting all these phases we obtain

$$A_\psi(U_1,U_2|\emptyset_1,\emptyset_2) = \frac{2^{-\frac{1}{2}}}{2}[\exp i(s_1+\emptyset_1+r_1+s_2'+t_2') + \exp i(s_1'+t_1'+s_2+\emptyset_2+r_2)]. \tag{6}$$

Hence,

$$P_\psi(U_1,U_2|\emptyset_1,\emptyset_2) = 1/4[1 + \cos(\emptyset_1 - \emptyset_2 + w)], \tag{7}$$

where w is a total fixed phase shift, independent of the variable phase shifts $\emptyset_1$ and $\emptyset_2$, specifically,

$$w = s_1+r_1+s_2'+t_2'-s_1'-t_1'-s_2-r_2. \tag{8}$$

Likewise,

$$P_\psi(U_1,L_2|\emptyset_1,\emptyset_2) = \tfrac{1}{4}[1 + \cos(\emptyset_1 - \emptyset_2 + w')], \tag{9}$$

where

$$w' = s_1+r_1+s_2'+r_2'-s_1'-t_1'-s_2-t_2, \tag{10}$$

and expressions similar to Eqs. (7) and (9) can be given for $P_\psi(L_1,U_2|\emptyset_1,\emptyset_2)$ $P_\psi(L_1,L_2|\emptyset_1,\emptyset_2)$. In short, the probability of joint entrance of particles 1 and 2 into any of the four possible pairs of channels depends sinusoidally upon the difference $\emptyset_1-\emptyset_2$ of the variable phase shifts. Thus quantum mechanics predicts two-particle interference fringes in the experimental arrangement that has been described. What is extraordinary is that there are no one-particle interference fringes in this arrangement, as one can see by adding Eqs. (7) and (9) to obtain the probability that particle 1 will enter channel U_1, regardless of the behavior of particle 2:

$$P_\psi(U_1|\emptyset_1,\emptyset_2) = P_\psi(U_1,U_2|\emptyset_1,\emptyset_2) + P_\psi(U_1,L_2|\emptyset_1,\emptyset_2) =$$

$$\tfrac{1}{2} + \cos(\emptyset_1 - \emptyset_2 + w) + \cos(\emptyset_1 - \emptyset_2 + w') = \tfrac{1}{2}, \tag{11}$$

because by Eqs. (8), (10), and (5),

$$w' = w + (r_2'-t_2-t_2'r_2) = w + \pi (\text{mod } 2\pi). \tag{12}$$

In fact, no matter what the values are of the variable phase shifts $\emptyset_1$ and $\emptyset_2$, the single-particle probabilities are the same, namely $\tfrac{1}{2}$. This result is at first very surprising, not only because of the sinusoidal behavior of the two-particle probabilities but also because in the arrangement of Fig. 1 each of the particles 1 and 2 seems to be subjected separately to a Mach-Zehnder interferometric experiment.

The quantum mechanical explanation for the absence of single-particle interference fringes is obtained by returning to the entangled state of Eq. (1) and inquiring what it implies about the state of particle 1 by itself and the state of particle 2 by itself. Neither 1 nor 2 is in a pure quantum state, but both can be described by statistical or density operators W_1 and W_2, as discussed, for example, by Beltrametti and Cassinelli (1981), 66, where

$$W_1 = \tfrac{1}{2}(|A\rangle\langle A| + |D\rangle\langle D|), \tag{13}$$

$$W_2 + \tfrac{1}{2}(|B\rangle\langle B| + |C\rangle\langle C|). \tag{14}$$

All predictions concerning particle 1 alone, neglecting correlations with particle 2, can be obtained from Eq. (13) and will be in exact agreement with those obtained from Eq. (1); and all predictions concerning particle 2 alone can be obtained from Eq. (14) and will agree with Eq. (1). Now W_1 is the statistical operator that would correctly describe an ensemble, of which half of the members are in quantum state $|A\rangle$ and half are in quantum state $|D\rangle$, though infinitely many other ensembles (so-called "mixtures") are correctly described by W_1. And likewise, W_2 is the statistical operator that would correctly describe the ensemble of which half are in state $|B\rangle$ and half are in state $|C\rangle$. Of course, neither of these ensembles would exhibit interference fringes, since each particle in each ensemble travels from source to output channel by only one path. Hence, neither ensemble takes advantage of the Mach-Zehnder interferometer to bring together contributions by two different paths, with definite phase relations, as required for single-particle interference fringes. Another way to put the matter is to say that the entangled state of Eq. (1) shows a definite phase relation between two two-particle states, namely $|A\rangle_1|C\rangle_2$ and $|D\rangle_1\ |B\rangle_2$, but no definite phase relations between single-particle states.

An obvious question is how one can know that the quantum state of a pair of particles emerging from the source has the form of Eq. (1). There are two ways to answer this question, one hard and one easy. The hard way is to describe quantum mechanically the process which gives birth to the two-particle pair and show that the sresulting quantum state of !+2 has the desired form. The (relatively) easy way is to do two-particle interferometry, in order to see whether two-particle interference fringes are exhibited, for it is straightforward to show that if the quantum state of each pair emerging from the source is a product of single-particle states, then the two-particle fringe behavior of Eqs. (7) and (9) will not be exhibited. So far, the only realizations of two-particle interferometry have used pairs of photons produced by the interaction of single photons with an appropriate crystal[1], and in these experiments the observation of two-particle interference fringes provides decisive evidence for the entangled state of the emerging two-photon system.

At the conclusion of the lecture "An Exposition of Bell's Theorem" in this volume it was noted that there is no intrinsic reason why a polarization experiment is necessary for the purpose of testing Bell's Inequality. Indeed, the arrangement of Fig. 1 of the present lecture is a special case of the schematic arrangement of Fig. 1 of that lecture and can be used to test an Inequality, when the following identifications are made: the outcomes of analysis of particle 1 are passage into channels U_1 and L_1, and the conventional values s_m assigned to these two outcomes are 1 and -1 respectively; likewise the outcomes of analysis of particle 2 are passage into channels U_2 and L_2, and the values t_n assigned to these are 1 and -1 respectively; and the variable parameters a and b are taken to be the variable phase shifts ϕ_1 and ϕ_2. Then Inequality (4) of "An Exposition of Bell's Theorem" can be rewritten as

$$-2 \le E_w(\phi_1',\phi_2') + E_w(\phi_1',\phi_2'') + E_w(\phi_1'',\phi_2') - E_w(\phi_1'',\phi_2'') \le 2. \qquad (15)$$

The quantum mechanical expectation value of the products of outcomes, when the variable phase shifts are ϕ_1 and ϕ_2, is

$$\begin{aligned} E_\psi(\phi_1,\phi_2) &= P_\psi(U_1,U_2|\phi_1,\phi_2)\cdot 1 + P_\psi(U_1,L_2|\phi_1,\phi_2)\cdot(-1) + \\ &\quad P_\psi(L_1,U_2|\phi_1,\phi_2)\cdot(-1) + P_\psi(L_1,L_2|\phi_1,\phi_2)\cdot 1 = \\ &\quad \tfrac{1}{4}[1 + \cos(\phi_1 - \phi_2 + w)]\cdot 1 + \tfrac{1}{4}[1 - \cos(\phi_1 - \phi_2 + w)]\cdot(-1) \\ &\quad + \tfrac{1}{4}[1 - \cos(\phi_1 - \phi_2 + w)]\cdot(-1) + \tfrac{1}{4}[1 + \cos(\phi_1 - \phi_2 + w)]\cdot 1 \\ &= \cos(\phi_1 - \phi_2 + w). \end{aligned} \qquad (16)$$

Now choose the variable phase shifts as follows:

$$\phi_1' = \tfrac{1}{2}\pi,\ \phi_2' = \tfrac{1}{4}\pi + w,\ \phi_1'' = 0,\ \phi_2'' = (3\pi/4)+w. \qquad (16)$$

Then,

$$\begin{aligned} \cos(\phi_1'-\phi_2'+w) &= \cos(\phi_1'-\phi_2''+w) = \cos(\phi_1''-\phi_2'+w) = -\cos(\phi_1''-\phi_2''+w) \\ &= 0.707, \end{aligned} \qquad (17)$$

and

$$E_\psi(\phi_1',\phi_2') + E_\psi(\phi_1',\phi_2'') + E_\psi(\phi_1'',\phi_2') - E_\psi(\phi_1'',\phi_2'') = 2.828, \qquad (18)$$

in disaccord with Inequality (15). The quantity w which enters into the choice of the variable phase shifts in Eq. (16) is determinable experimentally, by varying one or the other of ϕ_1 and ϕ_2 until the joint probability for photon 1 to enter U_1 and photon 2 to enter U_2 becomes 0, and then using Eq. (7).

As discussed in "An Exposition of Bell's Theorem," the detection loophole can be blocked if sufficiently efficient photodetectors are developed. It may be easier to block this loophole in the experimental arrangement of Fig. 1, which is based upon the linear momentum correlation of the two photons, than in a polarization correlation experiment, because in the latter there are two competing demands on the efficiency of the apparatus: both the polarization analyzers and the photodetectors must be sufficiently efficient, and these demands are best fulfilled in different energy ranges of the photons.

In order to achieve a test of Bell's Inequality as decisive as that of Aspect et al. (1982), it would be necessary to vary the phase shifts ϕ_1 and ϕ_2 very rapidly, in time intervals of the order of 10 nanoseconds.[1] It is, of course, very difficult to satisfy this desideratum experimentally, but in principle it is possible, either by using acousto-optical switches, like those of Aspect et al., or by electro-optical devices.

Quite apart from the potentiality of our proposal for achieving improvements over previous tests of Bell's Inequalities, it may be pedagogically valuable. The proposed arrangement is simpler than that of the polarization correlation experiments, and opens the possibility of performing a test of Bell's Inequality as a demonstration in an undergraduate class. Furthermore, the demonstration of two-photon interference fringes in the absence of one-photon fringes would be a vivid illustration of quantum mechanical nonlocality.

FOOTNOTES

[1]Two-particle interferometry using pairs of photons produced by parametric down-conversion was reported by Ghosh and Mandel (1987), Hong, Ou, and Mandel (1987), Ou and Mandel (1988a) and (1988b), Alley and Shih (1986), and Shih and Alley (1988). The last three of these references report tests of Bell's Inequality, but in these tests quarter wave plates are introduced into the beams for the purpose of transforming momentum correlation into polarization correlation. In the proposal of the present lecture, which was briefly mentioned in Horne, Shimony, and Zeilinger (1989) and will be developed in more detail in a later paper by us, polarization correlation is completely avoided. Two-particle interferometry using pairs of photons produced in positronium annihilation was proposed by Horne and Zeilinger (1985), (1986), and (1988), but there are great obstacles in the way of realizing their proposal. Rarity and Tapster (1989) have also proposed a test of Bell's Inequality without polarization analysis, using the momentum correlation of photon pairs produced by parametric down-conversion,and had already obtained preliminary results by July, 1989.

REFERENCES

Alley, C. and Shih, Y.H., 1986, in "Proceedings of the Second International Symposium on Foundations of Quantum Mechanics in the Light of New Technology," M. Namiki et al., eds., Physical Society of Japan, Tokyo, 47.

Aspect, A., Dalibard, J., and Roger, G., 1982, Physical Review Letters, 49: 1804.

Beltrametti, E. and Cassinelli, G., 1981, "The Logic of Quantum Mechanics," Addison-Wesley, Reading, MA.
Bohm, D. and Aharonov, Y., 1957, Physical Review, 108:1070.
Einstein, A., Podolsky, B., and Rosen, N., 1935, Physical Review, 47:777.
Ghosh, R. and Mandel, L., 1987, Physical Review Letters, 59:1903.
Hong, C.K., Ou, Z.Y., and Mandel, L., 1987, Physical Review Letters, 59:2044.
Horne, M.A., Shimony, A., and Zeilinger, A., 1989, Physical Review Letters, 62:2209.
Horne, M.A. and Zeilinger, A., 1985, in "Proceedings of the Symposium on the Foundations of Modern Physics," P. Lahti and P. Mittelstaedt, eds., World Scientific, Singapore, 435.
Horne, M.A. and Zeilinger, A., 1986, "New Techniques and Ideas in Quantum Measurement Theory," D. Greenberger, ed., New York Academy of Sciences, New York, 469.
Horne, M.A. and Zeilinger, A., 1988, in "Microphysical Reality and Quantum Formalism," A. van der Merwe, et al., eds., Kluwer, Dordrecht, vol. 2, 401.
Ou, Z.Y. and Mandel, L., 1988a, Physical Review Letters, 61:50.
Ou, Z.Y. and Mandel, L., 1988b, Physical Review Letters, 61:54.
Rarity, J.C. and Tapster, P.R., 1989, "Spatially separated two color photons in fourth order interference," unpublished preprint.
Shih, Y.H. and Alley, C.O., 1988, Physical Review Letters, 61:2921.
Wu, C.S. and Shknov, J., 1950, Physical Review, 77:136.
Zeilinger, A., 1981, American Journal of Physics, 49:882.

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