Down-conversion Photon Pairs: A New Chapter In the History of Quantum Mechanical Entanglement

Michael HORNE, Abner SHIMONY, and Anton ZEILINGER

Department of Physics, Stonehill College
North Easton, MA 02356

Departments of Philosophy and Physics
Boston University, Boston, MA 02215

Atominstitut der Österreichischen Universitäten
Schüttelstrasse 115, A-1020 Vienna, Austria

ABSTRACT

An entangled state of a many-particle system is one which cannot be expressed as a product of single-particle states. We summarize the major steps in the history of this concept, notably Schrödinger's original recognition of entanglement, its role in the Gedankenexperiment of Einstein-Podolsky-Rosen, and the Bohm-Aharonov demonstration that photon pairs from positronium annihilation are entangled. We then show that interferometric phenomena of photon pairs generated in parametric down-conversion are manifestations of entanglement, and we analyze two experiments in which different types of entanglement appear: that of Ghosh and Mandel, in which there is entanglement of *directions* of wave-vectors, and that of Kwiat et al., in which there is entanglement of *magnitudes* of wave-vectors (or wave-numbers).

1. Introduction

This paper is two years late. It should have been presented in 1987 at the Conference on the Foundations of Quantum Mechanics to Celebrate the Thirtieth Anniversary of the Demonstration by Bohm and Aharonov of the Existence of Entangled States of Spatially Separated Systems.[1] That conference did not take place – or, in the language of quantum mechanics, it was a virtual conference – and therefore we wish to use the occasion of the present conference, honoring Yakir Aharonov and David Bohm for their illumination of the effect that bears their name, to recognize an earlier contribution of theirs, which in our opinion is of the same order of magnitude.

But first, since we are evidently preoccupied with history, we wish to ensure that due recognition be given to two other physicists who made a pioneering contribution to the subject of the present conference. In 1949 W. Ehrenberg and R.W. Siday noticed the effect on an electron's wave function due to a magnetic field which is present only in the region where the wave function is zero.[2] The formula which they wrote was essentially that which Aharonov and Bohm[3] investigated. Although Ehrenberg and Siday were struck by the peculiarity of the effect, they made no attempt to analyze its conceptual significance, whereas Aharonov and Bohm pointed out that the effect implies either that quantum mechanics endows the electromagnetic potentials with a kind of reality over and above that of the fields, or that it implies a kind of nonlocality, or both. Science is not a zero-sum game, and therefore a large element of glory is due to each of these two pairs of pioneers, without detraction from the other.

2. Some History of Entanglement

We now turn to the concept of entanglement and to its history. Schrödinger recognized in his pioneering series of papers of 1926 that the quantum state of an n-particle system can be entangled, even though at the time he did not give a name to the concept.

> We have repeatedly called attention to the fact that the ψ-function itself cannot and may not be interpreted directly in terms of three-dimensional space – however much the one-electron problem tends to mislead us on this point – because it is in general a function in configuration space, not real space.[4]

Since in general a function of 3n variables (configuration space) does not factor into a product $u_1(\mathbf{r}_1)\ \ldots\ u_n(\mathbf{r}_n)$, $\mathbf{r}_i$ being the position variable of the i^{th} particle, Schrödinger has effectively recognized the kinematical possibility of entangled states. To our knowledge, it was not until 1935 that Schrödinger used the

English word "entangled"[5] and the German word "verschränkt"[6] to characterize a wave function that cannot be expressed in product form, and also to characterize the knowledge encapsulated in such a wave function; and he also uses the nouns "entanglement" and "Verschränkung" to refer to the property which such wave functions share. At that time Schrödinger could hardly have been more emphatic about the significance of entanglement:

> When two systems, of which we know the states by their respective representation, enter into a temporary physical interaction due to known forces between them and when after a time of mutual influence the systems separate again, then they can no longer be described as before, viz., by endowing each of them with a representative of its own. I would not call that *one* but rather *the* characteristic trait of quantum mechanics.[5]

For clarity in our subsequent discussion, it will be useful to give a more abstract definition of "entangled" than Schrödinger's, by using the terminology of Hilbert space theory. Suppose that $H_i(i = 1, 2)$ is a Hilbert space in which the states of system i are represented. Each state of system i is associated with a unique ray (or one-dimensional subspace) of H_i, but one can also speak of the state being represented by any normalized vector of the ray. (In fact, for short, we shall often use the locution "the state $|u\rangle$," where $|u\rangle$ is a vector in H_i). The states of the composite system 1+2 are represented in the tensor product Hilbert space $H_1 \otimes H_2$. If a state of 1+2 is represented by a vector $|\Psi\rangle$ of $H_1 \otimes H_2$ such that

$$|\Psi\rangle = |w\rangle_1 |z\rangle_2 \quad , \quad \text{where} \quad |w\rangle_1 \in H_1 \ \text{ and } \ |z\rangle_2 \in H_2 \ , \tag{1}$$

the state is called "a product state." On the other hand, if the state is represented by a vector that cannot be expressed in the form of Eq. (1) for any choice of vectors $|w\rangle_1$ and $|z\rangle_2$ then the state is called "entangled," and it is then sometimes also convenient to apply the adjective "entangled" to the system itself. Finally, all that has been said about a composite system with two parts can be naturally generalized to a system with more than two parts.

That entanglement is not merely kinematically possible but is indispensable for a realistic quantum mechanical description of many-particle systems was shown by early analyses of systems with several electrons. Heisenberg[7] explained the splitting of the helium term scheme into orthohelium and parahelium terms by postulating that in the former the spins combine into triplet states and in the latter they combine into a singlet state (the spatial wave function being anti-symmetrized or symmetrized in the respective cases in order to yield over-all anti-symmetrization). The singlet spin state and one of the three triplet basis states are entangled, as is the spatial wave function associated with the triplet spin state. Dirac[8] showed more generally that the Pauli exclusion principle can be recovered quantum mechanically by anti-symmetrizing the total many-electron wave function, and the Bose-Einstein statistics can be recovered quantum mechanically by symmetrizing the many-boson wave function, which automatically entails entanglement if more than one single-boson state is present in the many-boson state.

There was, of course, a wealth of applications of these ideas to many-body systems in the years immediately following the discovery of quantum mechanics. In none of these, however, was entanglement exhibited for a pair of particles which are spatially well separated over macroscopic distances. To be sure, the entanglement of the valence electrons in a metallic crystal, or of the photons in a black-body enclosure, is manifested over the macroscopic *extent* of these systems, but neither case is an instance of spatial *separation.*

A very important abstract discussion of entanglement appears in von Neumann's book in 1932 on the foundations of quantum mechanics.[9]. In chapters 5 and 6 von Neumann analyzes the measuring process quantum mechanically, taking system 1 to be a microscopic object and system 2 to be a macroscopic apparatus. The initial quantum state of 1+2 evolves under the time-dependent Schrödinger equation into a final state which in general is not a product state (see especially section 2 of chapter 6). If von Neumann's abstract formulation were realized in a concrete physical situation, then in principle the object and the apparatus could be spatially separated after ceasing to interact. However, we find no mention in von Neumann's book of the oddity of the resulting entanglement of spatially separated systems, for his preoccupation seems to be with aspects of the measurement problem, especially with understanding in what sense definite measurement results are finally achieved in spite of entanglement (see especially sections 1 and 3 of chapter 6).

It is only in the 1935 paper of Einstein, Podolsky, and Rosen[10] (hereafter abbreviated by EPR) that explicit attention is paid to the entanglement of a spatially separated pair of systems. EPR consider a pair of spinless particles constrained to move in one dimension, in the quantum state

$$|\Psi\rangle = \int\int dx_1\, dx_2\, \delta(x_1 - x_2 - 2a)\, |x_1\rangle|x_2\rangle\ . \tag{2}$$

If the momentum rather than the position basis is used, then $|\Psi\rangle$ can be re-written as

$$|\Psi\rangle = \int\int dk_1\, dk_2\, e^{ia(k_1+k_2)}\delta\,(k_1 + k_2)\, |k_1\rangle|k_2\rangle\ . \tag{3}$$

In the state $|\Psi\rangle$ both the positions of 1 and 2 and their momenta are strictly correlated, so that if the position of one is determined by measurement, that of the other can be inferred with certainty, and likewise with the momenta. Schrödinger[5,6] pointed out in comments on EPR that $|\Psi\rangle$ fails to be in product form not only in the position and momentum bases but in any basis – i.e., it is an entangled state.

Furthermore, it is easy to see that any state that would serve EPR's critical purposes – by implying strict correlation both of A_1 with A_2 and of B_1 with B_2, where A_i and B_i ($i = 1, 2$) are observables of particle i represented by non-commuting operators, hence not simultaneously measurable – must be entangled. Of course, it is also essential for EPR's critical purposes that particles 1 and 2 be spatially separated, in order that considerations of no action-at-a-distance can be brought to bear on the measurements performed upon the particles.

EPR's analysis suggested to Schrödinger the question of whether nature ever permits a pair of spatially separated particles to be in an entangled state. Either answer would be highly significant. A positive answer would accentuate the peculiarity of the quantum mechanical phenomenon of entanglement, while a negative answer would entail a limitation upon the scope of quantum mechanics. In the following remarkably penetrating passage Schrödinger says that he regards the question to be (as of 1935) an open one.

> It seems worth noticing that the paradox [of EPR] could be avoided by a very simple assumption, namely if the situation after separating were described by the expansion (12) [essentially an expression of correlations], but with the additional statement that the knowledge of the *phase relations* between the complex constants a_k has been entirely lost in consequence of the process of separation. That would mean that not only the parts, but the whole system, would be in the situation of a mixture, not of a pure state. It would not preclude the possibility of determining the state of the first system by *suitable* measurements in the second or *vice versa*. But it would utterly eliminate the experimenter's influence on the state of that system which he does not touch.
>
> This is a very incomplete description and I would not stand for its adequateness. But I would call it a possible one, until I am told, either why it is devoid of meaning or with which experiments it disagrees. My point is, that in a domain which the present theory does not cover, there is room for new assumptions without necessarily contradicting the theory in that region where it is backed by experiment.[11]

Schrödinger's profound question was not answered until 1957, when Bohm and Aharonov[1] analyzed the scattering data obtained by Wu and Shaknov[12] in 1950 on coincident scattering of pairs of photons produced by positronium annihilation. Bohm and Aharonov showed that the correlations of scattering could be explained accurately by ascribing an entangled state (of zero angular momentum and negative parity) to the photon pair, but could not be explained by any statistical mixture of quantum mechanical product states. Neither Bohm nor Aharonov was aware (as we have determined by asking them) that Schrödinger had raised the question that they answered, and indeed they attributed the question to Furry, even though the latter did not present the question as an open one, as Schrödinger had done. Consequently, although Schrödinger was still alive in 1957, there is no evidence that any one informed him of the remarkable answer to the question that he had posed more than two decades earlier. It is also noteworthy that Bohm and Aharonov avoided the need to perform a new experiment by brilliantly exploiting the results of an experiment which had been performed for an entirely different purpose – an exemplary case of quantum archaeology!

To our knowledge, no further example of the entanglement of spatially separated systems was exhibited until 1972, when the first test of a Bell's Inequality was completed by Freedman and Clauser.[13] Bell[14] and his followers[15] had derived inequalities for the purpose of testing the family of local hidden variables theories against quantum mechanics, but when one examines the various derivations one sees that Bell-type Inequalities also follow if the two-particle system under examination is described by a quantum mechanical product state. The reason is that such a product state ensures that the probability of joint behavior of the two particles factors into the product of single probabilities, which is the crucial premiss for deriving Bell-type Inequalities. We believe that most of the specialists in Bell's Theorem were aware of the fact that a quantum mechanical product state implied this factorizability of the joint probability, but there seems to have been no notice of this fact in print before a 1989 paper of Werner[16] (brought to our attention at this conference by L. Ballentine). In almost all of the dozen or so tests of Bell's Inequalities, the experimental data conflicted with the Inequality, and hence by the argument just summarized no quantum mechanical product state could account for the data. This conclusion is not as dramatic as the one that is usually cited, namely, the refutation of the family of local hidden variables theories, but it is nevertheless a conclusion of considerable conceptual significance. It is worth remarking that the so-called "detection loophole," which determined defenders of local hidden variables theories may invoke to ward off experimental refutation,[17] does not protect the hypothesis of a quantum mechanical product state from refutation, because there is no information encoded in a quantum mechanical product state (as there might conceivably be in a putative hidden variables state) that could account for a biased detection of photon pairs.

3. Down-Conversion Pairs of Photons

A new field for striking exhibitions of entanglement was opened up since 1987 by the application of interferometric techniques to pairs of photons generated in parametric down-conversion.[18] In retrospect, one sees that entanglement was implicit in all down-conversion phenomena since the pioneering experiment of Burnham and Weinberg[19] in 1970. They showed that the illumination of certain crystals by a pumping laser beam gives rise to pairs of photons which are generated simultaneously (as shown by coincidence counting) and have frequencies which sum to that of the pumping photons. To see that entanglement is implicit, consider the down-conversion radiation coming through two pinholes in a screen some distance downstream from the crystal, so that two well-defined beams A and B are formed, if the luminous region in the crystal is small. With suitable filters there approximately is only one mode in each beam, characterized by the wave-vectors $\mathbf{k}_A$ and $\mathbf{k}_B$. Since the two-photon state must be symmetrized, the only possible quantum mechanical description of the radiation into the beams A and B is

$$|\Psi\rangle = \tfrac{1}{\sqrt{2}}\left[|\mathbf{k}_A\rangle_1|\mathbf{k}_B\rangle_2 + |\mathbf{k}_B\rangle_1|\mathbf{k}_A\rangle_2\right] , \qquad (4a)$$

or equivalently one can write $|\Psi\rangle$ as a state in Fock space, which automatically guarantees symmetrization:[20]

$$|\Psi\rangle = |\mathbf{k}_A \, , \, \mathbf{k}_B\rangle \, . \qquad (4b)$$

Until interferometric experiments were performed using down-conversion pairs, however, there were no data in which the entanglement of the two-photon state was immediately exhibited, and physicists could (and sometimes did) fall into the non-quantum mechanical habit of thinking of one photon in beam A and the other in beam B, which would be formally expressed as

$$|\Phi\rangle = |\mathbf{k}_A\rangle_1|\mathbf{k}_B\rangle_2 \, . \qquad (5)$$

Furthermore, we think that the notation of Fock space in Eq. (4b), by making symmetrization automatic, hides the essential role of entanglement. For this reason, in our subsequent exposition we shall use the tensor product notation of Eq. (4a) in order to demonstrate the direct connection between entanglement and two-particle interference.

A warning is in order at this point against regarding symmetrization to be the essence of entanglement. To see this, one need merely contemplate the entanglement of non-identical particles, which are not subject to the rules of symmetrization and anti-symmetrization. What is essential to entanglement is the multiplicity of ways in which a source can communicate via two particles with two detectors.

We shall illustrate our thesis about entanglement by commenting on two beautiful experiments – that of Ghosh and Mandel of 1987[21], which was a pioneering interferometric experiment with down-conversion pairs, and that of Kwiat, Vareka, Hong, Nathel, and Chiao of 1990[22], which has been described by Chiao at this conference. Although these two experiments have in common the fundamental feature of exploiting entanglement, they differ radically and instructively in several respects. In the former there is entanglement of wave-vectors (directions), whereas in the latter there is entanglement of wave-numbers (or equivalently, frequencies).

Furthermore, in the latter, but not in the former, the placement of ideally narrow filters in front of the photodetectors, so as to admit only a single frequency, would eliminate the two-photon interference fringes. Finally, in the former, but not in the latter, beams of down-conversion radiation are "mixed" by means of beam-splitters before illuminating the detectors.

A simplified version of the experiment of Ghosh and Mandel is shown in Figs. 1a and 1b.

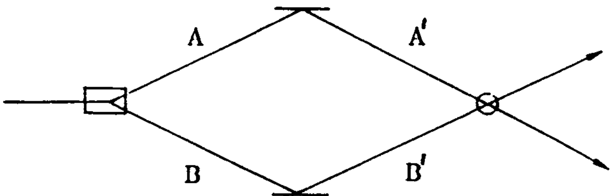

Fig. 1a

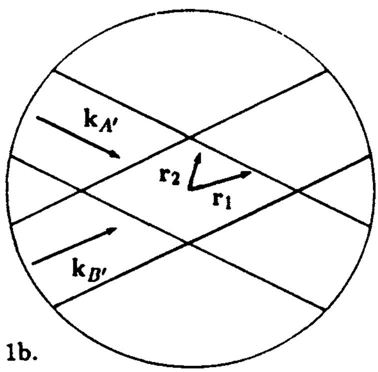

Fig. 1b.

Fig. 1a shows two beams A and B of down-conversion radiation, directed by mirrors through wave-number filters and pinholes (not shown) to a region of overlap, which is shown enlarged in Fig. 1b. Let A′ and B′ label the overlapping beams. If these beams are described by the two-photon state of Eq. (4a) or (4b), then the amplitude for joint detection at $\mathbf{r}_1$ and $\mathbf{r}_2$ (both points in the overlap region) is

$$\Psi(\mathbf{r}_1, \mathbf{r}_2) \sim e^{i(\alpha+\beta)} \left[e^{i\mathbf{k}_{A'}\cdot\mathbf{r}_1} e^{i\mathbf{k}_{B'}\cdot\mathbf{r}_2} + e^{i\mathbf{k}_{B'}\cdot\mathbf{r}_1} e^{i\mathbf{k}_{A'}\cdot\mathbf{r}_2} \right] , \qquad (6)$$

where α and β are the over-all phases associated with the A beam and the B beam respectively. Hence the probability of joint detection at $\mathbf{r}_1, \mathbf{r}_2$ is

$$P_{12}^{(\Psi)}(\mathbf{r}_1, \mathbf{r}_2) \sim [1 + \cos\{\mathbf{K} \cdot (\mathbf{r}_2 - \mathbf{r}_1)\}] \ , \qquad (7)$$

where

$$\mathbf{K} = \mathbf{k}_{A'} - \mathbf{k}_{B'} \ . \qquad (8)$$

The probability of joint detection given by Eq. (7) shows the genuine two-photon interference fringes obtained by Ghosh and Mandel. The fringes sinusoidally depend on the projection of $\mathbf{r}_2 - \mathbf{r}_1$ upon the fixed vector $\mathbf{K}$ in a manner that cannot be derived from single-photon interference, since the counting rate of single photons at either $\mathbf{r}_1$ or $\mathbf{r}_2$ (obtained by integrating over the other position variable) yields a constant. The two-photon interference fringes result directly from the fact that the entanglement in Ψ provides two different contributions to the probability amplitude. Ghosh and Mandel very emphatically made this point by writing, "In essence there is an interference between two two-photon amplitudes, because the apparatus cannot distinguish between photons from A and B being detected at x_1 and x_2 respectively, or vice versa."

Eq. (7) can be compared with the probability of joint detection implied by the unentangled or product state Φ of Eq. (5):

$$P_{12}^{(\Phi)}(\mathbf{r}_1, \mathbf{r}_2) \sim |e^{i\mathbf{k}_{A'} \cdot \mathbf{r}_1} e^{i\mathbf{k}_{B'} \cdot \mathbf{r}_2}|^2 = const. \qquad (9)$$

This result is not changed by allowing the joint detection to be due either to photon 1 being detected at $\mathbf{r}_1$ and photon 2 at $\mathbf{r}_2$ or vice versa, for then even though there are two contributions to the total probability there is no interference. Furthermore, the non-existence of two-particle interference fringes expressed by Eq. (9) is not due to the idealization to a single mode in each of the beams A and B. For suppose that the product form of Eq. (5) is kept, but $|\mathbf{k}_A\rangle$ and $|\mathbf{k}_B\rangle$ are each replaced by superpositions of states with different wave-vectors. The resulting product state, which we label $|\hat{\Phi}\rangle$, would imply a probability of joint detection of $\mathbf{r}_1$, $\mathbf{r}_2$ of the form

$$P_{12}^{(\hat{\Phi})}(\mathbf{r}_1, \mathbf{r}_2) = f(\mathbf{r}_1) g(\mathbf{r}_2) \ , \qquad (10)$$

where f and g may each depend sinusoidally on $\mathbf{r}_1$ and $\mathbf{r}_2$ respectively, but their product would not depend sinusoidally on $\mathbf{r}_2 - \mathbf{r}_1$. Returning again to the prediction from the entangled state $|\Psi\rangle$ of Eq. (7), we emphasize that it should not be construed as the interference of two photons with each other. Dirac's famous dictum, "Each photon interferes only with itself. Interference between two different photons never occurs,"[23] is not violated. The dictum must be generalized, however, without loss of the original spirit: "Each entangled system interferes with itself. Interference between subsystems of an entangled system never occurs."

Eq. (7) is the same as the joint probability function which Ghosh and Mandel obtained by a quantum optical analysis of the down-conversion field, which is not surprising, since they restricted their attention to a two-photon subspace of Fock space, with guaranteed symmetrization. We note that they compared their result with the prediction of a classical description of the down-conversion field, which gives a fringe visibility of no more than half of the 100% visibility shown in Eq. (7). We have been interested here in an entirely different comparison. We took for granted that the battle between a classical and a quantum mechanical characterization of the radiation field has been won by the latter. The question of product versus entangled state is a further battle within the victorious quantum point of view. Our foregoing argument, together with the observation of two-photon interference fringes by Ghosh and Mandel, unequivocally settles this battle in favor of entanglement.

The experiment of Ghosh and Mandel was followed by many others[24] that demonstrated the type of entanglement exhibited in Eq. (4a) – i.e., entanglement of wave-vector *directions*. With a suitable arrangement two-photon fringes can also be produced by exploiting the entanglement of wave-vector *magnitudes* (i.e., wave numbers,or, equivalently, frequencies or "colors"). This phenomenon was exhibited in an experiment of Kwiat et al.[22], who carried out a proposal of Franson[25] but adapted it to down-conversion pairs. A diagram of their arrangement is shown in Fig. 2.

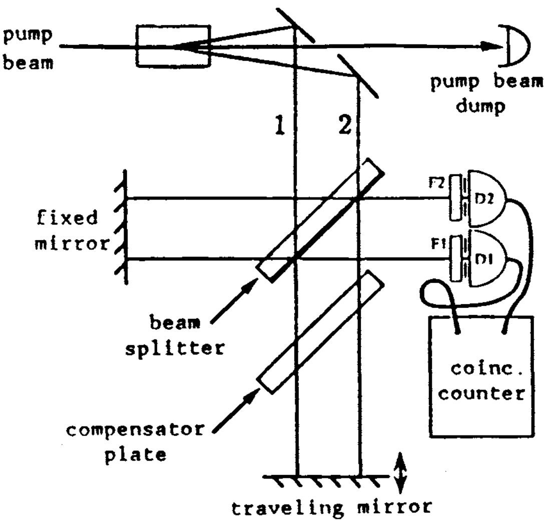

Fig. 2.

Two beams, 1 and 2, selected by pinholes (at the detectors) and made parallel by mirrors, were fed into a Michelson interferometer and then into the filter-detector assemblies, also labeled 1 and 2. Notice that beams 1 and 2 were never "mixed" before reaching their respective detectors, unlike in *all* of the experiments of Ref. 24. Also, because the beams do not meet, the entanglement of Eq. (4a) is irrelevant to the phenomenon of Kwiat et al. Nevertheless, as we shall explain, the coincidence count rate at the detectors exhibited fringes as the Michelson mirror was moved, provided that Δ, the optical path-length difference in the two routes through the Michelson interferometer, exceeded the coherence length associated with the filters. Note that Δ is *twice* the difference of the lengths of the interferometer arms.

In order to show how the fringes arise, we must characterize the state of the two-photon radiation incident upon the Michelson interferometer. Consider a single-photon in the pump beam. Suppose, for simplicity, that this photon is ideally monochromatic, with wave-number $2k_o$, so that its wave function in wave-number space is simply

$$\Phi(k) = \delta(k - 2k_o) \ . \tag{11}$$

If this photon converts in the crystal into a pair of photons, and this pair subsequently enters the beams 1 and 2, then the wave function of the pair in the two-dimensional wave-number space is simply

$$\Phi(k_1, k_2) = \delta(k_1 + k_2 - 2k_o)\,\phi(k_1) \ , \tag{12}$$

where the distribution $\phi(k_1)$ is determined by the details of the down-conversion process and by the beam selection apertures. Note that the wave-numbers k_1 and k_2 of the pair are not fixed, but must satisfy $k_1 + k_2 = 2k_o$ (energy conservation). For conceptual clarity suppose now that the wave-number filters are placed upstream of the Michelson interferometer, instead of just before the detectors. Also suppose, as is commonly done, that within the bandwidth of these filters $\phi(k)$ is essentially a constant. Then downstream from the filters the two-photon wave function of Eq. (12) is "clipped" to become

$$\Phi(k_1, k_2) = \delta(k_1 + k_2 - 2k_o)\, e^{-(k_1-k_o)^2/2\sigma^2}\, e^{-(k_2-k_o)^2/2\sigma^2} \ , \tag{13}$$

where for simplicity we have assumed that the two filters are gaussian with identical widths σ and centers k_o, as was the case in the experiment of Kwiat et al. (If the filters would have different centers, then among the resulting modifications of Eqs. (19)-(20) below are beating fringes of the sort observed by Ou and Mandel[26], even though the beams 1 and 2 in Fig. 3 never mix.) Since the Michelson interferometer operates in real space, it is convenient to Fourier transform the k_1, k_2-dependent wave function, with the time-dependence inserted, into the configuration space wave function

$$\Psi(x_1, x_2, t_1, t_2) = \int\!\!\int dk_1\, dk_2 \{\Phi(k_1, k_2)\, e^{-ic(k_1 t_1 + k_2 t_2)}\} e^{ik_1 x_1} e^{ik_2 x_2} \ . \tag{14}$$

For the Φ of Eq. (13) this yields

$$\Psi(x_1, x_2, t_1, t_2) = e^{ik_o(x_1 - ct_1)} e^{ik_o(x_2 - ct_2)} e^{-\frac{\sigma^2}{4}[(x_1 - ct_1) - (x_2 - ct_2)]^2} \ . \tag{15}$$

Note that both the general Eq. (14) and the specific Eq. (15) explicitly exhibit entanglement. In Eq. (14) one sees that the entangled state is a superposition of products of one-photon "color" states; and in Eq. (15) the non-factorizability of the two-photon wave function is apparent in the third exponential factor.

The wave function (15) describes the photons of the pair before they enter the Michelson interferometer, that is, $\Psi^*\Psi dx_1\, dx_2$ is the joint probability of finding photon 1 within dx_1 at position x_1 at time t_1 and photon 2 within dx_2 at position x_2 at time t_2, where x_1 (x_2) is a coordinate along beam 1(2). In stating this interpretation of the configuration space wave function of the photon pair we are not oblivious of the well known literature concerning the localization of a photon,[27] but we wish to delay a discussion of this matter to another paper. If detectors are placed in the beams equally distant from the source, so that $x_1 = x_2$, the joint probability density takes the particularly simple form

$$\Psi^*\Psi = e^{-\sigma^2(c\tau)^2/2} , \tag{16}$$

where $\tau = t_2 - t_1$ is the delay time in the arrival of the photons. Note that the probability of detecting a pair is essentially zero unless the delay time is of the order of $(c\sigma)^{-1}$ or smaller.[28]

Now, with the Michelson interferometer in place, each photon has two routes to its respective detector, the "short" route via the "near" fixed mirror and the "long" route via the "far" movable mirror. Consequently, the total wave function illuminating the detectors at times t_1 and t_2 is a superposition of *four* amplitudes, each of the type of Eq. (15):

$$\begin{aligned} \Psi_{tot}(t_1,t_2) &= \Psi(0,0,t_1,t_2) + \Psi(\Delta,\Delta,t_1,t_2) \\ &\quad + \Psi(\Delta,0,t_1,t_2) + \Psi(0,\Delta,t_1,t_2) \ , \end{aligned} \tag{17}$$

where Δ is the path difference between the "long" and "short" routes, and, for simplicity, the detectors have been positioned at $x_1 = x_2 = 0$.[29] From Eq. (17) the joint probability density of detecting photon 1 at t_1 and photon 2 at t_2 is proportional to

$$\begin{aligned} \Psi^*_{tot}\Psi_{tot} &= [2 + 2\cos(2k_o\Delta)]\, f^2(c\tau) \\ &\quad + 4\cos(k_o\Delta)\, f(c\tau)\,[f(\Delta - c\tau) + f(-\Delta - c\tau)] \\ &\quad + [f(\Delta - c\tau) + f(-\Delta - c\tau)]^2 \ , \end{aligned} \tag{18}$$

where $\tau = t_2 - t_1$ and $f(u) = e^{-[\sigma^2 u^2/4]}$. Note that this joint probability density, like that of Eq. (16), depends on the times t_1 and t_2 only through their difference. This result is expected, since we modeled the pump photon by an everlasting monochromatic wave. For comparison with experiment the joint probability density of Eq. (18) must be integrated from $\tau = -\frac{T}{2}$ to $\tau = +\frac{T}{2}$, where T is the coincidence window of the experimental arrangement. The result is that the probability for a coincidence count is

$$P_{12}^{(\Psi)} \sim \frac{\sigma c}{\sqrt{2\pi}} \int_{-\frac{T}{2}}^{\frac{T}{2}} \Psi^* \Psi d\tau = A(1 + \cos 2k_o\Delta) + B \cos k_o\Delta + C \ , \qquad (19)$$

where

$$A = 2\,\mathrm{erf}\left[\frac{cT\sigma}{2\sqrt{2}}\right] \qquad (20a)$$

$$B = 4e^{-\frac{\sigma^2\Delta^2}{8}}\{\mathrm{erf}\left[(cT+\Delta)\frac{\sigma}{2\sqrt{2}}\right] + \mathrm{erf}\left[(cT-\Delta)\frac{\sigma}{2\sqrt{2}}\right]\} \qquad (20b)$$

$$C = \mathrm{erf}\left[(cT+2\Delta)\frac{\sigma}{2\sqrt{2}}\right] + \mathrm{erf}\left[(cT-2\Delta)\frac{\sigma}{2\sqrt{2}}\right] + 2e^{-\frac{\sigma^2\Delta^2}{2}}\,\mathrm{erf}\left[(cT)\frac{\sigma}{2\sqrt{2}}\right], \qquad (20c)$$

and where $\mathrm{erf(u)} = \frac{2}{\sqrt{\pi}} \int_0^u e^{-u'^2}\, du'$ is the error function. In order to make a clear comparison with experiment, it is useful to examine the behavior of Eq. (19) in two cases: (1) $\Delta \ll \frac{1}{\sigma}$, and (2) $\Delta \gg \frac{1}{\sigma}$. Of course, all intermediates between these two cases are also contained in Eqs. (20a)-(20c).

Case 1: This is the case where the path difference Δ is less than the coherence length $\frac{1}{\sigma}$ of the radiation, as determined by the filters, so that the *single* count rate in each detector will exhibit ordinary single-photon Michelson fringes as Δ is varied. To examine the behavior of the *coincidence* count rate, assume for simplicity that $cT \gg \frac{1}{\sigma}$, so that each error function in Eqs. (20a)-(20c) is +1. Then $A = 2$, $B = 8$, $C = 4$, so that

$$P_{12}^{(\Psi)} \sim 2(1 + \cos 2k_o\Delta) + 8\cos k_o\Delta + 4 = [2 + 2\cos k_o\Delta]^2 \ . \qquad (21)$$

The last expression is proportional to the square of the single-photon probability when there are ordinary single-photon Michelson interference fringes. In other words, in this case all variation of the coincidence count rate as a function of Δ is simply the product of the single rates at the two detectors. No true two-photon fringes are present.

In case 2 the path difference Δ is greater than the coherence length $\frac{1}{\sigma}$. In this case, the ordinary single-photon Michelson fringes disappear, and the fringes which will appear are genuine two-photon interference fringes. Note that, in the limit of ideally narrow filters the coherence length $\frac{1}{\sigma}$ is greater then the path difference Δ, case 1 always holds, and hence the two-photon fringes do not appear. It will be convenient to subdivide case 2 into subcases: (2a) $\frac{cT}{\Delta} \gg 1$, (2b) $\frac{cT}{\Delta} \ll 1$. Of course, all intermediates between these two extreme subcases are also contained in Eqs. (20a)-(20c).

Case 2a: $\Delta \gg \frac{1}{\sigma}$, $\frac{cT}{\Delta} \gg 1$, as in the experiment of Kwiat et al, where $cT \approx 1.5\, m, \Delta \approx 240 \mu m$, and $\frac{1}{\sigma} \approx 50 \mu m$. Then $A = 2$, $B = 0$, and $C = 2$, so that

$$P_{12}^{(\Psi)} \sim 4 + 2\cos 2k_o\Delta \, , \tag{22}$$

Thus, the two-photon fringes have a visibility of 0.5, as observed by Kwiat et al. Physically, the extra "background" coincidence counts (i.e., the $C = 2$ contribution) arise because with the large coincidence window the electronics responds not only to long-long and short-short amplitudes but also to long-short and short-long amplitudes.

Case 2b: $\Delta \gg \frac{1}{\sigma}$, $\frac{cT}{\Delta} \ll 1$. Then $A = 2$, $B = C = 0$, so that

$$P_{12}^{(\Psi)} \sim 2 + 2\cos 2k_o\Delta \, , \tag{23}$$

That is, in this ideal limit the two-photon fringes have a visibility of unity. The narrowed coincidence window no longer accepts the long-short or short-long amplitudes, hence $C = 0$. We note that our derivation of Eq. (23) via Eqs. (19)-(20) is unnecessarily complicated, for once one realizes physically that there is no contribution from the long-short and short-long routes in Case 2b, one sees that the last two terms in Eq. (17) are negligible, and then Eq. (23) follows immediately[22]. Finally, it should be pointed out that an experimental effort to realize Case 2b is underway by greatly increasing the path difference Δ and modestly decreasing the coincidence window T, thereby improving the two-photon fringe visibility.[30]

We conclude with some remarks about procedure and substance. Our analysis of down-conversion photons has employed only elementary quantum mechanical procedures, and specifically we have refrained from using the field theoretical methods common in quantum optics. We have been able to derive completely by elementary means (largely in unpublished calculations) the results of all the two-photon interferometric experiments to date, including a recent one of Ou et al.,[31] in which a single pair of photons is generated by the superposed operation of two down-conversion crystals. The crucial point is that in all these experiments, including the last, the down-conversion radiation can be adequately treated in the two-particle subspace of Fock space.

Substantively, we call attention to the curious type of entanglement shown in the two-photon wave packet of Eq. (15). Each of the first two factors on the right hand side of this equation is a monochromatic wave ideally present throughout the longitudinal extent of the respective beam and throughout all time, and the third factor is a real Gaussian depending on a space-time point (x_1,t_1) in beam 1 and a space-time point (x_2,t_2) in beam 2. The phase differences that account for interferences are only determined by the first two factors, for there is no phase in the third. But the third factor, which is not a product of a function of x_1 and t_1 and another function of x_2 and t_2, determines a spatio-temporal correlation of the two particles – i.e., it tells at what *pairs of points in space-time* the particles can be found. It is this non-factorizability that accounts for the remarkable feature of the experiment proposed by Franson and carried out by Kwiat et al.: that two-photon interference can be produced without any mixing of beams.

After the completion of this paper we received a preprint from Ou, Zou, Wang, and Mandel,[32] the publication of which was delayed, reporting an independent realization of the experiment proposed by Franson. Their results agree essentially with those of Kwiat et al.

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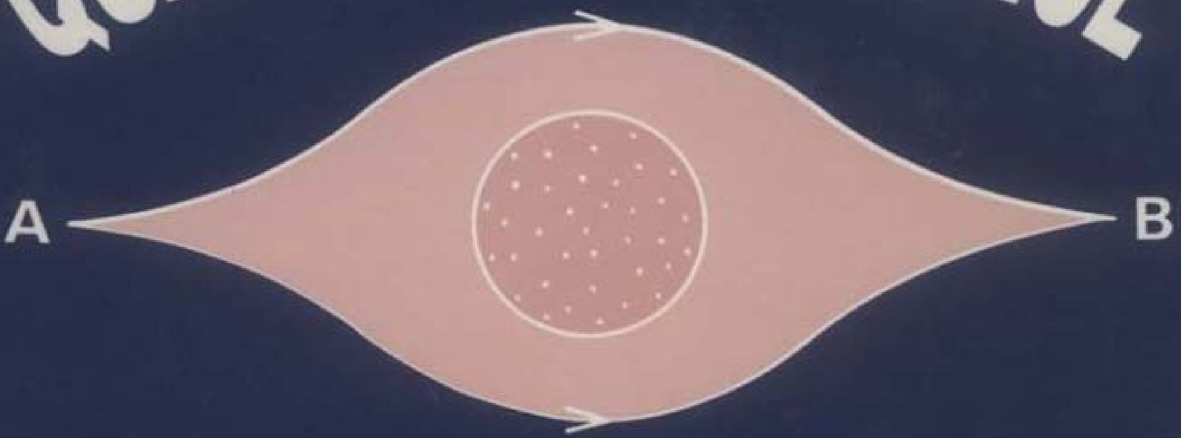

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Yakir Aharonov (left) and David Bohm at the Banquet
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