Physica B 151 (1988) 147–155
North-Holland, Amsterdam

NEUTRON FOCUSING EFFECTS IN PERFECT-CRYSTAL SYSTEMS

Anton ZEILINGER* and Michael A. HORNE**
Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA

Four different focusing effects in two-crystal systems are discussed. (i) Focusing of monochromatic radiation at the back face, (ii) a geometric focus of polychromatic radiation, (iii) pure-wavefield foci due to lenses between the crystal plates and (iv) effective-mass enhanced focusing inside crystals due to the action of inhomogeneous external forces. We present some new experimental results on these foci and point out their relevance for the development of novel neutron interferometry systems.

1. Introduction

The smallness of the various neutron interactions available in the laboratory makes it very difficult experimentally to focus thermal neutrons through lenses or mirrors whose action is based on the refractive index. This problem is relaxed for very cold and ultra-cold neutrons for which, due to their small kinetic energy, rather large deflections and focusing effects can be obtained.

Another way of achieving focusing effects is the subject of the present note. Here [1] we draw attention to the fact that the propagation direction of neutrons in perfect crystals can change very dramatically upon small changes of the neutron energy or the neutron momentum. These changes can be the result either of earlier crystal diffraction or of lenses inserted into the beam or even of forces acting on the neutron while propagating through the crystal medium. All these cases have found experimental verification as will be shown below.

There are in general at least two different ways to view the physics behind focusing. Firstly, one can regard the action of a lens or a lensing medium on the basis of its effects on the ray trajectories and thus define a focus as that point where rays meet after having travelled along different paths. Secondly, one can define a focus as that point in space where the variation of phase over different paths vanishes. In the present note we will encounter both viewpoints.

* Permanent address: Atominstitut der Österreichischen Universitäten, Schüttelstr. 115, A-1020 Vienna, Austria.
** Permanent address: Stonehill College, North Easton, MA 02356, USA.

2. Focussing in a large two-crystal interferometer system

It is known from the beginning of two-crystal interferometry [2] that for monochromatic radiation a focal point arises on the back face of the second crystal. This focus may easily be understood on the basis of the fact that the phase of the radiation leaving the first crystal plate within the Borrmann triangle in Bragg diffracted direction contains the phase

$$\Delta\phi_{\alpha,\beta} = \pm(\pi t/\Delta_0)\sqrt{1-\Gamma^2} \tag{1}$$

where t is the crystal thickness, Δ_0 is the Pendellösung length and Γ is a parameter describing the in-crystal propagation direction and hence the exit point of a ray. Γ is zero for propagation along the lattice planes and $+1$ or -1 respectively for propagation along either edge of the Borrmann triangle. The subscript α and the positive sign in eq. (1) refer to neutrons which propagated in the α-wavefield state through the crystal. The α- wavefield is the weakly absorbed one whose dispersion surface is closer to the Laue point while the β-wavefield is more strongly

0378-4363/88/$03.50

absorbed than in ordinary optical transmission through the same material.

Phenomenologically, these anomalous absorption properties can be understood from the fact that the α-wavefield has its nodes at the lattice planes and hence interacts less with the nuclear potential wells located there than the β-wavefield which has its antinodes there. For most materials and thermal neutrons these absorption effects are very small, though they have been demonstrated for very thick Si-crystals [3]. Yet there certainly is always a corresponding phase effect present implying that due to their different interaction with the nuclei of the crystal, neutrons in the α-wavefield or in the β-wavefield experience different phase changes as they propagate through the crystal. This is the physical reason of the phase in eq. (1).

It follows from eq. (1) that the phase shift the neutron experiences upon crystal propagation can be cancelled if the neutron propagates along the same direction Γ but as a different wavefield through another crystal of the same thickness because then

$$\Delta\phi_{\alpha}(\Gamma) + \Delta\phi_{\beta}(\Gamma) = 0 . \qquad (2)$$

Since this cancellation takes place for each propagation direction Γ separately it follows that on the back face of the second crystal a focal point* arises (fig. 1).

The position of this focal point varies with wavelength due to the dispersive feature of Bragg diffraction. This implies that the trajectories of the neutrons with different wavelengths separate on propagation between the first and the second crystal even if they originate at the same source point on the front face of the first crystal. Yet it can easily be understood geometrically (see Fig. 1) that a focusing even of neutrons of different wavelengths occurs at the distance D_2 behind the second crystal such that it fulfils the condition

* In usual geometries the defining entrance slit is close enough to be the first crystal plate and the distance between the two crystals is small enough such that the broadening of that single-wavelength focus due to the divergence of the incident radiation may be neglected.

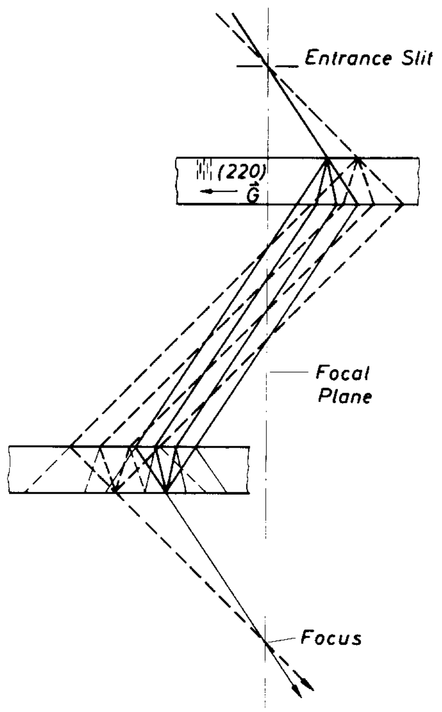

Fig. 1. In a two-crystal Laue system consisting of two plates of equal thickness a focus arises for each wavelength (—— or ---) separately at the back face of the second crystal plate. For a set of different wavelengths a focal plane exists containing a multi-wavelength focus.

$$D_1 + D_2 = D_0 , \qquad (3)$$

where D_1 is the distance of the defining entrance slit from the front face of the first crystal and D_0 is the distance between the crystal plates, all measured along the lattice planes; we assume symmetric Laue geometry, the generalization to other geometries is trivial.

This focusing feature was already demonstrated to exist in a small two-crystal system [4] where the effect was noticeable but small. Here we report a demonstration with a much larger two-crystal system intended for use as an interferometer with extremely long neutron paths. The interferometer was cut out of a 485 mm long and 54 mm thick Si-crystal grown in (111)-direction. Both crystal plates (ears) were 20.57 mm thick and the overall distance D_0 measured parallel to the (400) lattice planes was 334 mm resulting in a beam path length between the crystal plates of 409 mm for the $\lambda = 1.56$ Å neutrons. The divergence of the incoming beam was $\delta\theta = 10^{-2}$ rad as defined by the dimensions of the collimator channel in front of the entrance slit. The crystal was isolated from ambient rotational vibration noise by placing it on a table which could freely rotate and which was coupled to the outside by very weak springs damped by a small metal paddle immersed in oil.

The entrance slit and the exit slit of the interferometer system were mounted in focusing position as given by eq. (3). The resulting intensity distribution in the plane of the exit slit clearly exhibits the expected focus (fig. 2). This focus had a FWHM of only 2.4 mm which is in perfect agreement with the expectation based only on the width of the entrance slit (2.4 mm) and of the exit slit (2.0 mm). We note that the width of the beam at the exit slit simply due to the broadening as given by the divergence of the radiation incident on the entrance slit would be 8.2 mm after the neutron path length of 818 mm through the interferometer system from entrance slit to exit slit. Thus, the focusing effect is clearly seen in the experimental results and there is no evidence within experimental accuracy (<0.1 mm) for any broadening of the focus.

In subsequent experiments an Al step of such a height that it introduces a $3\pi/2$ phase shift was scanned across the beam between the crystal ears at the focal plane in order to search for interference action. No variation of the intensity in the focus was detected. This implies that due to some disturbance neutrons propagating through the system at different heights or with different Γ values recombine at the focus with different phases and thus appear to be incoherent. We point out that this is not in conflict with the definition of the focus as the point with constant phase upon trajectory variation since this definition need only be satisfied for sufficiently large subsets of rays separately.

The eventual detection of interferometer action may be quite fortuitous as shown in fig. 3. There, in an analogous measurement with the first two-crystal interferometer system (for its dimensions see refs. [2] or [4]), no clear effect of the expected type – a variation of intensity, complementary in the two beams, as a function of phase step position – was seen initially, yet sudden violent variations of intensity occurred when the senior experimenter to whom the present paper is dedicated arrived at the experiment setup early in the morning thus disturbing the quiet ambient temperature conditions. Further

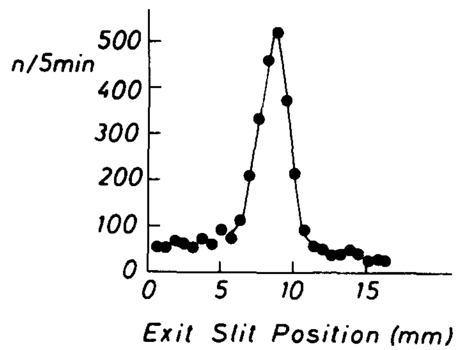

Fig. 2. Intensity distribution in the multi-wavelength focus of a large interferometer with 41 cm beam path length. Without focusing action the peak would have a FWHM of 8.2 mm.

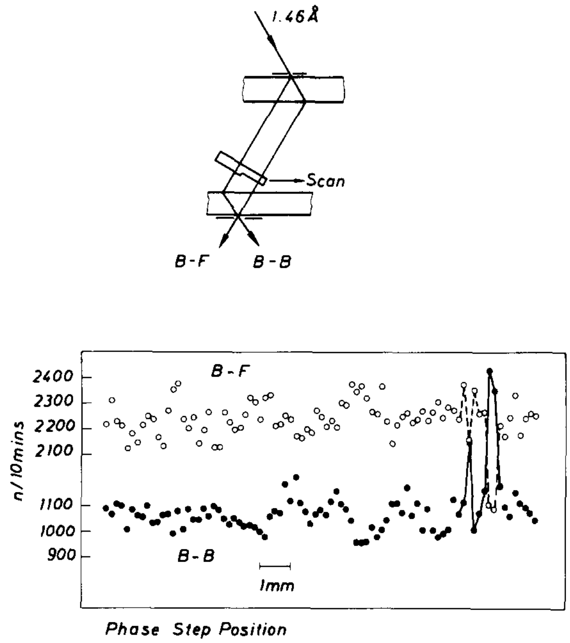

Fig. 3. Interferometer action in a small two-crystal system. While scanning of a $3\pi/2$ phase step does not demonstrate interference convincingly, external temperature disturbance results in complementary erratic oscillations of both beams, indicative of interference.

experimentation then established in detail the conditions for proper operation of such an interferometer.

3. Focusing of single-wavefield radiation

It may be noted from eq. (2) that radiation reaching the focus at the back face of the second crystal plate has travelled as different wavefields through the two crystals, i.e. in one crystal as the α-wavefield and in the other crystal as the β-wavefield in either order because only for those neutrons the total phase upon crystal passage is

$$\Delta\Phi_{\alpha\rightarrow\beta} = \Delta\Phi_{\beta\rightarrow\alpha} = 0\,. \qquad (4)$$

Neutrons propagating as the same wavefields through the two crystals acquire the phase

$$\Delta\Phi_{\alpha\rightarrow\alpha,\beta\rightarrow\beta} = 2\Delta\phi_{\alpha,\beta}\,. \qquad (5)$$

This phase is trajectory-dependent through Γ and hence the corresponding neutrons do not end up in the focus.

In an earlier publication [5] we have pointed out that it is possible to achieve a pure-wavefield focus if a proper lens is introduced into the beam path between the two crystal ears. Such a lens has to cancel the phase of eq. (5). If we view the parameter Γ as a parameter describing a positional coordinate within the neutron beam (with $\Gamma = 0$ being the beam center) then eq. (5) defines an ellipsoidal shape of the lens. Assuming the same sign for the potential of the lens material and of the crystal material it follows that $\alpha \rightarrow \alpha$ radiation will be focused by a full lens while $\beta \rightarrow \beta$ radiation by a hollow one, i.e. a cylindrical hole with elliptical cross section cut into a phase shifter plate. This may also be understood on the basis of the fact that α-wavefield neutrons have interacted less with the crystal nuclear potential which fact has to be compensated by the lens action, and vice versa for the β-wavefield neutrons.

Our experimental conditions, namely $\lambda = 1.56$ Å neutrons and Si(400) planes, were such that the phase shift of eq. (1) just defines a cylindrical lens of circular cross section if the lens material used is Al. Thus for compensation of the Γ-variation of the phase as defined in eq. (5), two circular lenses, one for each crystal plate traversed by the neutrons, have to be used. They can be either full or hollow depending on whether one wishes to focus $\alpha \rightarrow \alpha$ or $\beta \rightarrow \beta$ wavefield radiation.

With no lens in the beam a focus was clearly observed on the back face of the second crystal plate (see ref. [5] and for the dimensions of the system again ref. [2] or [4]). Fig. 4 then shows that this back-face focus disappears if either one full or one hollow circular lens is inserted into the beam path. It reappears (fig. 5) with insertion of a second lens of the same type as can be seen by inspecting figs. 4 and 5. Having the same phenomenon appear for either hollow or full lenses is quite unusual and, to our knowledge, it is not encountered in ordinary optics. The physical difference that in one case (i.e. two hollow lenses) we focus $\beta \rightarrow \beta$ radiation while in the

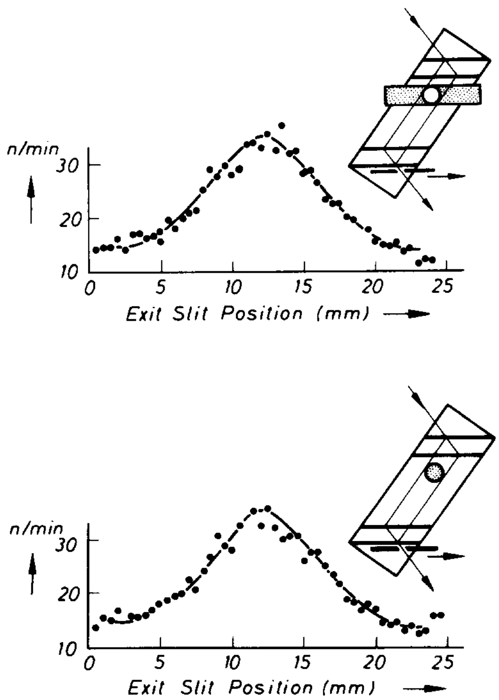

Fig. 4. One cylindrical lens (full or hollow) positioned in the beam of a two-crystal interferometer destroys the back-face focus.

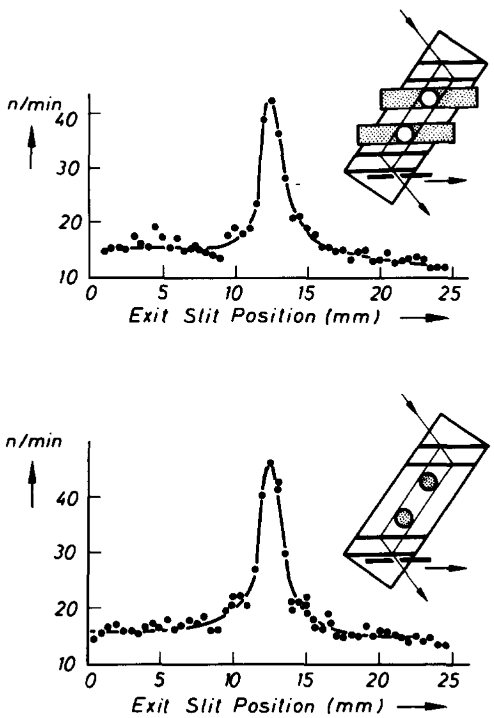

Fig. 5. A second cylindrical lens of the same type restores the focus. The same focus appears for full or hollow lenses.

other one (i.e. two solid lenses) $\alpha \rightarrow \alpha$ radiation does not show up in the experimental results. This is expected because both Si and Al are rather transparent for the neutrons used. We would expect a significant difference between the two cases if the crystal were made of a heavily absorbing material, e.g. InSb, and the lenses still of some weakly absorbing substance. Then the $\alpha \rightarrow \alpha$ focus peak is expected to be of much higher intensity than the strongly absorbed $\beta \rightarrow \beta$ focus peak or even the mixed $\alpha \rightarrow \beta$, $\beta \rightarrow \alpha$ focus.

In our nonabsorbing case the intensity in the pure-wavefield foci is only half the intensity of the mixed wavefield focus. This is a consequence of the fact that the latter consists of both $\alpha \rightarrow \beta$ and $\beta \rightarrow \alpha$ neutrons while the former contain only either $\alpha \rightarrow \alpha$ or $\beta \rightarrow \beta$ ones. This intensity level can be seen in fig. 6 which demonstrates rather dramatically the focusing action of the double lens system. The intensity of the mixed focus is that of the shoulders of that pattern since there the co-moving double lens system is outside the beam. In the experiment the two lenses were connected rigidly to each other and positioned such that one was exactly downstream from the other. As soon as the double lens system enters the beam the focus intensity is drastically reduced by refraction in the lenses nearly down to the background level of about 15 neutrons/min. Yet, as soon as the lens system reaches the center beam position a sharp peak due to the focusing action arises up to half the shoulder height. It is remarkable that the

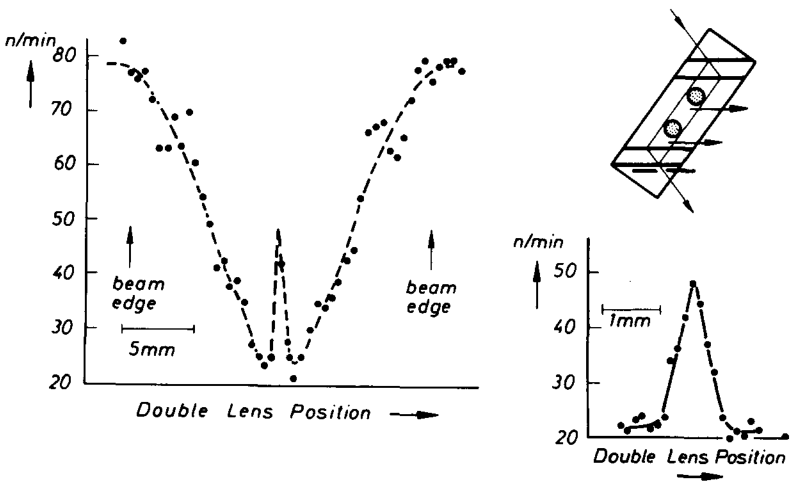

Fig. 6. With the exit slit positioned at the focus, that focus is destroyed by scanning two co-moving lenses through the beam. Yet, a very sharp peak arises (see insert) when the double lens is centered in the beam.

FWHM of that peak is 0.6 mm and therefore only half the width of the entrance slit as measured along the scan direction. Assuming that peak to be infinitely sharp with a δ-function entrance slit we expect it to be smeared out accordingly if that slit is widened up. A separate experiment has shown that if the same scan is made with just one lens a very similar intensity variation results with the central peak missing.

4. Focusing by fields inside a crystal

Neutrons propagating inside a crystal can be easily deflected by modest forces acting on them. This property is most satisfactorily explained [6] by attributing to the neutron an effective inertial mass m^* which is defined as

$$(1/m^*)_{\mu\nu} = \partial^2\omega(\boldsymbol{K})/\partial K_\mu \partial K_\nu \,, \tag{6}$$

where $\boldsymbol{K}$ is an in-crystal wavevector, $\omega(\boldsymbol{K})$ is the unperturbed dispersion relation and μ and ν are Cartesian coordinates. The resulting tensorial effective mass has a number of interesting properties [6]. For our considerations, relevant is the fact that the effective mass reduces to

$$m^* = \pm 2mV_G/E_G \tag{7}$$

for neutrons very closely fulfilling the Bragg condition at the set of lattice planes $\boldsymbol{G}$ and for the force directed along $\boldsymbol{G}$. Here, V_G is the neutron-crystal interaction potential, E_G is the kinetic energy of a neutron with wavelength equal to the spacing of the set of lattice planes $\boldsymbol{G}$ and m is the rest mass of the neutron. For Si(220) planes it follows that the magnitude of the effective mass is smaller than the rest mass of the neutron by a factor of 2.1×10^5.

Various predictions of the effective mass concept have been verified experimentally already [6] and in the present paper we present explicit results of an interesting focusing effect. In the

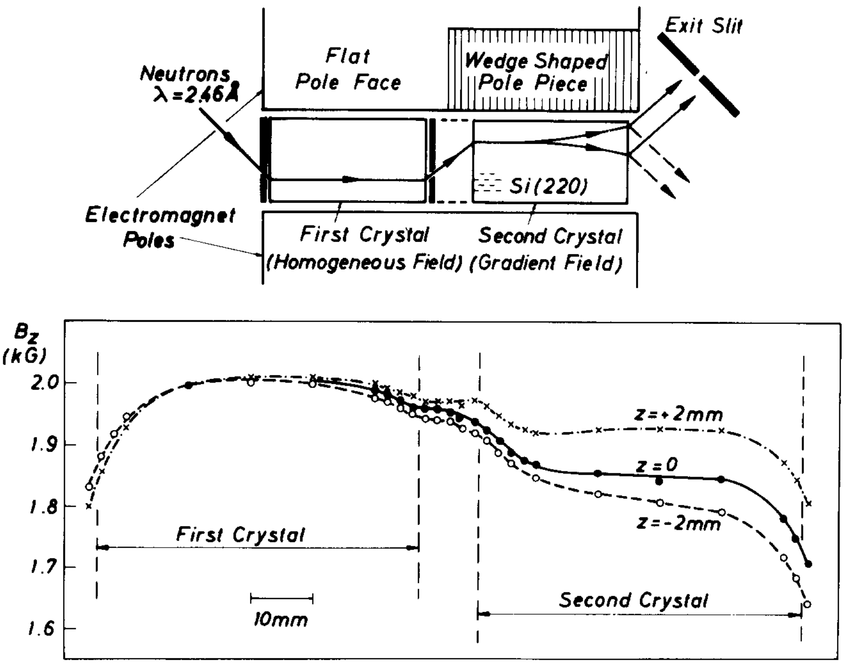

Fig. 7. Beam path through a thick two-crystal system (top) and magnetic field measured at a magnet current of 5.5 A at three different parallel trajectories (bottom). Note the field gradient in the second crystal.

experiment, a two-crystal system was again used where the first crystal served as a crystal collimator in order to define on-Bragg radiation which was then admitted into the second crystal (fig. 1).

The force on the neutron was provided by an externally-applied, inhomogeneous magnetic field of the Stern–Gerlach type acting over the spatial region of the second crystal, while over the first crystal we had a homogeneous magnetic field. This homogeneous field was of approximately the same magnitude as that acting on the average neutron trajectory within the second crystal in order to reduce refraction effects of the neutron during passage through the gap between the crystals.

Consequently, the magnetic field is very nearly the same along different parallel paths through the first crystal (fig. 7), while it varies strongly with path position in the second crystal. The point we make here is that the existing difference in field strength in the second crystal between the $z = -2$ mm and the $z = 0$ mm trajectories differs from that between the $z = 0$ mm and the $z = +2$ mm trajectories. In other words, the field gradient and hence the force $\boldsymbol{F} = -\text{grad}(\boldsymbol{\mu} \cdot \boldsymbol{B})$ acting on the neutron varies from trajectory to trajectory inside the crystal. Such a variation results in a different deflection of radiation propagating along different trajectories and, if the parameters are well chosen, in a focusing effect.

Numerical calculation of the trajectories for our specific magnetic field configuration using the effective mass concept explicitly predicts such a focus to arise for one mass-sign/polarization state (fig. 8) and likewise a defocusing of the other mass-sign/polarization state.* The width of the focus varies with magnet current, i.e. with overall field strength. We point out that the calculations presented in fig. 8 hold only for neutrons incident exactly on-Bragg onto the second crystal. In the experiment, the small divergence of the neutrons leads to a broadening of the focused peak and hence to a smaller variation with field strength of the width of the observed focus.

The focusing effect was clearly seen in the experiment as shown in fig. 9. We point out that the FWHM of the focused peak was 2.3 mm and that of the defocused peak 4.1 mm. These numbers, if compared with the width of the peak without magnetic field (3.2 mm) very nicely de-

* The details of the relations between the neutron trajectories, the neutron polarization and the sign of the effective mass are not relevant for the point we make in this paper. They have been treated elsewhere both theoretically and experimentally [6].

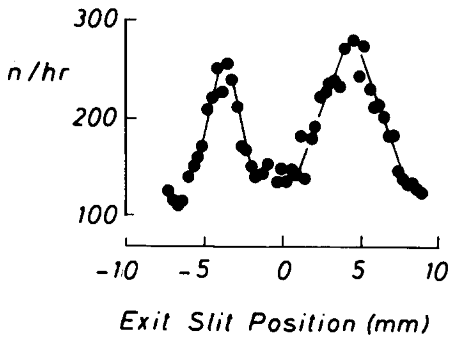

Fig. 9. Focused (left) and defocused (right) peaks after neutron passage through the gradient-field crystal (magnet current 15 A).

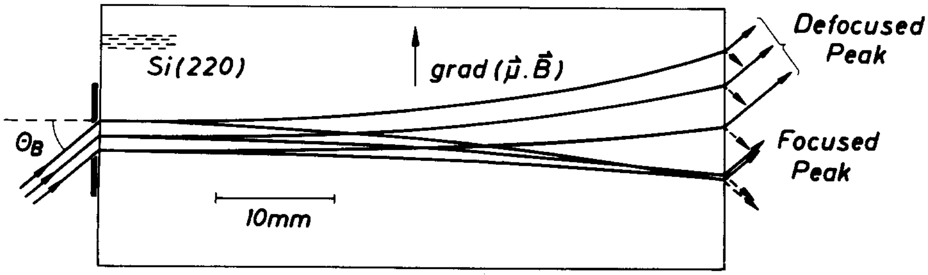

Fig. 8. Calculated trajectories in the second crystal (magnet current 7.5 A). Variation of the field gradient results in focusing effects.

monstrate the focusing/defocusing action, particularly in view of the fact that our magnet system was not specifically designed for that purpose.

5. Concluding comments

In this paper we have reviewed some unique focusing effects arising in perfect-crystal diffraction of neutrons. Though due to intensity limitations they are unlikely to lead to any application in improving neutron intensities in routine experiment, they may nevertheless give rise to interesting new experiments. Of those we would simply mention here that each of the focusing effects may lead to new types of neutron interferometers. This possibility exists in principle whenever two or more trajectories starting from the same point meet again at another point in space–time.

In the second section two different types of foci were discussed. Of these, the focus for a single wavelength arising at the back face of the second crystal plate is the basis of the two-crystal Laue-case interferometer. On the other hand, the geometric focus for different wavelengths is clearly important for intensity reasons because it allows the use of a broad wavelength band. Yet, to observe interference effects between these different wavelengths, experiments with wave packets are necessary. Only in wavepackets which have to be defined by suitably chopping the beam, does coherence between different wavelengths exists, which again can only be detected in a time-resolved way [7, 8]. Thus, this type of focusing may find applications in future novel experiments combining quantum chopping [9] with perfect-crystal interferometry.

The focusing effects utilizing lens action as presented in section 3 should prove to be useful if one attempts to construct an interferometer using an absorbing perfect crystal. In such a case only the $\alpha \rightarrow \alpha$ peak could lead to reasonable intensities in a two-crystal interferometer. Yet in order to observe interference in such an arrangement one has to ascertain that the radiation emerging from the entrance slit is sufficiently coherent over the part of the Borrmann fan used. This can be achieved through an entrance slit comparable in dimensions to the Pendellösung length [4].

The focusing effect arising through external forces acting on the neutron as presented in section 4 may find a very interesting application in one-crystal interferometry. There, neutrons originating at the same point on the front face of a crystal plate may arrive at a focus point on the back face of the same crystal plate after having followed different trajectories through the crystal. Clearly, in such an interferometer it would be impossible to insert material phase shifters into either interferometer beam, but interference fringes due to, e.g., gravitational interaction or crystal rotation (Sagnac effect) should result in detectable phase shifts and, hence, intensity variations.

Acknowledgements

This paper is dedicated to Professor Clifford G. Shull, colleague and friend, who brought us together ten years ago in 1977 at the M.I.T. Neutron Diffraction Laboratory, and who has supported our work over the years both intellectually and financially. We also wish to thank Dr. Kenneth D. Finkelstein for his help during the magnetic field measurements and the trajectory calculations. The work reported here has been supported partially by the U.S. National Science Foundation under grant number DMR-87 · 13559 and partially by the Austrian Science Foundation, project no. S42-01.

References

[1] This paper is part of a review presentation by the first author at the Conference on 'Matter-Wave Interferometry' of neutron optics and neutron interferometry activities at M.I.T. This complete review would surpass the space available here. Therefore the present article covers a small subset only, focusing on more recent work.

[2] A. Zeilinger, C.G. Shull, M.A. Horne and G.L. Squires, Two-crystal neutron interferometry, in: Neutron Interferometry, U. Bonse and H. Rauch, eds. (Oxford Univ. Press, London, 1979), p. 48.

[3] C.G. Shull, A. Zeilinger, G.L. Squires, M.A. Horne, D.K. Atwood and J.R. Arthur, Phys. Rev. Lett. 44 (1980) 1715.
[4] J. Arthur, C.G. Shull and A. Zeilinger, Phys. Rev. B 32 (1985) 5753.
[5] A. Zeilinger and M.A. Horne, Physica B 136 (1986) 140.
[6] A. Zeilinger, C.G. Shull, M.A. Horne and K.D. Finkelstein, Phys. Rev. Lett. 57 (1986) 3089.
[7] A.G. Klein, G.I. Opat and W.A. Hamilton, Phys. Rev. Lett. 50 (1983) 563.
[8] H.J. Bernstein and F.E. Low, Phys. Rev. Lett. 59 (1987) 951.
[9] J. Felber, R. Gähler and R. Golub, Physica B 151 (1988) 135, these Proceedings.