QUANTUM CONCEPTS IN SPACE AND TIME

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Testing quantum superposition with cold neutrons

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2.1. Introduction

The historical development of quantum mechanics, particularly its interpretation, has been accompanied and signposted by an extended discussion of various brilliant *Gedankenexperiments*. These were of particular interest whenever a strange or seemingly counter-intuitive prediction of quantum mechanics for the results of observations were at the focus of scientific discussion. A high point in that respect was certainly the famed Bohr–Einstein dialogue (Bohr 1949) where Einstein, through a series of ever more sophisticated *Gedankenexperiments*, purported to demonstrate a supposed internal consistency of quantum mechanics, a position which, in all cases, could be refuted elegantly by Bohr. Despite the fact that quantum mechanics today is probably the single most successful physical theory in terms of breadth, correctness and variety of its predictions, its foundations and the epistemological questions raised by it are increasingly attracting interest.

It is in this context of an increase in interest in the foundations of quantum mechanics that attention is again focusing on conceptually simple and basic experiments. A significant difference, as compared to the earlier situation, lies in the fact that due to various technological advances in the meantime many of these experiments, or basically very similar ones, could actually be performed i.e. be moved out of the domain of 'mere' *Gedankenexperiments* to that of real ones (see e.g. the series of experiments presented and discussed in Kamefuchi *et al.* (1984)). In a sense, these experiments therefore serve to demonstrate some of the strange features of quantum mechanics in a rather direct way. In addition, if performed in a sufficiently precise and controlled way, some of these experiments provide evidence against or put upper limits on other theories being alternative to or extensions of quantum mechanics.

This chapter deals with some of these experiments performed in recent years with cold neutrons. Also, further experiments will be proposed aimed at elucidating further basic points. The reasons why neutrons are particularly well suited for some investigations of this kind stem from various facts. For example, compared with electrons, the larger mass of the neutron may be important if experiments aimed at possible deviations from standard quantum mechanics due to gravitational effects are performed. The larger mass also implies a slower speed at a given wavelength as compared with electrons. The property of the neutron of having no electric charge, which has been tested experimentally to extremely low limits (Gähler *et al.* 1982), on the one hand implies some disadvantages as compared, again, with electrons in beam handling, yet it also implies a significant reduction of the sensitivity of an experiment to the ever-present stray electric and magnetic field, a feature not insignificant in precision experiments. Such a disturbance due to stray fields in the neutron case can only be due to an interaction with the neutron's magnetic moment which again is significantly smaller than that of the electron. Neutron experiments have been particularly facilitated by the development of cold neutron sources, which provide beams of slow neutrons of appreciable intensity.

2.2. Linearity versus nonlinearity

The linearity of quantum mechanics may be viewed as one of its basic axioms (d'Espagnat 1976). It is reflected in the linearity of the Schrödinger equation and the superposition principle. Yet it is just that linearity which leads to some of the epistemologically most complex issues, like the spreading of wave packets beyond limit, and to the questions relating to the problem of the reduction of the wave packet upon measurement (Wigner 1963). Therefore it is not surprising, that nonlinear generalizations of the Schrödinger equation have been proposed (Bialynicki-Birula and Mycielski 1976; cf. also Chapter 9) with the aim at eliminating these issues. It is also interesting that many physicists expect a successful quantum formulation of gravity theory to lead to nonlinear quantum mechanics (Chapter 9). It is quite evident that, in view of the success of linear quantum mechanics, any nonlinear deviations must be very small.

Starting from the observation that in physics many linear equations are only the limiting cases of more general nonlinear equations, Bialynicki-Birula and Mycielski (1976) investigated the class of equations obtained by adding to the standard linear Schrödinger equation a term which is a function of the probability density

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\boldsymbol{r},t) + F(|\psi|^2)\right]\psi(\boldsymbol{r},t) = i\hbar\frac{\partial}{\partial t}\psi(\boldsymbol{r},t). \tag{2.1}$$

These authors then find that many of the customary features of the solutions of the Schrödinger equation are still obeyed by their nonlinear generalization. Of particular interest is a variant of eq (2.1), where the nonlinearity is logarithmic:

$$F(|\psi|^2) = -b \ln(|\psi|^2 a^n). \tag{2.2}$$

Here, b has the dimension of energy and is a measure of the strength of the nonlinear term, a is an unimportant constant and n is the dimension of the definition space of ψ. The reason for a logarithmic nonlinearity of the kind of eqn (2.2) is that this specific form provides for the separability of non-interacting subsystems. Based on the impressive agreement of experimental results for the Lamb shift with theoretical predictions, Bialynicki-Birula and Mycielski were obliged to place an upper limit of 10^{-10} eV on the magnitude of b.

Considerations of the nonlinear equation as represented in eqns (1.1) and (1.2) led Shimony (1979) to predict the existence of a purely amplitude-dependent phase shift in a neutron interferometer experiment. The experiment proposed by Shimony was subsequently performed by Shull *et al*. (1980) using the MIT two-crystal interferometer (Zeilinger *et al*. 1979). The experiment consisted of searching for a phase difference between an attenuated beam and an unattenuated one. No such phase difference was found within experimental accuracy, which observation permitted Shull *et al*. (1980) to lower the upper limit of b to 3.4×10^{-13} eV.

As already pointed out by Bialynicki-Birula and Mycielski, a non-vanishing value for the quantity b would prevent wave packets from spreading beyond limit. In particular, it can be shown that there exist soliton-like solutions of the logarithmic nonlinear equation of width (Bialynicki and Mycielski 1976, 1979)

$$l = \hbar/(2mb)^{1/2}. \tag{2.3}$$

It is therefore reasonable to search for changes in the free-space propagation of neutrons effected by the nonlinear term in Schrödinger's equation. Since the nonlinearity of eqns (2.1) and (2.2) is a function of $|\psi|^2$, the largest effect is to be expected for the most abrupt change in $|\psi|^2$ possible. This is just the case in the diffraction at an absorbing edge. In detail, one can argue that a gradient of $|\psi|^2$ in a direction normal to the wavefront leads to a related gradient of the phase of ψ. Yet, as can easily be seen, such a gradient in the phase leads to a bending of the wavefront, i.e. to a deflection. Assuming that any deviation due to a nonlinear term is very small, a more rigorous derivation leads to the

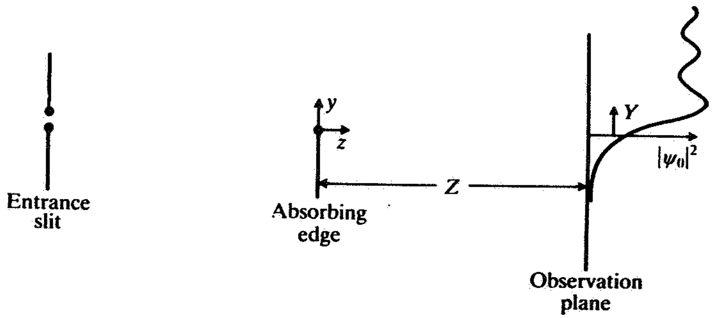

Fig. 2.1. Diffraction of cold neutrons at an absorbing straight edge: principle of the arrangement and co-ordinates used.

following expression for the deflection (Gähler *et al.* 1981):

$$Y = \frac{b}{E} \int_0^Z \frac{1}{|\psi_0|} \frac{\mathrm{d}|\psi_0|}{\mathrm{d}y} (Z - z)\,\mathrm{d}z. \tag{2.4}$$

Here, y designates a direction orthogonal to the absorbing edge, z is the neutron propagation direction, Y denotes a given position in the diffraction pattern, and Z is the distance between the absorbing edge and the observation plane (Fig. 2.1). E is the kinetic energy of the neutrons and ψ_0 is the solution of the linear Schrödinger equation. The integral is to be taken along the whole path of the neutron from absorbing edge to observation plane. It is immediately obvious from eqn (2.4) that any effect would be the smaller the larger the neutron wavelength λ i.e. the smaller its kinetic energy is.

The experiment was performed at a cold neutron beam at the high-flux reactor of the Institute Laue–Langevin in Grenoble. The wavelength used was 20 Å with a bandwidth of ±0.5 Å. The experimental set-up was arranged on a large optical bench with a distance between absorbing edge and detector of 5 m. The observed diffraction pattern did not show any evidence for a statistically significant deviation from the predictions of the linear Schrödinger equation (Fig. 2.2). Based on the experimental resolution one therefore arrives at a new upper limit for the nonlinear term (Gähler *et al.* 1981):

$$b < 3.3 \times 10^{-15}\,\mathrm{eV}. \tag{2.5}$$

We note that, using this limit, one can calculate the maximum to which an electron wave packet would spread, since b should be a universal constant (Bialynicki-Birula and Mycielski 1976). One thus obtains for the width of an electron gausson, which is the free-space soliton-like solution of the nonlinear Schrödinger equation, the value of 3 mm. It is

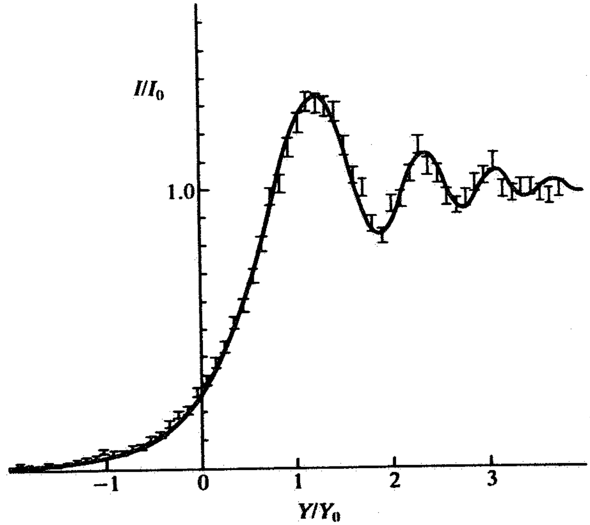

Fig. 2.2. Measured edge diffraction pattern compared with the prediction of standard linear quantum mechanics.

remarkable, that therefore even the nonlinear theory has to admit the existence of macroscopic quantum objects.

At present we plan to improve the lower limit for b obtained so far by exploiting a neutron beam of about 80–100 Å wavelength available at the ILL in the future. The resulting decrease in neutron energy by a factor of 20 together with improved counting statistics should permit a limit of the order of 10^{-17} eV to be achievable.

2.3. Two-slit diffraction

As a *Gedankenexperiment*, two-slit diffraction has and is continuing to serve as one of the basic paradigmatic examples for demonstrating peculiar features of quantum mechanics (see e.g. Feynman *et al.* 1965). This is due to the fact that in that conceptually simple experiment some of the most fundamental features of the interpretation of quantum mechanics can be demonstrated directly. These are (1) the superposition of probability amplitudes, (2) the complementarity between different kinds of information, and (3) the reduction of the wave packet.

For massive particles, two-slit diffraction was first successfully shown by Jönnson (1961, 1974) with a lengthy series of experiments on electron diffraction at various slit assemblies. The results observed are in excellent

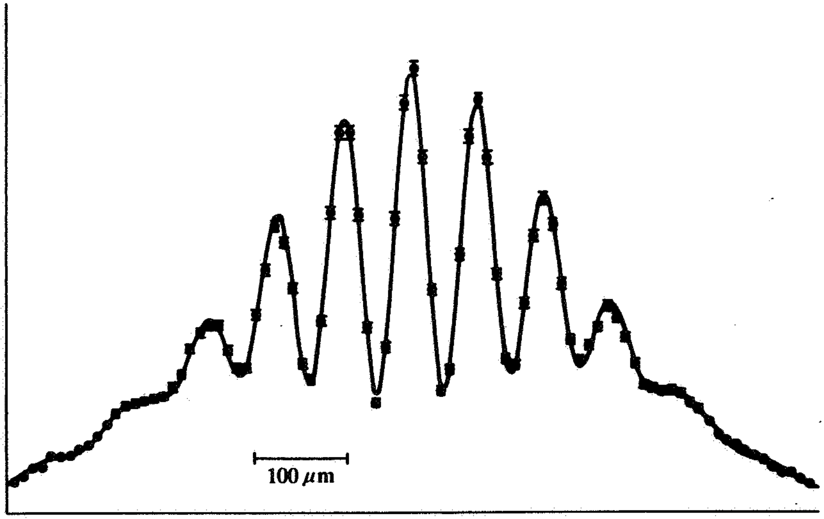

Fig. 2.3. Measured two-slit diffraction pattern compared with the prediction of standard linear quantum mechanics.

agreement with theoretical prediction as regards the spacing of the interference fringes. Yet, to my knowledge, there exists no detailed comparison of the intensity distribution seen in the electron interference pattern with detailed theoretical calculation, although qualitative agreement has certainly been established.

Having had the optical bench set-up available for cold neutrons as mentioned in the last paragraph, the classic two-slit experiment was also performed for neutrons of wavelength 18.45 Å with a bandwidth of ±1.4 Å (Zeilinger *et al.* 1982). In that experiment, the centre-to-centre separation between the two slit openings was 126 μm and the combined width of the slits was 44 μm, making each slit about 22 μm wide. Due to experimental difficulties there was a slight asymmetry between the two slits which showed up as a corresponding asymmetry in the interference pattern (Fig. 2.3). It may be noted that, because of the fact that the wavelength of the neutrons used was so many orders of magnitude smaller than the size of the diffracting two-slit assembly, the angles of diffraction were only of the order of seconds of arc. Hence, a separation distance between object and observation plane of 5 m had to be employed.

In order to compare the experimentally obtained interference pattern with theoretical expectation, a detailed calculation based on scalar

Fresnel–Kirchhoff diffraction theory was performed (see e.g. Born anc Wolf 1975). In that approach one has to include the various experimental details very carefully. First, the source has to be defined, which, foi precision calculations, is by no means a trivial procedure. It was found that it suffices to regard the entrance slit as the effective source. This i: due to the fact that the width of that slit is very narrow (20 μm) and thu: the slit effectively operated as the source of a beam coherent over botl slits in the two-slit diaphragm. An interesting point is the treatment o the radiation right at the source, i.e. inside the source slit, since th radiation incident on that slit consists of both different direction an different wavelength components—in principle treatable as bein either coherent or incoherent with respect to each other. In extensiv numerical calculations it was found that the difference between treatin different incident directions as being coherent or incoherent to each othe did lead to differences in the interference pattern too small to b detectable in the experiment performed. Also, since the experiment wa not performed in a time-dependent mode, any coherence betwee different wavelength, i.e. energy, components would be undetectable.

Therefore, the amplitude at an observation point P is

$$u_P = \text{const} \iint e^{ik(r+s)} \, dS_1 \, dS_2 \qquad (2.\text{(}$$

where r is distance from a source point to a point in the diffraction scree and s is the distance from that latter point to P. The integrations are to b performed over the source and over the two-slit diaphragm. The intensi is then found by incoherent integration over the directions and wav lengths incident on the entrance slit

$$I_P = \iint |u_P|^2 \, d\alpha \, d\lambda. \qquad (2.$$

The smooth curve in Fig. 2.3 is the result of numerical calculatio performed according to this equation.

The excellent agreement thus obtained between experiment a standard Schrödinger wave mechanics can be interpreted as implyi upper limits on deviations from that linear theory. In particular, the fa that the interference contrast is less than 100 per cent can be fu accounted for by the wavelength spread of the incident radiation, whi implies a coherence length of

$$L_c = \lambda^2 / \Delta\lambda = 243 \text{ Å} \qquad (2$$

for the present experiment. Any deviation of the experimentally (served interference contrast from the quantum mechanically predict one therefore has to be smaller than the experimental accuracy. T implies that, if such a deviation actually existed, it could not be larger than $6 \cdot 10^{-3} = \Delta C/C$, where C is the interference contrast defined as

$$C = (I_{max} - I_{min})/(I_{max} + I_{min}) \tag{2.9}$$

with I_{max} and I_{min} being the maximum and minimum intensities of the interference pattern at the innermost maxima and minima respectively.

We note that this agreement indicates that any presently unknown mechanism for a further reduction of the interference contrast is restricted to bounds which are given by the present experiment. Such additional mechanisms have been proposed explicitly and independently by Pearle (1976, 1979, 1982) and Hawking (1982). In both cases it is proposed that there may exist intrinsic mechanisms breaking the unitary evolution of the state vector. Such an evolution would then lead to a dynamically describable reduction mechanism which, for the two-slit experiment, could result in a loss of interference contrast. Such a loss of interference contrast should then depend on the time the state vector spends evolving.

Following Pearle (1984) we parametrize such a dynamic reduction mechanism by an exponential law as

$$\Delta C/C = e^{-t/\tau}, \tag{2.10}$$

where t is the time available for evolution of the state vector and τ is a characteristic reduction time, the properties of which depend on the particular theory chosen. Using the neutron speed of 217 m s^{-1} we find the flight time for the distance of 5 m between the slits and the detector to be 2.3×10^{-2} s. Hence, with the maximum possible interference contrast reduction of 6×10^{-3} which would not be in disagreement with experiment we obtain as a lower limit for the spontaneous reduction time (Zeilinger *et al*. 1984)

$$\tau > 4\text{s}. \tag{2.11}$$

Looking again into the future, it can be estimated that with a neutron beam of a wavelength of 100 Å, as may be available in the near future at the high-flux reactor of the ILL in Grenoble, an improvement of the lower limit of the reduction time by possibly as much as two orders of magnitude could be achievable. This assumes that an experiment dedicated explicitly to the search for an unknown reduction mechanism would be carried out. In such an experiment one could think of using a flight path of about 10 m length and of counting neutrons in the central portion of the interference pattern only in order to improve the counting statistics in the determination of the interference contrast. We hope that such an experiment may be performed in the near future.

2.4. Two-slit complementarity experiments

The two-slit experiment is usually regarded as providing a beautiful explicit example for complementarity in quantum mechanics. In that experiment it is the complementarity between the interference pattern and the information which slit the particle passed through (Wooters and Zurek 1979). Yet, to our knowledge, no experiment exists which shows explicitly this complementarity feature in a two-slit set-up. Here we will propose rather simple extensions of the experiment reported in the previous paragraph which would permit an explicit demonstration of two-slit complementarity.

First, it is evident, yet still important, that the width of the scanning slit in front of the detector is crucial. In the conventional way of analysing the two-slit experiment, it is usually said that the scanning slit has to be narrower than the characteristic width of the features in the interference pattern, i.e. the minimum-maximum distance. This often is connected to the implied, yet incorrect, mental picture of the interference pattern as having some kind of *a priori* reality independent of whether a detector is actually placed there to detect it. Analysing in detail the operation of the detector slit—and for that of a narrow detector too—we have to investigate the diffraction taking place at that slit. Doing that we assume that the slit width is just equal to the distance between the interference minima and maxima

$$\delta = \frac{\lambda L}{2s}. \tag{2.12}$$

Here, L is the distance between the two-slit diaphragm and the detector slit and s is the centre–centre distance between the two slits in the two-slit diaphragm itself. Diffraction at a slit of that width of incident unidirectional radiation results in a single slit pattern of angular full width at half maximum of

$$\Delta\theta = \lambda/\delta = 2s/L. \tag{2.13}$$

Yet, we note that this is just twice the angular separation between the two slits as seen from the detector slit. Hence, observation of the interference pattern using a narrow enough detector (slit) results in destruction of the information about the path the particle took when passing through the two-slit assembly. It is, we submit, just this latter property which is responsible for the creation of the interference pattern in the first place.

If, on the other hand, the detector slit is much wider than the width given in eqn (2.13), diffraction at that slit is negligible and it is possible to determine the particle path even after passage through the detector slit.

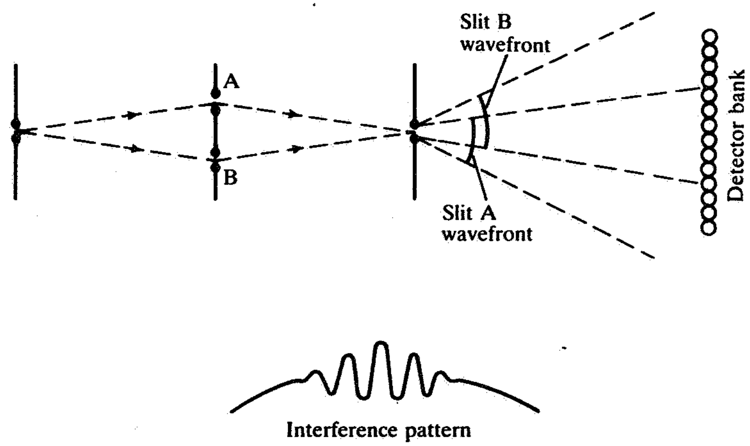

Fig. 2.4. Proposed two-slit complementarity experiment: diffraction at the narrow exit slit destroys the information about passage through slit A or B and hence permits the two-slit diffraction pattern to be observed upon variation of exit slit position.

Therefore, an experimental arrangement is proposed with a variable-width detector slit and a detector bank behind that slit (Figs. 2.4 and 2.5). If the slit is narrow (Fig. 2.4), diffraction at that slit creates the two-slit pattern in the detector bank due to overlap between the two wavefronts originating from the two different openings in the two-slit diaphragm. Such an overlap will no longer occur if the detector slit is

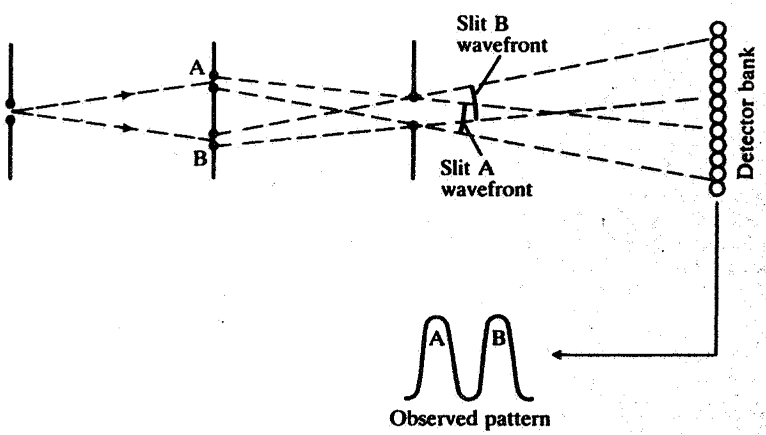

Fig. 2.5. Proposed two-slit complementarity experiment: wide exit slit. Here, diffraction at that slit is negligible and hence no two-slit pattern appears, yet slit passage through A or B may still be discriminated.

made much wider (Fig. 2.5). Then, the firing of a given detector would allow determination of the slit passed. Evidently this latter observation can be interpreted as a particle property, while the first one is a wave property. Also, we note that any intermediate type of result is possible.

An interesting extension of these experiments, which are readily performable with both neutrons and photons, results if we consider changing the detector slit width at a time after the particle has already passed the two-slit diaphragm. Then we can still decide to observe the interference pattern or to determine the particle's path. This clearly is a very explicit variant of a delayed choice experiment as proposed by Wheeler (1978). A most simple variant of these experiments results by replacing the detector bank by just two properly positioned detectors (Zeilinger 1984).

2.5. Concluding comments

The experiments presented here provide a nice example of technological progress—here the development of intense sources of cold neutrons—leading to fundamental experiments of hitherto unachieved precision. Whether these experiments will ever show a deviation from physics as known today is open. Yet, we note that increases in the precision of experimentation repeatedly in the history of physics have led to the observation of unexpected phenomena. An interesting field of work in that spirit will also be provided by the possibility of doing experiments, in the near future, aimed at testing the time-dependent Schrödinger equation prediction in great detail with precision far surpassing present capabilities. This again will be done with intense cold neutron beams.

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