GENERALIZED AHARONOV-BOHM EXPERIMENTS WITH NEUTRONS

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INTRODUCTION

The Aharonov-Bohm effects[1] are generally regarded as direct manifestations of the property, that potentials are affecting quantum systems in a way significantly different from the classical case. This stems from the fact, that potentials enter the classical Newtonian equations of motion only through their derivatives, while in quantum mechanics the potentials themselves enter the Schrödinger equation. The Aharonov-Bohm effects often are said to be also manifestations of the nonlocal character of quantum mechanics, because there the observed phase shift depends on fields in regions which are inaccessible or practically inaccessible to the interfering electron. It is this latter point which has raised a considerable body of discussion in the scientific literature[2] indicating, that the epistemological significance of the Aharonov-Bohm effects is not yet understood.

In the present paper I do not wish to enter into these discussions, I rather address the question, whether one can find generalisations for the neutron case, where still some of the features of the original Aharonov-Bohm effects are maintained. This will be based on operational analogy and it will not involve any interpretive or epistemological questions.

THE OPERATIONAL SIGNATURE OF THE AHARONOV-BOHM EFFECTS

From the experimental point of view it is important to identify operationally significant features of the Aharonov-Bohm (AB) effects. By such a term we mean features which are insensitive to the interpretation of the experiments. Therefore, a thorough operational analysis unavoidably implies, that the electric and the magnetic AB effects are analyzed separately. This procedure will lead to the identification of common features.

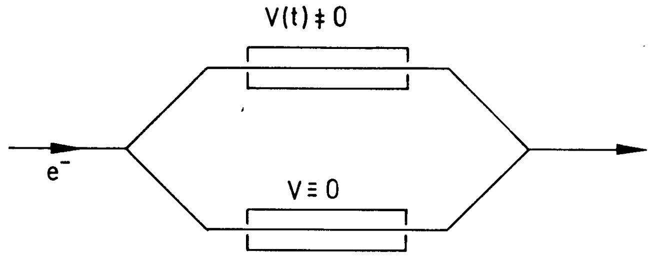

Fig. 1: The electric AB experiment: a time-dependent potential leads to a non-dispersive phase shift.

In the electric AB effect, an incoming electron beam is split into two beams in an electron interferometer and each of the two resulting coherent beams is then passed through a Faraday cage (Fig. 1). The electric potential on one of the Faraday cages is then raised and lowered while the electron wave packet is inside the cage. The potential applied to the other cage is kept constant and equal to zero.

If the potential as a function of time is $\Phi(t)$, a phase difference

$$\Delta\phi_{AB} = \frac{e}{\hbar} \int \Phi(t)dt \qquad (1)$$

between the two beams results. This despite the fact, that classically no force acts on the electron at any time or, equivalently, no effect observable on either beam by itself results.

This latter property may also be seen by comparing the dispersion relation in the zero-potential region

$$\frac{\hbar k_o^2}{2m} = \omega_o \qquad (2)$$

where k_o denotes the electron wave number and ω_o is its frequency disregarding the rest mass contribution, with the dispersion relation in a time-dependent potential

$$\frac{\hbar k_o^2}{2m} + e\Phi(t)/\hbar = \omega(t). \qquad (3)$$

Here we have used the property, that for a purely time-dependent potential momentum is conserved.

Calculation of the group velocity

$$v_g = \partial\omega/\partial k \qquad (4)$$

evidently leads to the same results for either case, i.e. no time delay of the wave packet is caused by the time-dependent potential.

It is interesting to compare this latter result with the static case, where we assume the constant potential Φ to be applied to one of the Faraday cages all the time. Then the dispersion relation inside that cage is

$$\frac{\hbar k^2}{2m} + e\Phi/\hbar = \omega_o \tag{5}$$

because energy is now conserved. Clearly, the group velocity resulting from Eq. (5) is now different from that resulting from Eq. (2). This implies, that for a static experiment there is an operationally accessible, i.e. measurable, time delay of the wave packet (Fig. 2).

The time delay is also reflected by the property, that in a static experiment the phase shift

$$\Delta\phi_{stat} = (k-k_o).d = [\sqrt{k_o^2 - 2me\Phi/h^2} - k_o]d \tag{6}$$

due to a potential region of length d is dispersive, i.e.

$$\frac{\partial}{\partial k}\Delta\phi \neq 0. \tag{7}$$

In contrast to that case the AB phase shift is nondispersive, i.e.

$$\frac{\partial}{\partial k}\Delta\phi_{AB} = 0. \tag{8}$$

Figure 3 demonstrates the relationship between wave packet shift and phase shift for the two cases discussed.

We finally point out, that the electric AB effect has not yet found an experimental verification. This is primarily due to the fact, that electrons commonly used in electron interferometry experiments have very high velocities e.g. for 50 keV electrons the speed is approximately a tenth of the speed of light. This implies, that the potential applied to the Faraday cage would have to be switched extremely rapidly (with fre-

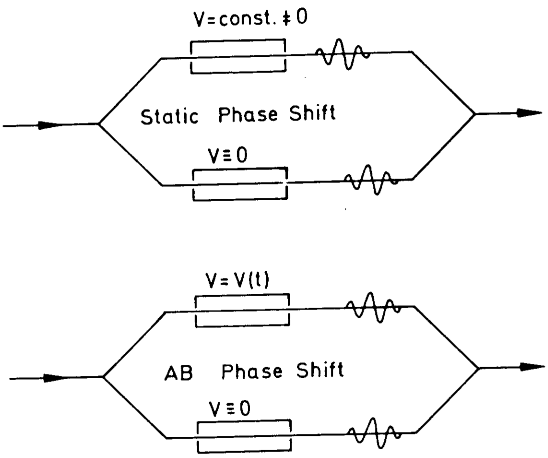

Fig. 2: In contrast to a static phase shift experiment the two wave packets are not shifted in an AB experiment

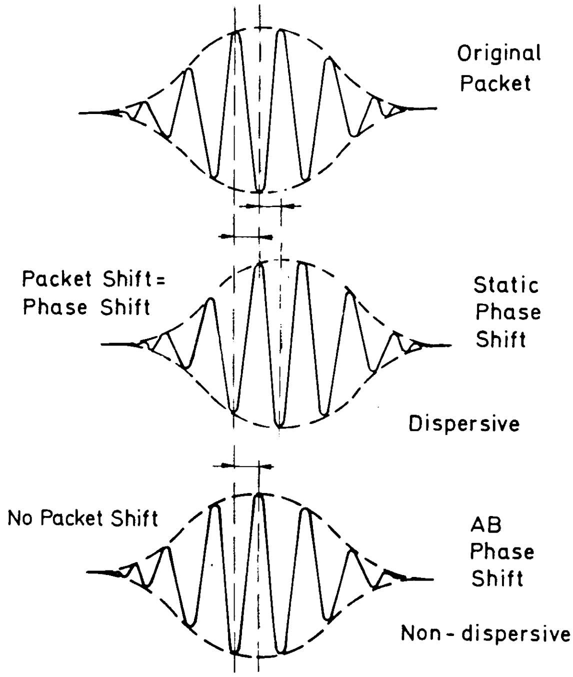

Fig. 3: Detailed comparison of the dispersive static phase shift with the non-dispersive AB phase shift.

quencies of the order of 10 GHz). Besides the practical problem this poses for the experimental realizability, it definitely implies, that for these conditions an adiabatic charging/discharging procedure cannot be maintained.

The situation is quite different for the magnetic AB effect, which has found numerous experimental[3] realizations . Analysing the magnetic case in a similar spirit as the electric case above, we note, that here again no change in group velocity occurs. This feature results from the property, that the change which occurs due to the magnetic vector potential $\vec{A}$ is a change of canonical, not kinetic, momentum, i.e. a wavevector change

$$\vec{k}(\vec{r})' = \vec{k} - \frac{e}{\hbar}\vec{A}(\vec{r}) \tag{9}$$

The AB phase shift obtained from this equation then is

$$\Delta\phi_{AB} = \frac{e}{\hbar}\oint \vec{A}(\vec{r})d\vec{r} \tag{10}$$

which again is non-dispersive. We point out that despite the many experimental verifications of the magnetic AB-effect, this latter feature has not yet been demonstrated, though it would represent rather convincing evidence of the special nature of the AB effect.

Summarizing we point out, that the significant operational feature of the AB effect is that there exists no observable defined on either beam separately, which is influenced by the electric or magnetic field. This is related to the property, that the AB phase shifts are non-dispersive. Only a constant overall phase change for the wave packet arises, which clearly is observable only in an interference experiment.

THE NEUTRON CASE

Considering generalisations to the neutron case, we realize that due to the electric[4] and the magnetic[5] neutrality of the neutron no direct interaction with either the scalar or the vector potential of electrodynamics exists. Thus, at first sight, there is no reason to expect to observe AB effects for neutrons. For the magnetic AB effect this latter point has even been tested experimentally[6] with the predicted negative results.

In principle there exists the possibility of realizing the AB situation in a gravity experiment. For example, one could think of moving around properly arranged masses in the vicinity of a neutron interferometer in order to change the Newtonian gravitational potential acting on the neutron in one beam (Fig. 4). With due care this can be done such as to avoid any additional force acting on the neutron and therefore the neutron wave packet would neither be accelerated nor delayed by changing potential. It is interesting, that nevertheless we seemingly are able to observe a local effect in that situation, namely the change in proper time

$$\Delta\tau = \frac{\Delta\Phi}{c^2} \qquad (11)$$

as caused by the potential difference $\Delta\Phi$. Clearly, for the neutron case this difference in proper time is only observable in an interference experiment. In fact, it may be viewed as being the cause of the AB phase shift observed[7]. Yet, this change in proper time leads to the possibility of performing a classical AB experiment by sending a real macroscopic clock along the path subject to the time-dependent potential $\Phi(t)$. This clock would then read a different time as compared to other clocks. But even this classical effect still is a non-local one of the same kind as the other AB effects, since locally no effect is observable, the observation of the change in proper time necessitates a comparison with other clocks. An observer travelling with the clock would never sense anything special happening.

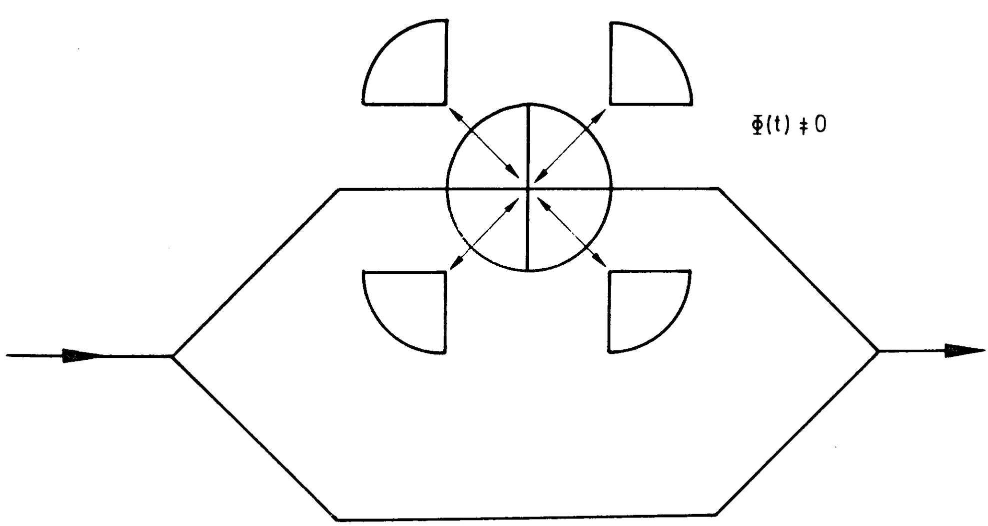

Fig. 4: Principle of a gravity AB experiment, where a mass shell is closed and opened up for producing a time-dependent gravitational potential $\Phi(t)$.

Contemplating the realisability of the gravity AB experiment discussed above we note, that a very useful precursor would be a successful Cavendish experiment with neutrons, i.e. the demonstration of the action of laboratory gravitational forces on the neutron. This latter experiment is feasible with very cold neutrons (VCN) with a speed of 100 $m\ s^{-1}$ or below and will certainly be performed in the near future. The realization of a gravity AB experiment most likely necessitates the use of even colder neutrons because of the time scales involved.

Above we pointed out, that the operational signature of the AB phase shift is the absence of an observable effect on either interferometer beam separately. This is related to the property, that the AB phase shifts are non-dispersive, the consequence of which is the fact, that the flight times are not changed. This operational signature can be used as a guidance tool in search of generalizations of the AB effects[8]. One possible generalization arises, if we employ some other time-dependent interaction in a neutron interferometer. Of these, the interaction with a time-dependent magnetic field is the most promising candidate. Thus we may arrange in one beam of a neutron interferometer a field (Fig. 5) described by the Hamiltonian

$$H = -\mu\vec{\sigma}.\vec{B}(t) \tag{12}$$

where in addition we require the magnetic field to change only its magnitude not its direction as a function of time. In this equation μ is the neutron magnetic moment and $\vec{\sigma}$ is the Pauli spin pseudovector. If we switch the magnetic field in such a way, that it vanishes when the neutron enters or leaves the magnetic field region, no force will act on the neutron and the phase shift

$$\Delta\phi = \pm\frac{\mu}{\hbar}\int B(t)dt \tag{13}$$

is non-dispersive. Yet, in a magnetic field there will generally be a torque acting on the neutron magnetic dipole which leads to a spin rotation. This property shows in Eq. (13) as two different signs of the phase shift for the two spin states.

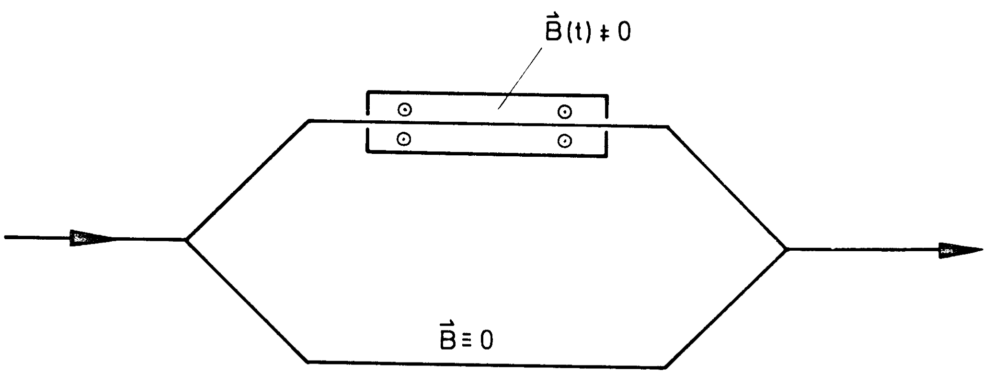

Fig. 5: Principle of a generalized neutron AB experiment, where a time dependent magnetic field $\vec{B}(t)$ results in a dispersion-free phase shift.

In order to arrive at an AB situation we therefore have to perform the experiment such, that the spin rotation is unobservable. One way to achieve this would be to use unpolarized neutrons described by the density matrix

$$\rho = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (14)$$

No unitary operator applied to this density matrix is able to change it, hence no observable effect on the individual beam exists. Another possibility is to arrange the incoming neutron beam in an eigenstate of the Hamiltonian Eq. (15). Then again the only observable effect is the relative phase between the two interferometer beams.

From an operational point of view, one would in an actual experiment demonstrate, that the phase shift arising from a time-dependent magnetic field is non-dispersive. This implies, that the number of observable interference fringes is not restricted by the coherence length of the beam, as it is an experiment with static fields.

Recently it has been pointed out[9], that the neutron spin-orbit scattering also leads to an AB situation. The spin-orbit scattering Hamiltonian of neutrons in an electric field $\vec{E}$ is

$$H = - \frac{\mu\hbar}{mc} \vec{\sigma}.(\vec{E}x\vec{k}) \quad (15)$$

where $\vec{k}$ is the neutron wave vector. This may be viewed as the fact, that relativistically the rest frame electric field $\vec{E}$ in the moving frame gives rise to a magnetic field

$$\vec{B}' = \frac{\hbar}{mc} \vec{k}x\vec{E} \quad (16)$$

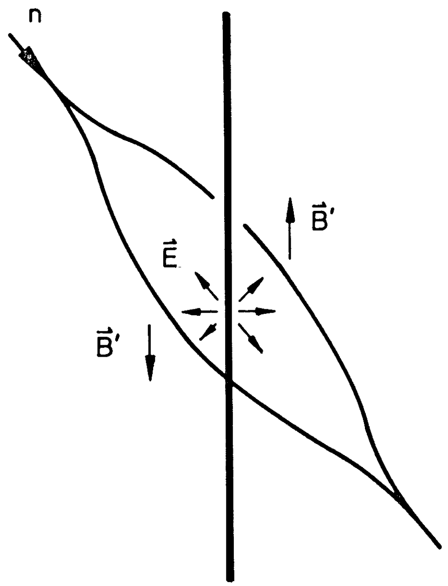

Fig. 6: Interaction of a neutron interferometer beam with an electrical changed wire leads to a generalization of the magnetic AB effect.

This could be realized (Fig. 6) by arranging electric charges at rest between the beams of a neutron interferometer. It is then easy to see, that the phase shift caused by the Hamiltonian Eq. (15) is non-dispersive. In fact, this latter conclusion holds for any time-dependent Hamiltonian which is proportional to $\vec{k}$. Since the velocity of the neutron beam used is irrelevant, the most important design feature of such an experiment is the amount of charge which may be arranged between the beams. We point out, that it is possible to construct neutron interferometer[10] for every cold neutrons with enclosed areas of the order of 1000 cm^2.

As a final point we mention, that neutron interferometry may also be used to search for unknown interactions by looking for any observable AB effect. An experiment aiming[11] at an unknown interaction tied to isospin[12] gave the expected null results.

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