

COMPLEMENTARITY IN NEUTRON INTERFEROMETRY

Anton ZEILINGER

*Department of Physics, Massachusetts Institute of Technology, Cambridge MA 02139, USA and
Atominstut der Oesterreichischen Universitaeten, Schuettelstrasse 115, A-1020 Wien, Austria*

The complementarity between the interference pattern and the neutron's path through the interferometer is formulated and studied in a quantitative information theoretic way. This leads to an understanding of intermediate cases of partial knowledge of complementary quantities. The role of complementarity is also analysed for the case of interference experiments where the neutron's spin plays a crucial role and a direct connection between different classes of complementary observables is found.

1. Introduction

Ever since the invention of quantum physics, its epistemological peculiarities have drawn significant attention. At the heart of these considerations is usually the double slit experiment as the most simple realization of quantum interference. Despite the intrinsic claim of quantum physics to be universally valid on microscopic as well as macroscopic scales, most working physicists, using hand waving arguments, tend to assign quantum physics to the microworld of elementary particles and fields. With such a viewpoint the question obviously arises, where the line of distinction between "microphysics" and "macrophysics" is to be drawn. In order to demonstrate the universal validity of quantum mechanics, it is therefore of significance that the realm of interferometry be extended to increasingly heavier objects. In that context, it is interesting to observe that interferometry, after having started with visible light in the eV range, has been extended to the keV range (X-rays) to nearly an MeV (electrons) and finally up to the GeV range of total mass-energy. This latter limit having been achieved with neutrons is the present-day limit.

In the development of this technique and its applications Cliff Shull and his M.I.T. Laboratory have played a significant role and it is therefore appropriate to give here a brief review particularly

from the point of view of its relation to the fundamental concept of complementarity.

2. Interferometry and Pendellösung structure

It is the specific characteristic of any interferometric experiment that a wave is coherently split into at least (and often just) two mutually coherent waves, which travel along macroscopically different paths such that the phase of each may be changed by the experimenters freely and independently of the other beam. It is this latter feature of distinctly different paths which distinguishes interferometric experiments from other quantum experiments also exhibiting interference effects. In an interferometer experiment, the spatially separated beams are then coherently recombined such that they form at least two (for reasons of particle number conservation) emerging beams. Today, for thermal neutrons the only mechanism available for this coherent splitting and recombination is diffraction at perfect crystals, which process permits the production of coherent beams separated from each other by distances on the centimeter scale. With respect to the details of the various crystal interferometer systems which have been developed [1–3] we refer the reader to a recent review [4]. An important precursor for the development of perfect crystal interferometry is the work by Cliff

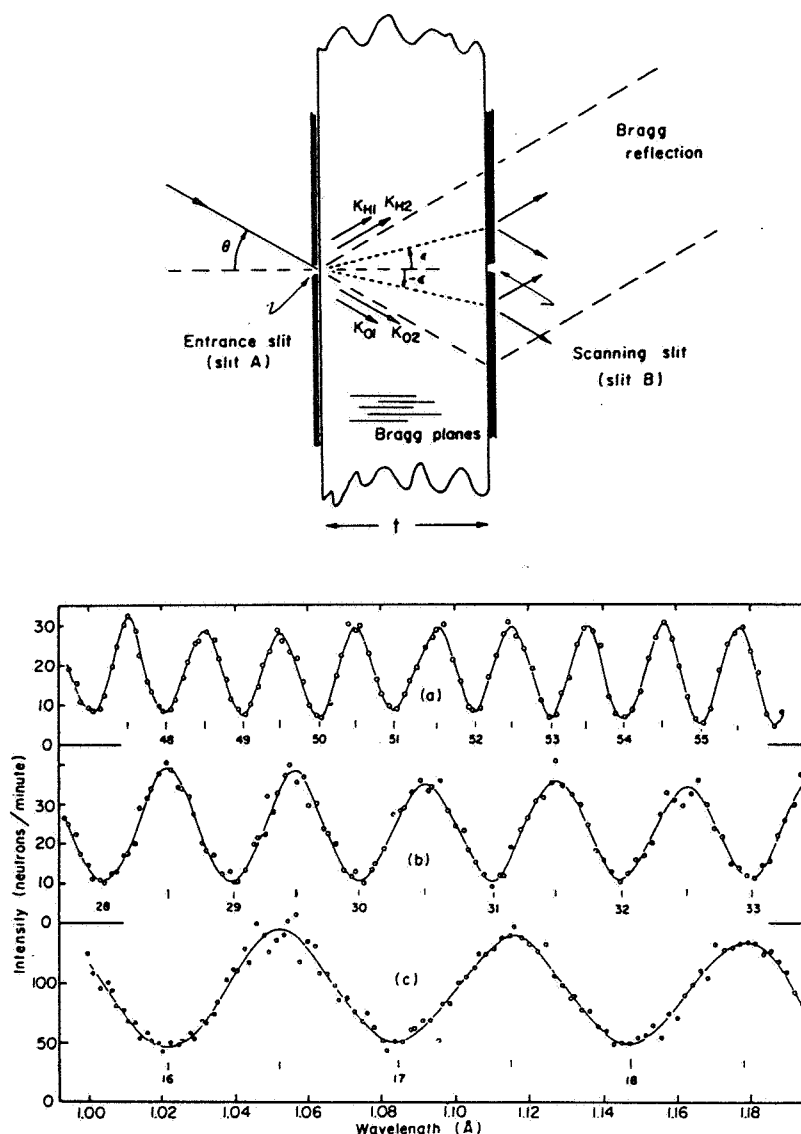


Fig. 1. Experimental scheme (top) and result (bottom) of the determination by C.G. Shull [5] of neutron Pendellösung oscillations in perfect Si crystals demonstrating explicitly phase coherence between wavefields inside the crystal. The curves are for crystal thickness of (a) 10.000 mm (b) 5.939 mm, and (c) 3.315 mm respectively.

Shull demonstrating in great detail the applicability of the dynamical diffraction theory to neutrons. There, he could show that the predictions of that theory are fulfilled in a very detailed quantitative way [5,6]. In particular, the observation of Pendellösung fringes did demonstrate explicitly the fact that in-crystal neutrons could retain their

coherence properties over macroscopic distances (fig. 1). These Pendellösung fringes indicate that the two wavefields propagating along a common path within a perfect Si-crystal stay mutually coherent. As always in quantum physics the property that two amplitudes are coherent with each other implies that the corresponding paths are

indistinguishable. Here it means that whenever we observe the Pendellösung fringes we do not know, not even in principle, whether the neutron traveled as α -wavefield or as β -wavefield radiation through the crystal.

This demonstration of Pendellösung fringes was an important prerequisite for the mutual coherence between waves propagating along different paths within a crystal as required for interferometry. This has been pointed out by Cliff [6], and the

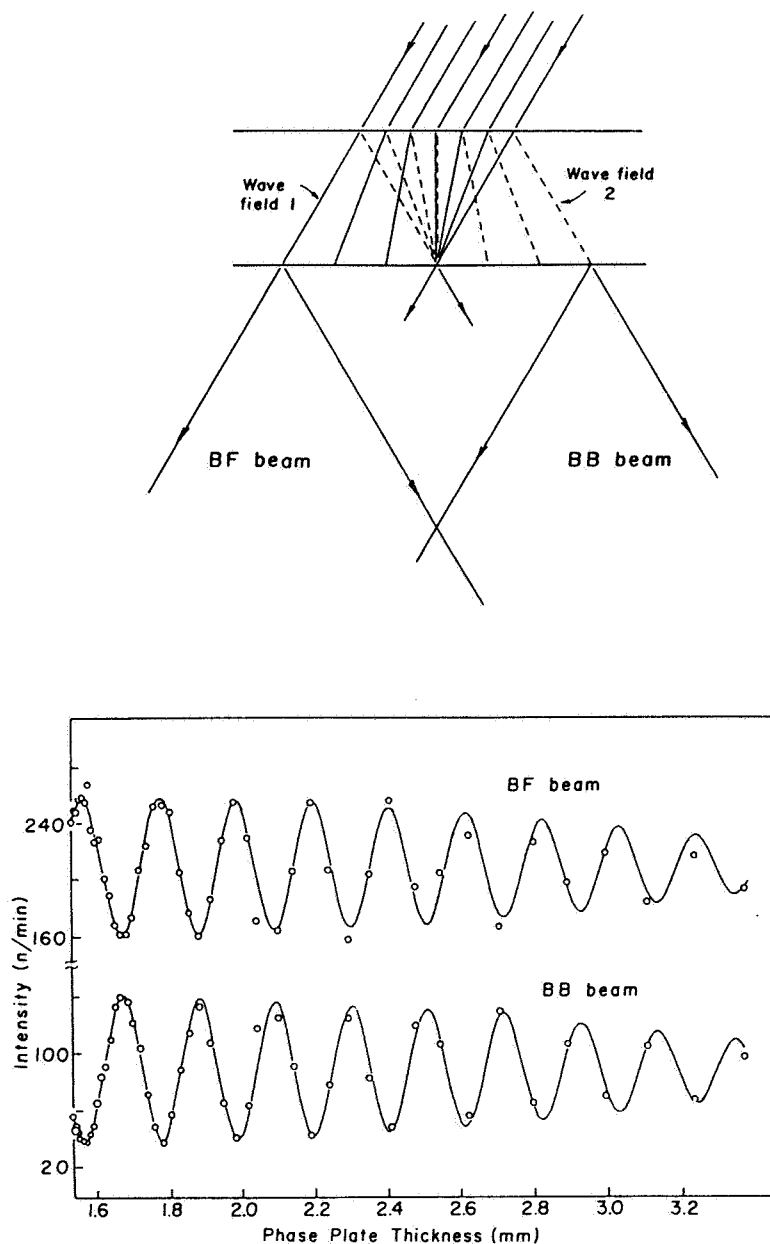


Fig. 2. Operational characteristic (top) of the two-crystal neutron interferometer: a second crystal plate (shown here) is used to recombine the beam separated by Borrmann fanning in a first crystal plate. Insertion of a phase shifter plate into half the beam then results in the observation of interference fringes [2].

successful development of the two-crystal interferometer [2] did demonstrate this point explicitly (fig. 2).

3. Complementarity and interference

A very important fact which cannot be overemphasized is the property that quantum mechanics predicts interference for each individual neutron, because the wave function describes an ensemble of identically prepared particles. One signature of the property that therefore each neutron carries the coherence information is the fact that there are always emerging beams or spots in the interference pattern which can, at least in principle if not in practice, be made of arbitrarily low intensity by properly adjusting the phase difference between the interfering beams, i.e. each neutron carries enough information to not end up there. From the experimental point of view, this single-neutron self-interference property is already guaranteed by the low intensities available in all experiments. Given a typical count rate of 1–10 neutrons/s and given a typical neutron flight time of the order to $10 \mu\text{s}$ through the crystal interferometer it is obvious that most of the time there is no neutron in the interferometer and sometimes one but virtually never (only about once a day) two neutrons.

As is well known, observation of interference and the knowledge which path the particle took inside the interferometer are two mutually exclusive classes of information. This property of a mutual exclusiveness of classes of information is also called complementarity, a term introduced by Bohr into the interpretation of quantum mechanics. In an interference experiment one can observe either the interference pattern or one can determine the path of the particle but never both at the same time. It has been pointed out by Wootters and Zurek [7], for the classic double slit experiment, that there are also intermediate cases possible where we can obtain some, yet not complete, knowledge about the particle's path and still observe an interference pattern of reduced contrast as compared to the ideal interference situation. Therefore, complementarity not only implies total knowledge of one variable at the expense of

total ignorance about the complementary one, but it also applies to intermediate cases.

In applying this concept to neutron interferometry one runs into the obvious problem that, at least for slow neutrons, no detector exists which permits the observation of a neutron in one interferometer arm without absorption of the neutron itself. This seems to imply a trivial realization of the complementarity principle since a neutron once observed in a beam path is not available for further experimentation anyway and therefore cannot contribute to the interference pattern. Yet, as we will argue, use of a partial absorber in one beam path allows us to realize in practice just those intermediate cases.

For reasons of simplicity we consider an idealized Mach–Zehnder arrangement (fig. 3). Such an arrangement has the advantage over the double slit arrangement that both the particle state inside the interferometer and the state after the interferometer may each be described by two-state wavefunctions. This significantly simplifies the formal treatment of quantum interferometry.

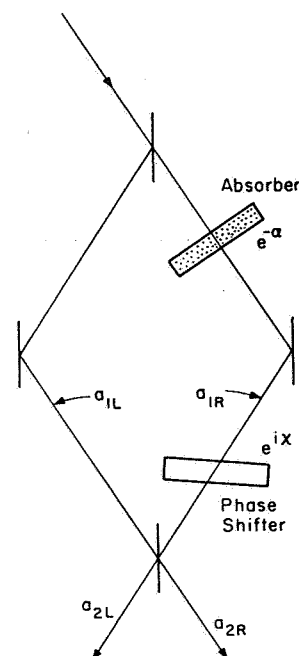


Fig. 3. Principle sketch of the arrangement to obtain partial information about the particle's path through the interferometer.

In order to analyze the complementarity in that situation, we assume the two mirrors used for splitting and recombining the beams to be identical half-silvered mirrors. In such a case, for the well adjusted interferometer (i.e. equal path lengths between the mirrors and with no absorption of either in-interferometer beam and in the absence of a phase shifter) only one exit beam is bright. In fig. 3 this is the exit beam on the right-hand side. The beam on the left-hand side has zero amplitude due to destructive interference resulting from the fact that for symmetric beam splitters the phase shift of a reflected wave with respect to a transmitted one is $\pi/2$ (see ref. 8). If we now introduce the phase shift χ between the two beams inside the interferometer the outgoing intensity can be shifted from the right to the left-hand beam. For the specific case of a π shift the right-hand exit beam will be completely dark and all intensity will be in the left-hand beam. We therefore note that in this case we observe 100% interference contrast in each of the outgoing beam at the expense of the fact that we have absolutely no knowledge which of the two paths the particle took inside the interferometer.

In contrast, if we completely absorb one of the two beams inside the interferometer, both outgoing beams will be equally bright and introduction of a phase shifter does not change these intensities. In this case we know precisely the path those particles which were not absorbed took through the interferometer, at the expense of complete loss of interference contrast.

The two situations discussed above are just the two extreme cases where we either have complete knowledge of the path the particle took or where we obtain an interference pattern with 100% modulation. Inserting now a partial absorber into either beam path in the interferometer, intermediate cases may be obtained. In such cases we have some partial knowledge of the path those particles took which were not absorbed. This knowledge is the more definite the lower the transmittivity of the absorber is. Yet, it simultaneously implies a less-than-perfect interference contrast.

In order to quantify this intermediate situation of partial knowledge of two mutually exclusive types of information, we make use of Shannon's

[9] notion of the lack of information.

In an experiment with n possible outcomes occurring with the normalized probabilities p_i , the information gained by a specific individual outcome is

$$h_i = -\log_n p_i, \quad (1)$$

where \log_n is the logarithm to the basis n . Therefore the total lack of information in the situation characterized by the individual p_i 's is defined as the weighted sum of the individual h_i 's:

$$H = -\sum_i p_i \log_n p_i. \quad (2)$$

This definition has the property of resulting in a zero value of H if one of the p_i 's equals one and all others are zero. This reflects the fact that in such a situation we have complete a priori knowledge of the outcome of the experiment, i.e. no lack of information or, equivalently, no information is to be gained by performing the experiment. The other remarkable property is that H attains its maximum value of unity if all p_i 's are equal ($p_i = 1/n$). This again is sensible since there we have no a priori knowledge about the outcome of the experiment.

We now apply the definition given above to quantitatively measure the lack of information about the particle's path through the interferometer. There, if we did an experiment to determine that path (fig. 3), the particle will either be found in the right-hand path with probability p_{1R} or in the left hand path probability p_{1L} ($p_{1L} + p_{1R} = 1$). Hence, the total lack of information about the particle's path through the interferometer is

$$H_1 = -p_{1L} \text{ld } p_{1L} - p_{1R} \text{ld } p_{1R}, \quad (3)$$

where ld designates the dual logarithm, i.e. the logarithm to the basis two*. Evidently, these probabilities are connected to the respective probability amplitudes via $p_{1L} = |a_{1L}|^2$ and $p_{1R} = |a_{1R}|^2$.

These amplitudes are both of magnitude $1/\sqrt{2}$

* Von Neumann's definition of the entropy of a quantum system as $-\rho \log \rho$ leads to a zero value for any pure state. This is operationally not significant for the present situation, because it does not distinguish between cases with different amplitudes of the interferometer beams.

after the half-silvered beam splitter. Therefore, after the absorber they are

$$\begin{aligned} a_{1L} &= i/\sqrt{1+e^{-\alpha}}, \\ a_{1R} &= 1/\sqrt{1+e^{\alpha}}, \end{aligned} \quad (4)$$

normalized to unity. Here, $e^{-\alpha}$ is the intensity attenuation of the right-hand beam due to the partial absorber. Inserting this into eq. (2) we obtain a quantitative measure for the lack of information on the particle's path through the interferometer as a function of the amplitudes of the two beams. At extreme cases, the lack of information is maximal and equal to one bit if both amplitudes are equal and it is zero if either amplitude vanishes.

We will now also use the lack of information concept to characterize quantitatively the interference contrast. From the in-interferometer amplitudes of eq. (4) the amplitudes of the outgoing beams may be calculated as

$$a_{2L} = \frac{i}{\sqrt{2}} (a_{1L} - i e^{i\chi} a_{1R}) \quad (5)$$

for the left-hand beam behind the interferometer and

$$a_{2R} = \frac{1}{\sqrt{2}} (a_{1L} + i e^{i\chi} a_{1R}) \quad (6)$$

for the right-hand beam. In these equations χ is the phase shift introduced onto the right-hand beam inside the interferometer. Varying χ both amplitudes can be changed and the resulting variation of these intensities clearly is a measure of the interference contrast. Specifically, we obtain for the maximum and the minimum amplitudes of either beam

$$\begin{aligned} |a_{2\max}| &= \frac{1}{\sqrt{2}} (|a_{1L}| + |a_{1R}|), \\ |a_{2\min}| &= \frac{1}{\sqrt{2}} (|a_{1L}| - |a_{1R}|), \end{aligned} \quad (7)$$

these extreme values being achieved by choosing either $\xi = 0$ or $\chi = \pi$. In either case we may define the probability $p_{2\max} = |a_{2\max}|^2$ as the probability for finding the particle in the maximum intensity beam and $p_{2\min} = |a_{2\min}|^2$ as the probability for

finding the particle in the minimum intensity beam. Therefore, the quantity

$$H_2 = -p_{2\max} \text{ld } p_{2\max} - p_{2\min} \text{ld } p_{2\min} \quad (8)$$

is a useful measure of the information content of the interference pattern. Specifically, it is the lack of the information about the path the particle will be found in after the interferometer with the relative phase χ adjusted such as to give maximum difference of the intensities of the outgoing beams.

It follows that the two types of information characterized by eqs. (3) and (8) respectively are interdependent since they are connected via eq. (7). The resulting functional dependence of H_2 on H_1 is shown in fig. 4. This figure is a quantitative representation of the continuous correspondence principle.

The solid line in fig. 4 corresponds to the ideal situation where the particle's state can be described by a state vector and where eq. (7) applies. In the general cases of a real experiment deviations from this situation will arise resulting in a characterization of the particle's state by a density matrix instead of a state vector. If such is the case, the resulting points will lie in the area above the solid curve in fig. 4, i.e. further lack of information will result. This latter lack of information now is

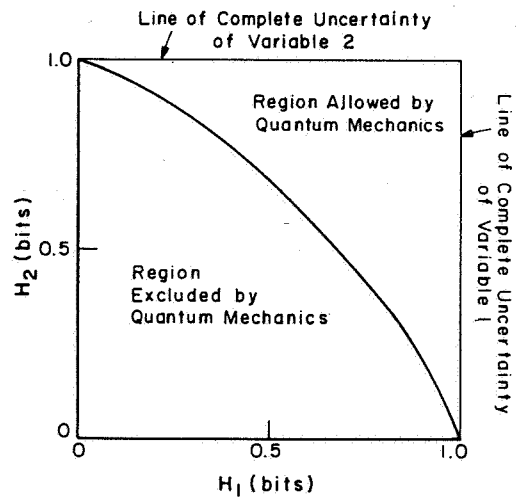


Fig. 4. The continuous quantitative correspondence principle: the lack of information H_2 contained in the interference pattern in relation to the lack of information about the particle's path through the interferometer.

related to a more classical lack of knowledge as contrasted to its quantum counterpart discussed above which is a consequence of complementarity. In that context it is worthwhile to point out the meaning of the two extreme borderlines in fig. 4. The line $H_1 = 1$ bit describes the case where we have complete lack of information about the particle's path through the interferometer independent of interference contrast. Operationally, this means that both beams inside the interferometer are equally bright with the interference contrast being reduced by experimental disturbances. The line $H_2 = 1$ bit means that no interference contrast results no matter what the relative amplitudes of the beams inside the interferometer are, i.e. the interferometer is not working at all in that situation.

In a related experiment, Rauch and Summhammer [10] measured the interference contrast in an experiment with a partial absorber in the beam and obtained full agreement with the quantum mechanical description. Measuring the interference contrast with a chopper in one beam such that the beam was either open or closed at a given time results in a reduced interference contrast as compared to the continuous absorption case. Or, in the language of information theory, partial lack of information about each particle's path is not equivalent with no lack of information for a frac-

tion of the particles and total lack of information for the others.

4. Spinor symmetry and complementarity

Early neutron interferometry experiments concerned the verification of the phase change of a spinor wave function subject to a rotation by 2π , i.e. the 4π symmetry of spinors [11,12]. The experiments were done such that one of the two interfering beams was subject to a magnetic field causing Larmor precession (fig. 5). Various comments were raised with respect to the interpretation of these experiments [13,14]. One type of comment was implying that the experiment did not show that the neutron spin was actually rotating. According to that viewpoint, the whole reference to rotation would only be a result of interpreting the action of the magnetic field B as Larmor precession such as

$$\alpha = g \int B dt \quad (9)$$

and then writing the rotation operator as

$$R = e^{i\sigma \cdot \alpha / 2}, \quad (10)$$

where α is the rotation vector parallel to the magnetic induction. Thus, in that interpretation, rotation would only arise because of the implicitly

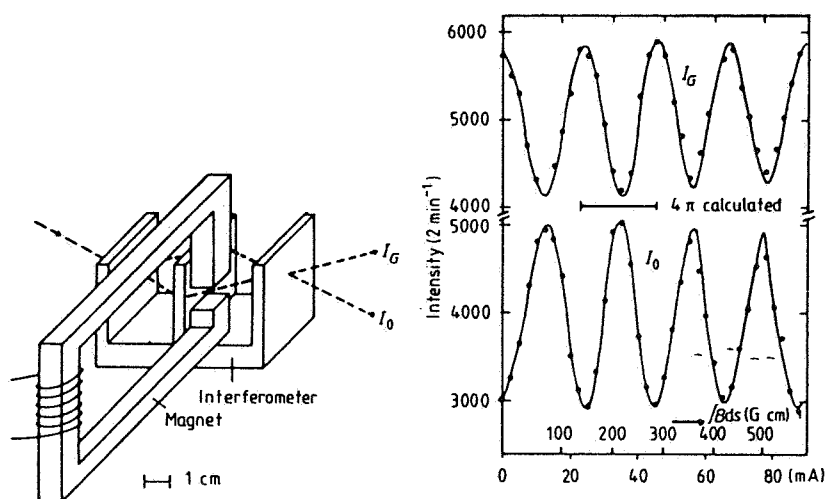


Fig. 5. Measurement of spinor rotation symmetry: experimental principle (left) and interference pattern (right) showing 4π -periodicity.

assumed validity of the quantum mechanical way of the description of rotations, the very feature to be tested by the experiment. Yet, we submit, the observation of the interference effect, here the superposition of a rotated and an unrotated spinor, is not complementary to the relative angle of rotation. This can be shown by dividing the total rotation into smaller individual ones (fig. 6) and by inserting after each rotation stage a spin-measuring device, conveniently a Stern–Gerlach setup on the gedanken level. Discussing this gedanken experiment in the language of information theory as presented above, we first analyze the simple case of a beam described by the general spinor

$$\Psi = \begin{pmatrix} a^+ \\ a^- \end{pmatrix} \quad (11)$$

entering a Stern–Gerlach apparatus oriented along the z -direction. In this case we can again define the lack of information about the z -component of the particle's spin as

$$H_S = -p^+ \text{ld } p^+ - p^- \text{ld } p^-, \quad (12)$$

where $p^+ = |a^+|^2$ and $p^- = |a^-|^2$. Hence, as expected, for a particle polarized within the x - y plane ($|a^+| = |a^-|$) the lack of information about the z -component of its spin is maximal ($H_S = 1$ bit), while for either p^+ or p^- equal to unity H_S vanishes, i.e. measurement of the z -spin does not lead to new information.

Analyzing the interferometer experiment with

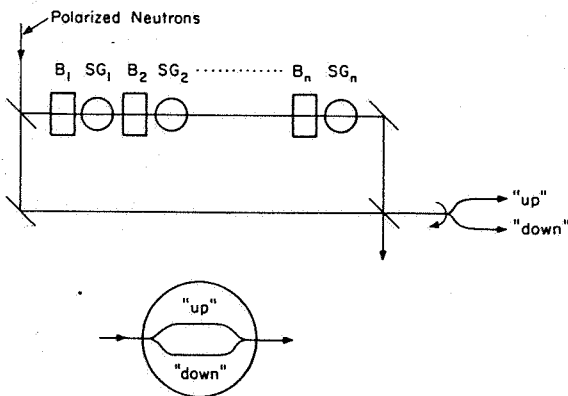


Fig. 6. Gedanken experiment arrangement for tracing the successive neutron spin rotation by the individual magnetic fields B_i using the Stern–Gerlach magnets SG_i .

one Stern–Gerlach analyzer in each of the beam paths, we may identify the amplitudes a_{1L}^+, a_{1L}^- as the spin-up and the spin-down amplitude in the left-hand beam respectively and analogously the amplitudes a_{1R}^+, a_{1R}^- for the right-hand beam. It is relevant to note that the analyzing direction of the Stern–Gerlach analyzers need not be the same for the two beams.

$$H_{1L} = -p_{1L}^+ \text{ld } p_{1L}^+ - p_{1L}^- \text{ld } p_{1L}^-, \quad (13)$$

where $p_{1L}^\pm = |a_{1L}^\pm|^2 / (|a_{1L}^+|^2 + |a_{1L}^-|^2)$, and analogously for the right-hand beam. It is now evident that this lack of information about the particle's spin can be made zero if the corresponding Stern–Gerlach apparatus is oriented such that the neutrons are in one of its spin eigenstates. This fact can even be operationally checked, because in that case a detector may be placed into the other path of the Stern–Gerlach apparatus which will never register a neutron. Therefore, if we then bring the neutron onto its original path, the interference pattern may still be observed. The lack of information about the neutron's path through the interferometer (right-hand vs left-hand) is still maximal. The spin determination procedure does not violate complementarity considerations because the essence of the determination of the relative spin rotation angle here is the non-detection of the neutron in the detector path of the modified Stern–Gerlach device.

The other comment raised concerns the property that the 4π rotational symmetry of spinors is a direct consequence of the spinor description of Larmor precession, i.e. already the two spin states experience phase shifts of $+\alpha/2$ and $-\alpha/2$ respectively in order to result in an α precession angle being the phase difference between the spin states. This is certainly correct since many of these fundamental experiments demonstrate a principle deduced from indirect evidence at a more complex level. Yet the objection raised above is certainly valid here, i.e. the non-observability of any relative rotation in Larmor precession certainly limits its usefulness as implication of spinor symmetry.

Also, as a matter of principle, possible future realizations of that experiment using helical fields would be subject to the same analysis and possible objections as the experiment already performed.

This follows, because even in such experiments in a strict sense the relation to rotation again is obtained via recourse to quantum mechanical analysis of the experiment, unless here too it is explicitly demonstrated that the neutron spin is actually rotating.

5. Spin superposition and complementary

In a series of rather involved experiments [16–18], the interferometer was used as a device to demonstrate the quantum spin superposition laws (fig. 7). There one observed the coherent superposition of two oppositely, say along the z -axis, polarized states and demonstrates that the resulting state is polarized along a direction orthogonal to it, i.e.

$$|+x\rangle = \frac{1}{\sqrt{2}}(|+z\rangle + |-z\rangle). \quad (14)$$

The experiment consisted of a spin flipper (either static [16,17] or dynamic [18]) in one beam of an interferometer operating with polarized neutrons.

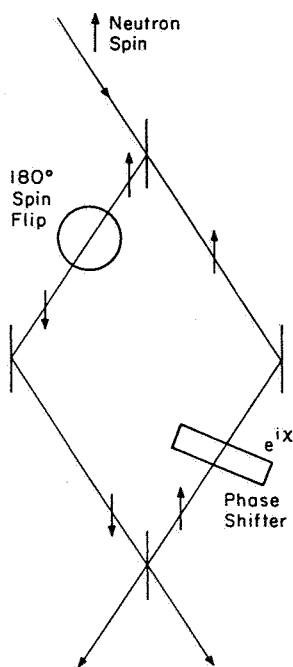


Fig. 7. The principle of the spin superposition experiment.

The polarization of one of the emerging beams is then measured as a function of an additional phase shift introduced between the $|+z\rangle$ and $|-z\rangle$ states. In agreement with quantum prediction, the neutron polarization vector is found in the emerging beam to be oriented within the x - y plane and the polarization along the z -axis is found to vanish.

We analyze now the possible spin measurements of the emerging beams in more detail. In particular we note that the experimenter has the choice of measuring the spin of the outgoing beams along any arbitrary axis. For simplicity we will focus on one of the outgoing beams only. Given the state of eq. (14), measuring the spin along the x -direction gives the result that, in the ideal case, all neutrons are in a spin eigenstate with respect to the x -axis. This is because such a measurement constitutes an observation of the interference effect between both routes through the interferometer and it is evident that here again we have no means of detecting which path the neutron actually took. The specific signature of the interference effect is the property, that the neutron spin can be rotated within the x - y plane by introducing a phase-shift χ between the two beams inside the interferometer [19,20]. The resulting intensity oscillation behind an x -spin analyzer is equivalent to the interference oscillations in a standard interferometer experiment.

On the other hand, if we measure the neutron spin along the z -axis, no interference effect will be observed. In particular, the measured count rates are independent of the phase shift χ as was also seen in the experiment [17]. Yet, the fact that an individual neutron is counted in the up or in the down channel of our spin analyzer contains the information about the neutron's path through the interferometer. For the specific case of the arrangement shown in fig. 7, a count in the up channel implies the right-hand path through the interferometer and a count in the down channel implies the left-hand path. This is an explicit example of how different classes of complementary observables are connected to each other. In our case it implies that the complementarity between different orthogonal spin directions is equivalent to the complementarity between path

determination and interference.

The identification of the measurement of the z -component of the neutron spin after the neutron has already left the interferometer with a determination of the path the neutron took through it can also be justified by operational reasoning. Suppose we measure the $+z$ spin count rate of the outgoing beam only. Then we will observe, that this count rate is independent of whether the left-hand beam path through the interferometer is open or not. Even more, if we close the right-hand beam path with an absorber the count rate for the ideal interferometer will be reduced to zero, independent of whether the left-hand beam path is open or not. Hence, we submit, it is operationally reasonable to conclude that all those neutrons which are counted by the spin-up detector actually follow the right-hand path through the interferometer and vice versa.

It is evident that the information theoretic approach proposed in the last few sections is also applicable to the present situation. Particularly, measuring the neutron spin along some arbitrary direction constitutes another experimental realization of partial simultaneous observation of two complementary quantities.

Finally we note that one could place the spin analyzer at any distance behind the interferometer. Therefore the particle's path can be determined with arbitrary time delay after the particle's passage through the interferometer system resulting in an explicit variant of a delayed-choice situation [21]. Evidently the very reason why this is possible is the complementarity feature pointed out above.

6. Concluding comments

The quantitative approach to complementarity presented in this paper is intimately tied to thorough operational analysis of physical concepts and their experimental meaning. This kind of operational analysis uniquely characterizes the intellectual atmosphere in Cliff Shull's neutron diffraction group at MIT. It is very much the result of his insistence on not using terms whose meaning is not clearly understood. The present author wishes

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References

- [1] H. Rauch, W. Treimer and U. Bonse, *Phys. Lett.* A47 (1974) 369.
- [2] A. Zeilinger, C.G. Shull, M.A. Horne and G.L. Squires, in: *Neutron Interferometry*, U. Bonse and H. Rauch, eds. (Oxford Univ. Press, London, 1979) p.48.
- [3] A. Zeilinger, C.G. Shull, J. Arthur and M.A. Horne, *Phys. Rev. A* 28 (1983) 487.
- [4] C.G. Shull, paper presented at Int. Conf. on Neutron Scattering, Sante Fe, August 1985, *Physica* 136B (1986) 126.
- [5] C.G. Shull, *Phys. Rev. Lett.* 21 (1968) 1585.
- [6] C.G. Shull, *J. Appl. Cryst.* 6, (1973) 257.
- [7] W.K. Wootters and W.H. Zurek, *Phys. Rev. D* 19 (1979) 473.
- [8] A. Zeilinger, *Am. J. Phys.* 59 (1981) 882.
- [9] C.E. Shannon, *Bell. Syst. Techn. J.* 27, (1948) 279.
- [10] H. Rauch and J. Summhammer, *Phys. Lett.* 104 (1984) 44.
- [11] H. Rauch, A. Zeilinger, G. Badurek, A. Wilfing, W. Bauspiess and U. Bonse, *Phys. Lett.* A54 (1975) 425.
- [12] S.A. Werner, R. Colella, A.W. Overhauser and C.F. Eagen, *Phys. Rev. Lett.* 35 (1975) 1053.
- [13] J. Byrne, *Nature* 275 (1978) 189.
- [14] F. Mezei, in: *Neutron Interferometry*, U. Bonse and H. Rauch, eds. (Oxford Univ. Press, London, 1979) p. 265.
- [15] A. Zeilinger, *Nature* 294 (1981) 544.
- [16] G. Badurek, H. Rauch, J. Summhammer, U. Kischko and A. Zeilinger, *J. Phys.* A16 (1983) 1133.
- [17] J. Summhammer, G. Badurek, H. Rauch, U. Kischko and A. Zeilinger, *Phys. Rev.* A27 (1983) 1.
- [18] G. Badurek, H. Rauch and J. Summhammer, *Phys. Rev. Lett.* 51 (1983) 1015.
- [19] A. Zeilinger, in: *Neutron Interferometry*, U. Bonse and H. Rauch, eds. (Oxford Univ. Press, London, 1979) p. 241.
- [20] A. Zeilinger, *Z. Physik* 325 (1976) 97.
- [21] A. Zeilinger, *J. Physique* 45 (1984) C3-213.