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A BELL-TYPE EPR EXPERIMENT USING LINEAR MOMENTA

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ABSTRACT

A new experiment is described for which Bell's theorem applies. Unlike existing experiments which use spin, the new proposal uses linear momentum.

1. INTRODUCTION

Bell's[1] theorem on the thought experiment of Einstein, Podolsky and Rosen[2] actually uses Bohm's[3] version of that experiment, which involves correlations in the polarizations of a pair of particles. Moreover, all realizations of the EPR experiment have been polarization-type experiments. Yet, as already pointed out by Bell, his theorem would also apply to other, suitably correlated, two-valued observables of spatially separated systems. Here we present an explicit example involving linear momenta of a pair of particles.

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2. PROPOSAL

Consider a source emitting pairs of correlated particles, labeled 1 and 2, whose state in momentum space is the following superposition of products of momentum eigenstates,

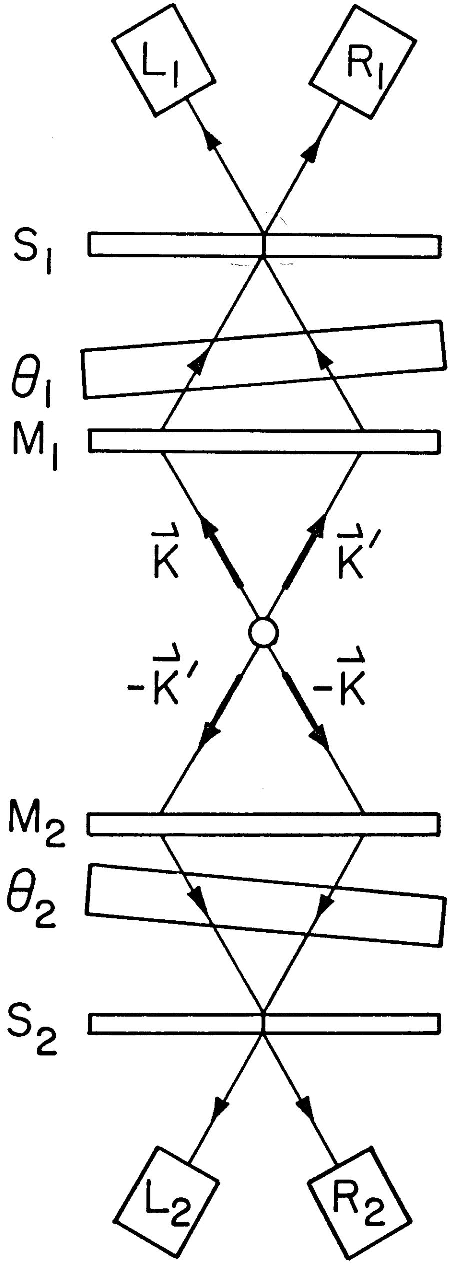
$$|\psi\rangle = \int d^3\vec{k}\ |\vec{k}\rangle_1 |-\vec{k}\rangle_2\ \delta(|\vec{k}| - k_o), \tag{1}$$

where δ is the Dirac delta function and k_o is a constant. Note that, although the wavevectors of the two particles have definite and equal magnitudes, k_o, and are definitely oppositely directed, the direction of emission is uncertain in that uniquely quantum mechanical sense: namely, the state (1) is a coherent superposition, not a statistical mixture, of the various emission directions. An example of such a source is provided by positronium annihilation at rest, which yields a pair of photons in state (1).

Suppose the source is placed at the center of the arrangement shown in the Figure. There, M_1, M_2, S_1 and S_2 are four crystal plates in Laue diffraction geometry with thicknesses chosen such that M_1 and M_2 are total Bragg mirrors for radiation incident at the Bragg angle and such that S_1 and S_2 are Bragg analogs to half-silvered mirrors for this radiation. The Figure also shows a phase plate arranged so as to introduce a relative phase θ_1 (θ_2) between the left and right routes through the top (bottom) portion of the apparatus. Let $|\vec{K}\rangle_1$ and $|\vec{K}'\rangle_1$ denote those momentum eigenstates of particle 1 which fulfill the Bragg condition at crystal M_1. With certain specific choices for the crystal thicknesses, the effects of diffraction by M_1 and S_1 and passage through the phase plate θ_1 are given by

$$|\vec{K}\rangle_1 \longrightarrow \frac{1}{\sqrt{2}}\left[-|L\rangle_1 + i|R\rangle_1\right] e^{i(\theta_1/2)} \tag{2}$$

$$|\vec{K}'\rangle_1 \longrightarrow \frac{1}{\sqrt{2}}\left[i|L\rangle_1 - |R\rangle_1\right] e^{-i(\theta_1/2)} \tag{3}$$


L1
R1
S1
θ1
M1
K
K′
-K′
-K
M2
θ2
S2
L2
R2

where $|L\rangle_1$ means particle 1 lands in detector L_1, etc. Similar evolutions hold for particle 2 upon passage through M_2, S_2, and θ_2. Since only radiation incident at the Bragg angle is diffracted by the plates, the continuous sum in state (1) collapses to just two terms so that the effective illumination of M_1 and M_2 is

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left[|\vec{K}\rangle_1 |-\vec{K}\rangle_2 + |\vec{K}'\rangle_1 |-\vec{K}'\rangle_2 \right] . \qquad (1a)$$

All other terms in (1) can be kept from contributing at the detectors by suitably placed shields.

The evolutions (2) and (3) for particle 1 and the similar evolutions for particle 2 imply that the state (1a) becomes, at the detectors,

$$|\psi\rangle = \frac{1}{\sqrt{2}} \Big[\{i \sin [(\theta_1-\theta_2)/2]\}\{|L\rangle_1|R\rangle_2 - |R\rangle_1|L\rangle_2\}$$
$$+ \{-i \cos [(\theta_1-\theta_2)/2]\}\{|L\rangle_1|L\rangle_2 + |R\rangle_1|R\rangle_2\} \Big] . \qquad (1b)$$

Thus the probabilities to register a particle unconditionally in a detector are

$$P(L_1) = P(R_1) = P(L_2) = P(R_2) = \frac{1}{2} . \qquad (4)$$

The conditional probabilities $P(R_2|R_1)$, etc., of registering a count in detector R_2 given a count in detector R_1, etc, are

$$P(R_2|R_1) = \frac{1}{2} \left[1 + \cos (\theta_1 - \theta_2) \right] , \qquad (5)$$

and similarly for the other conditionals. These probabilities are identical to those of the singlet spinstate considered by Bell[1)] except that the relative phases θ_1 and θ_2 play the role of the angular orientations of the Stern-Gerlach analysers.

3. CONCLUSION

Bell's theorem applies to the newly proposed experiment in its ideal form. The implications of non-ideal features of a real experiment (e.g. variation of Pendellosung length with off-Braggness, deviations from exact momentum anticorrelation due to motion of the positronium, polarization effects) will be discussed in a forthcoming paper.

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5. REFERENCES

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