Neutron Propagation in Moving Matter: The Fizeau Experiment with Massive Particles

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(Received 13 April 1981)

Using a two-slit neutron interferometer, we have measured the shift of the interference pattern induced by the motion of a quartz rod. The measured shift is found to be in agreement with the quantum mechanical prediction derived from a relativity argument. Some peculiarities of the case of massive particles as compared with the classic Fizeau photon case are discussed.

PACS numbers: 07.90.+c

Following the invention of the technique of neutron interferometry many interesting experiments were performed or proposed.[1] Among those proposed was an analog of Fizeau's famous experiment[2] carried out more than a century ago, in which he observed the phase shift of a light wave due to the motion of its medium. Its neutron analog may be discussed[3] with use of as a starting point the fact that the phase difference between the two beams of an interferometer has to be a relativistic invariant. Therefore, we analyze first the situation shown in Fig. 1, in which a phase shifting slab of thickness D and refractive index $n(k)$ is moving in one of the beams of an idealized interferometer. The following expression is obtained for the phase difference calculated in the rest frame of the slab:

$$\Phi(\vec{k}) = D\{[n^2(k)k^2 - k_p^2]^{1/2} - k_o\}. \tag{1}$$

Here $\vec{k}$ is the wave vector of the radiation in that frame, and k_p and k_o are its components parallel and orthogonal to the surface of the slab. With use of now the property of relativistic invariance of phase differences, the effect due to the motion of the slab is then found as the difference between two expressions of the type of Eq. (1), one with $\vec{k}'$, the wave vector in the frame of the moving slab, the other with $\vec{k}$, the wave vector in the frame of the slab at rest, i.e., the laboratory frame. For the case in which the velocity vector $\vec{v}$ of the slab is parallel to both the radiation $\vec{k}$ vector and to the surface normal of the slab, the Fizeau effect is given by

$$\Delta\Phi = D[n(k) - 1 + k\, dn(k)/dk](k' - k). \tag{2}$$

Equation (2) holds for slab velocities much smaller than the velocity of the radiation.

For neutrons, of kinetic energy $E = \hbar^2k^2/2m$, the refractive index may be written as $n = (1 - V/E)^{1/2}$, where $V = (2\pi\hbar^2/m)Nb_c$ is the mean potential inside the material, which results from the Fermi pseudopotential (b_c is the coherent neutron-nuclear scattering length, and N is the atom density). Hence we obtain for the Fizeau phase shift

$$\Delta\Phi = kD(v/v_n)(n-1)/n, \tag{3}$$

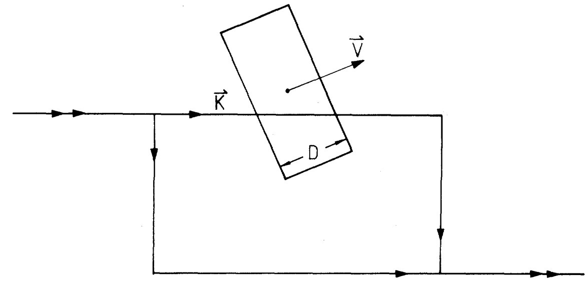

FIG. 1. Phase-shifting slab moving with velocity v in one of the beams of an interferometer.

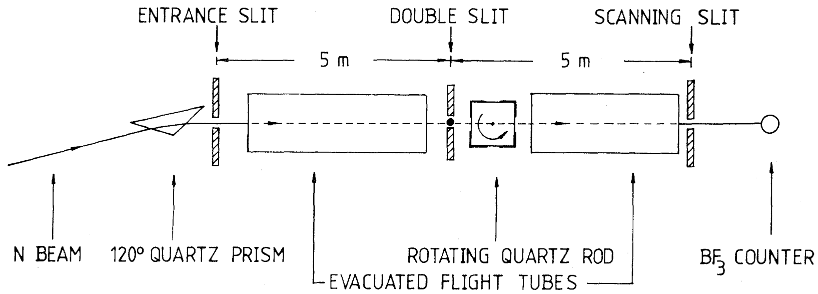

FIG. 2. Overall layout of experiment (not drawn to scale).

where v_n is the vacuum neutron velocity. For thermal and cold neutrons ($V/E \ll 1$) we finally obtain

$$\Delta\Phi = -kD(V/2E)v/v_n . \tag{4}$$

It is interesting to note that in deriving this result we found that, for neutrons, the dispersive term in Eq. (2) is twice as large as the other term and of opposite sign. This is in contrast to light waves, where the dispersive term is only a small correction, as first derived by Lorentz[4] and measured by Zeeman.[5]

Initially we attempted to demonstrate the neutron Fizeau effect by using a perfect crystal interferometer of the type developed by Rauch, Treimer, and Bonse.[6] The motion was produced by oscillating two plates in antiphase, one in each of the beam paths of the interferometer. Unfortunately, because of the extremely high sensitivity of that type of interferometer to phase disturbing influences, no interference pattern was observed with the plates in motion.

The experiment was then redesigned to be less sensitive to mechanical and thermal disturbances, namely for the optical bench assembly[7] set up on the cold-neutron beam H-18 of the high-flux research reactor of the Institut Laue-Langevin in Grenoble. On this optical bench a double-slit neutron-diffraction experiment was in progress.[8] The incoming neutrons were monochromated by prism refraction and entered the apparatus through a 20-μm wide entrance slit (see Fig. 2). The central diffraction peak of this slit coherently illuminated the double-slit assembly located 5 m downstream. The double slit assembly consisted of a single slit of 146 μm width in the center of which was mounted a boron wire of 102 μm diam. Thus two slits resulted, each of 22 μm nominal width. A further 5 m downstream the diffracted intensity profile was scanned with a BF_3 counter located behind a 20-μm exit slit.

The mean wavelength of the neutrons, measured by a time-of-flight technique, was 18.45 Å with a spread of ±1.40 Å.

Since in this type of interferometer the separation of the two beams is quite small, it would have been difficult to move material in one beam path without encroaching on the other. For this reason a quartz rod with a square cross section was designed to be rotated immediately behind the two-slit assembly (Fig. 3). As the Fizeau phase difference depends only on the velocity difference of the phase-shifting material in the two beams, the centering of the rod relative to the double-slit assembly is not critical. Thus we may use in the further analysis the effective velocity components $v = \mp \Omega y$ of the material traversed by the two beams. The component of the motion perpendicular to the direction of the neutron beams turns out to have no effect on the phase of the neutrons traversing it. According to Eq. (4), the extra phase introduced by virtue of the translatory motion is

$$\Delta\Phi = kD(V/2E)\Omega y/v_n , \tag{5}$$

i.e., a continuous lateral phase gradient is introduced by the rotating rod which manifests itself

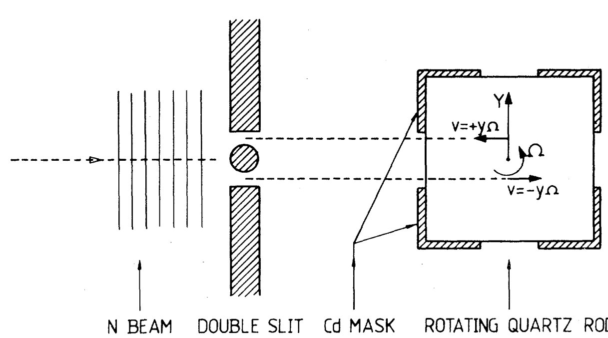

FIG. 3. Schematic of the rotating quartz rod and the double slit.

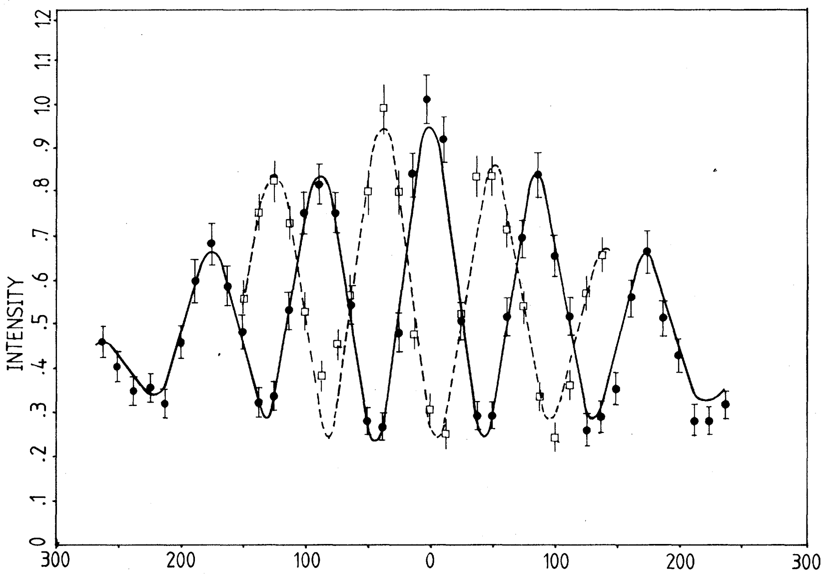

FIG. 4. Double-slit interence pattern shifted by rotation of quartz rod. Solid line, $\Omega = 0$; dotted line, $\Omega = 100$ Hz.

as an angular deflection, α, of the whole interference pattern given by

$$\alpha = k^{-1} d(\Delta\Phi)/dy = D(V/2E)\Omega/V_n . \tag{6}$$

Hence the rotating rod acts like a refracting wedge. As shown in Fig. 3, the corners of the prism were covered by cadmium to define the neutron paths and to restrict the effect of variation of the Fizeau phase with the angular position of the prism. The prism was $\simeq$10 mm square, made of "Optosil" quality quartz characterized by $n - 1 = V/2E = 1.88 \times 10^{-4}$ for 18.45-Å neutrons. Hence, at this wavelength, the expected displacement of the interference pattern was 28.05 μm/100 Hz. Figure 4 gives the experimental result of the two-slit diffraction pattern as measured with the quartz rod at rest together with one obtained with a rotation speed of 100 Hz. The shift of the interference pattern is clearly visible. Measurements were performed at rotational speeds of 0, 14, ±50, and ±100 Hz maintained to better than 1% during each run. Figure 5 shows the theoretical straightline relation between the shift of the central peak and the rotational speed, together with the experimental points, determined by cross correlating the displaced patterns with the 0-Hz pattern. The slope of the straight line fitted through the experimental points is 27.8 ± 2.5 μm/100 Hz which compares well with the theoretically expected value.

The result obtained above in a wave-optical approach (that the rotating rod acts like a refracting wedge) may also be derived in the particle picture. We observe that during the neutron transit time, $\Delta t = D/v_n$, the rod, and hence the exit face, has turned through an angle $\Delta\theta = \Omega\,\Delta t$. The resulting effective thin wedge leads to a deflection, $\alpha = (n-1)\Delta\theta$, in agreement with Eq. (6).

A remarkable consequence of these considerations is that no neutron Fizeau effect is to be observed if the geometric boundary of the phase shifter is not moving, e.g., in the classic Fizeau experiment with a liquid flowing in pipes, or for a sideways-moving plate as in the experiment by Macek, Schneider, and Salamon.[9] Deviations from that behavior and the invalidity of Eqs. (3) and (4) are to be expected for a velocity-dependent potential or near neutron resonances.

Equation (1) for the phase change is based on a relativity argument. Another calculation leading to the same result may be carried out by tracing the evolution of wave packets in the laboratory frame. We thus find that, in our experiment, the neutron undergoes a frequency (as well as a wavelength) change at the entrance and exit surfaces of the medium. In fact, the contribution of the frequency change to the phase change is of the same order of magnitude as that due to the wavelength change. The $\vec{k}$ and the ω changes may both be found directly by applying a Galilean transformation to the corresponding quantities in the moving frame.[10] Hence our experiment may also be viewed as a demonstration of the transforma-

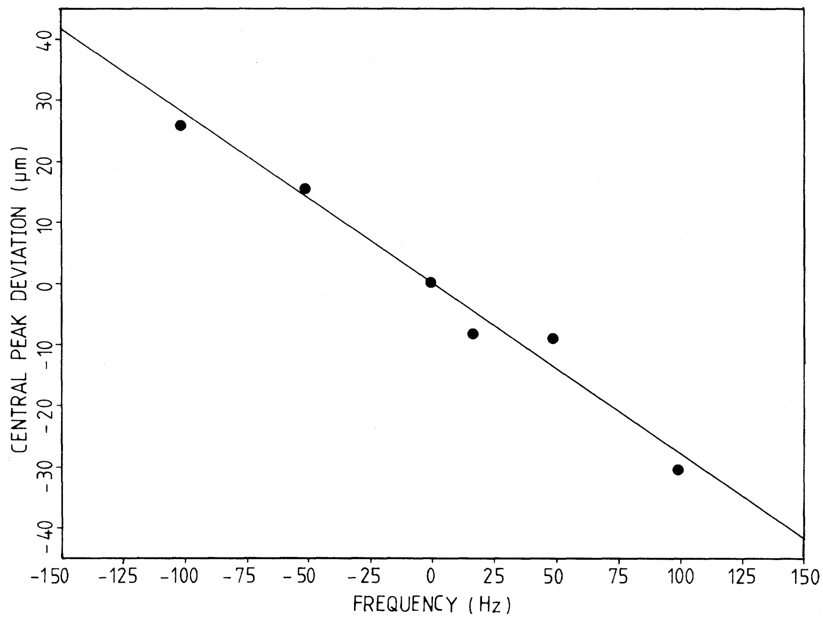

FIG. 5. Shift of interference pattern as a function of rotational speed. Theoretical line with experimental points.

tion laws of the de Broglie wavelength and frequency of a massive particle. In a similar way, Zeeman[5] interpreted his experiment as a demonstration of the corresponding transformation laws for light as used by Lorentz[4] in his derivation of the dispersion correction term.

We wish to thank M. A. Horne for valuable discussions and J. Kalus for permission to use his apparatus. We are grateful to the directors and staff of the Institut Laue-Langevin for their support and cooperation and particularly to M. Schlenker, U. Kischko, W. Mampe, and F. Gönnenwein for their help. The University of Melbourne group wishes to acknowledge support from the Australian Research Grants Committee. One of us (A.Z.) acknowledges support from the Fonds zur Förderung der wissenschaftliches Forschung (Austria) Project No. 3185; and one of us (W.T.) acknowledges support from Bundesministerium für Forschung und Technologie Project No. 41E0GP.

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