NEUTRON PHASE-ECHO CONCEPT
AND A PROPOSAL FOR A DYNAMICAL NEUTRON POLARISATION METHOD

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ABSTRACT

Experiments in the field of neutron interferometry can be interpreted in terms of a neutron phase-echo concept, which is closely related to the spin-echo concept. Phase echo analysis can be done even with unpolarized neutron beams if coherence between separate beams exists. Furthermore a method is proposed for a dynamical polarization of a neutron beam due to the combination of a momentum separation and a spin overlapping part. This system consists of a multistage h.f. flipper and one half of a spin-echo spectrometer and allows an optimal use of available neutrons.

1. Phase echo concept

In neutron spin-echo systems /1/ the Larmor precessions of the neutron spin within two separate parts of the instrument are balanced. In analogy to this technique we denote a system where the phases of neutron waves along two paths of an instrument compensate each other as a neutron phase-echo system. Because no phase sensitive reflections are known we need a reference beam to detect phase differences. First we assume an idealized system as sketched in Fig. 1 and plane waves throughout the instrument.

The incident plane wave ($\vec{k} = k_x\hat{x} + k_y\hat{y}$) is split into two waves with different k_y-vectors (k_y and $-k_y$) and mirrors change both y-components by 180 degree. The analyser is assumed to allow the observation of the interference pattern independent of its x-position. The focusing conditions are given by the geometrical optics. The maximum phase difference due to the different y-component ($\chi_y = k_y \cdot D$) is balanced at the analyser position for all wave lengths. A phase shift in the x-directions (χ_x) can be obtained by a phase shifting material with an index of refraction

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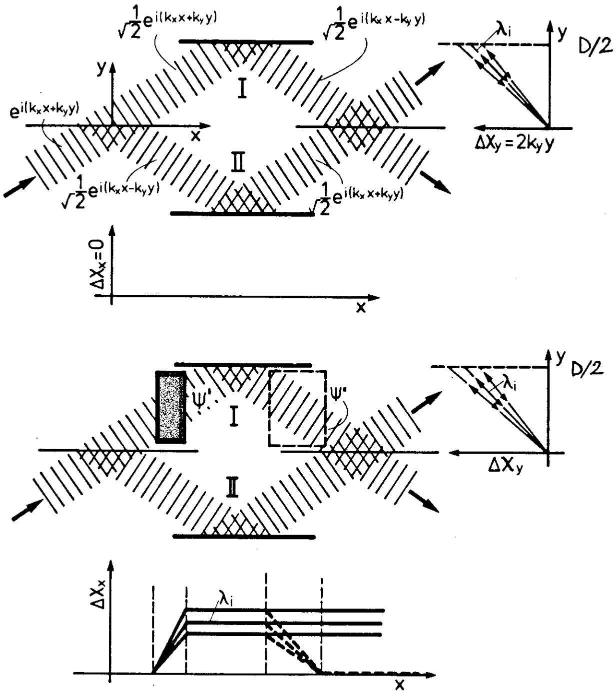

Fig. 1: Sketch of an idealised interferometer composed of conventional optical components without (above) and with samples in the beams (below)

$n = 1 - \lambda^2 N b_c/(2\pi)$ (N ... particle density, b_c ... coherent scattering length, λ ... neutron wave length). This phase shift changes the wave behind the material of thickness D_{eff} along the beam path according to:

$$\psi_o' = \psi_o\, e^{+i(k'-k_o)D_{eff}} = \psi_o\, e^{-ik_o(1-n)D_{eff}} =$$

$$= \psi_o\, e^{-iN_1 b_{c1} D_{eff,1}\lambda} = \psi_o\, e^{i\chi_x} \qquad (1)$$

A focusing condition can be obtained also for this x component if another phase shifting material with an opposite scattering length is inserted in a way that

$$\chi = \chi_1 + \chi_2 = 0$$

or $$b_{c1} N_1 D_{eff1} = - b_{c2} N_2 D_{eff2} \qquad (2)$$

One recognizes that there exists a focusing condition which is independent of the wave length - similar to spin-echo systems.

The experimental realization can be achieved with the perfect crystal interferometer /2-5/ (Fig. 2). The relevant transfer functions can be taken from the literature /6/

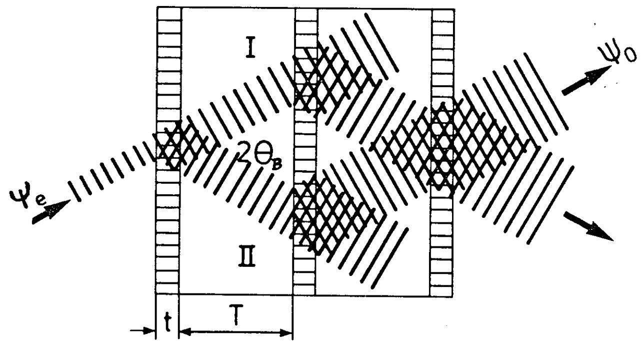

Fig. 2: Sketch of the perfect crystal interferometer

and are given below for the symmetric Si-(220) reflection and the beam in forward direction.

$$\psi_o = \psi_o^{I} + \psi_o^{II} = [v_o(y)\ v_H(y)\ v_{-H}(-y) + v_H(y)\ v_{-H}(-y)\ v_o(y)] \cdot$$

$$\cdot \exp\,[-\,2\,\pi i\ y\ (T+t)/\Delta o]\ \psi_e \tag{3}$$

with $v_o(y) = [\cos A\sqrt{1+y^2} + \frac{iy}{\sqrt{1+y^2}} \sin A\sqrt{1+y^2}]\ \exp(iPt)$

$$v_H(y) = -i\,\frac{\sin A\sqrt{1+y^2}}{\sqrt{1+y^2}}\ \exp\,(iPt)$$

$$P = -\frac{\pi y}{\Delta_o} - \frac{2\pi}{D_\lambda^{Si}\cos\theta_B}$$

$$y = \frac{(\theta_B - \theta)\,\pi \sin 2\,\theta_B}{\lambda^2\, N_c^{Si}\, b_c^{Si}}$$

$$\Delta_o = \frac{\pi t}{A} = \frac{D_\lambda \cos\theta_B}{2} = \frac{\pi \cos\theta_B}{b_c^{Si}\, N^{Si}\lambda}$$

For the balanced interferometer this relation reduces to

$$\psi_o^{I} = \psi_o^{II} \tag{4}$$

which remains still valid if spherical wave theory is used /7,8/. Equ. (4) shows the important feature that no y (or λ) dependence exists and therefore a divergent incident beam can be used.

A phase shifting material introduced into beam I changes the wave function to $\psi_o^{I'}$ (equ. 1) and one obtains from equ. (3) the intensity behind the interferometer as:

$$I = |\psi_o^{I'} + \psi_o^{II}|^2 = \frac{I_o}{2} (1 + \cos\chi) \qquad (5)$$

Due to the wave length spread a dephasing effect (D_ϕ) occurs ($\exp(i\chi) \rightarrow \exp[i(\chi + \Delta\chi)]$. For a Gaussian wave length distribution centered around λ_o and with a half width (fwhm) of $\Delta\lambda$ we get /9/:

$$I = \frac{I_o}{2} (1 + D_\phi \cos\chi_o) \qquad (6)$$

with $D_\phi = \exp\left[- A\,\chi_o^2\,(\Delta\lambda/\lambda_o)^2\right]$

$A^{-1} = 16 \ln 2$

This dephasing effect is shown in Fig. 3 for a beam with a half width of $\Delta\lambda/\lambda_o$ = 5 % and is experimentally observed up to high order interferences /10/. From such measurements the wave length distribution $f(\lambda)$ of the incident beam can be obtained by a simple Fourier transformation of the measured intensity modulation ($c = \chi/\lambda$)

$$f(\lambda) \propto \int_0^\infty I(c) \cos c\lambda \, dc \qquad (7)$$

which is again very similar to the spin-echo procedure /11/. The dephasing effect vanishes ($D_\phi = 1$) if the phase echo condition (equ. 2) is fulfilled.

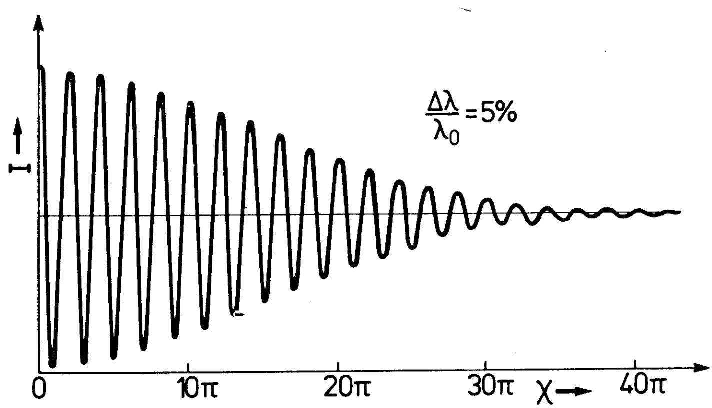

Fig. 3: Loss of contrast due to the dephasing effect of a broad incident wave length spectrum

The magnetic interaction of the neutron introduces an additional phase shift $\vec{\sigma}\,\vec{\alpha}/2$ where $\vec{\sigma}$ are the Pauli spin matrices and $\vec{\alpha}$ a rotation angle ($\alpha = \gamma BD/v$, γ ... gyromagnetic ratio, v ... neutron velocity) around the direction of the magnetic induction B. Characteristic beat effects occur if nuclear (χ) and magnetic phase shifts ($\alpha/2$) exist /12, 13/. Including the dephasing effect we get for unpolarized neutrons

$$I = \frac{I_o}{2}\{1 + \frac{1}{2}\left[D_{\phi_1} \cos(\chi_o + \frac{\alpha_o}{2}) + D_{\phi_2} \cos(\chi_o - \frac{\alpha_o}{2})\right]\} \quad (8)$$

with

$$D_{\phi_1} = \exp\left[-A\,(\chi_o + \frac{\alpha_o}{2})^2\,(\Delta\lambda/\lambda_o)^2\right]$$

$$D_{\phi_2} = \exp\left[-A\,(\chi_o - \frac{\alpha_o}{2})^2\,(\Delta\lambda/\lambda_o)^2\right]$$

and for polarised neutrons we obtain

$$I = \frac{I_o}{2}\left[1 + D_{\phi_1} \cos(\chi_o + \frac{\alpha_o}{2})\right] \quad (9)$$

These relations show that the focusing condition for unpolarized neutrons requires a separate compensation of the nuclear and magnetic phase shifts ($\chi_o = 0$ and $\alpha_o = 0$) while for polarized neutrons nuclear and magnetic phase shifts can compensate each other ($\chi_o + \alpha_o/2 = 0$). Similar effects can be discussed concerning gravitational interaction.

So far strictly elastic processes are discussed because experimental experience is available in this regime only. In future experiments with coherent beams may become feasible even in the inelastic (e.g. phonon) regime. In this case a generalized pair correlation function is related to this effect /14/ and Pendellösungseffects may become visible in the inelastic peaks, too /15/. In principle the large phase shift ($\sim 10^9$) inherent in the y-direction can be used with polylithic perfect crystal interferometers, which are tested up till now for X-rays only /16-18/. A disadvantage for a spectrometric application exists in the complicated response of the relevant parts of the interfermeter due to dynamical diffraction theory. No such difficulties should occur for interferometers for ultracold neutrons because nondispersive optical components can be used.

2. Dynamical neutron polarization

The rather low intensity of polarized neutrons limits their general use for solid state physics application. Various techniques are available to polarise thermal and cold neutrons /19/. In any case only neutrons with the desired spin direction are selected and these often with a rather low efficiency. Therefore the possibility of a dynamical polarization method is discussed. For the proposed system a momentum separation and a spin overlapping part is desired, which means a combination of flippers and of components of a spin-echo system /20/.

Many types of neutron spin-turners are used in polarised neutron physics /19/. Here we discriminate between flippers with a time dependent and a time independent magnetic field $\vec{B}$. The behaviour of the first ones are described by the time dependent Schrödinger equation

$$H\,\Psi(\vec{r},t) = i\,\hbar\,\frac{\partial\Psi}{\partial t} \tag{10}$$

with

$$H = -\frac{\hbar^2}{2m}\Delta - \mu\,\vec{\sigma}\,\vec{B}\,(\vec{r},t)$$

while for static flippers the time independent Schrödinger equation can be used

$$\left(-\frac{\hbar^2}{2m}\Delta - \mu\,\vec{\sigma}\,B(\vec{r}) - E\right)\Psi(\vec{r}) = 0 \tag{11}$$

μ is the magnetic moment, m the mass of the neutron and $\vec{\sigma}$ are the Pauli spin matrices. In most cases where the Zeeman splitting is small compared to the energy spread of the incident beam it is allowed to describe the spin precession in a field $\vec{B}\,(\vec{r},t)$ by the classical Bloch equation

$$\frac{d\vec{s}}{dt} = \gamma\,\vec{s}(t) \times \vec{B}(\vec{r}\,(t),\,t) \tag{12}$$

Due to the interference of the two energy eigenstates ($\pm\mu B_o$) the wellknown Larmor precession with an angular frequency ω_L appears

$$\omega_L = 2\,|\mu|\,B_o/\,\hbar \tag{13}$$

Existing spin-echo systems are described by equ. (12) /1, 21, 22/. The behaviour of the spin within a rotating (or oscillating) field superimposed to a constant guide field is calculated within the rotating frame and the resonance conditions for a complete spin turn are fulfilled if the rotational frequency $\omega = \omega_L$ and the amplitude B_1 of the rotating field is related to the length l of the rotating field and the neutron velocity v_o as

$$\gamma B_1\,l/v_o = \pi(2\,n + 1) \tag{14}$$

Oscillating fields are seen as a superposition of two rotating fields. If we take B_1 as the amplitude of the oscillating field we get for $B_1 << B_o$

$$\gamma B_1 \; 1/v_o = 2\pi \; (2 \; n + 1) \qquad (13 \; a)$$

Small deviations from this value are discussed in the literature /23/.

In the case of time dependent fields the total energy is not conserved and at resonance the energy $\Delta E = \hbar \omega_L = \pm 2 \; |\mu| \; B_o$ is transferred to or from the neutrons to change their potential energy according to the related spin turn. In that situation the change of the kinetic energy of the neutron ($E \rightarrow E \pm \mu B_o$) at the entrance into the guide field is enlarged at the exit to $E \pm 2 \; \mu \; B_o$, which was first mentioned by Drabkin and Zhitnikov /24/. This energy change is rather small compared to the energy of thermal neutrons ($\Delta E \sim 0,12$ µeV for $B_o = 1$ T) and has not been observed experimentally up to now. A multiplication of this effect ($\Delta E_n = n\Delta E$) can be achieved by a multistage system as shown in Fig. 4. Some "non-adiabatic" spin turns between the stages serve for a situation where a spin com-

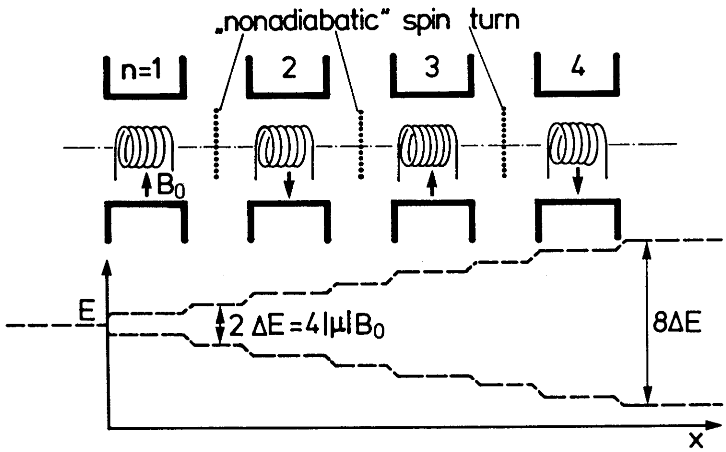

Fig. 4: Multistage system to increase the Zeeman splitting

ponent enters every stage with the same direction to the field. Guide tubes can be used to avoid any loss of luminosity.

A more quantum mechanical treatment describes the neutron by a spinor wave function throughout the device /25-28/. Using some results and a notation similar to that used by Krüger /27/ we write the wave function within the field B_o but before the h.f. field region as

$$\Psi = \begin{pmatrix} \cos\frac{\theta}{2}\, e^{-i\phi/2}\, e^{ik^+x} \\ \sin\frac{\theta}{2}\, e^{i\phi/2}\, e^{ik^-x} \end{pmatrix} e^{-iE\,t/\hbar} \qquad (15)$$

where we have neglected small additional contributions of reflected waves. Behind the h.f. field ($\omega = \omega_L$) but still within the B_o field we have

$$\Psi = \begin{pmatrix} \cos\frac{\theta}{2}\, e^{-i\phi/2}\, e^{ik^+x} \\ \sin\frac{\theta}{2}\, e^{i\phi/2}\, e^{ik^-x} \end{pmatrix} \cos\left(\frac{\mu B_1 l}{\hbar\, v}\right) e^{-iE\,t/\hbar}$$
$$-i \begin{pmatrix} \sin\frac{\theta}{2}\, e^{i\phi/2}\, e^{ik^-x}\, e^{-i(E+\Delta E/2)\,t/\hbar} \\ \cos\frac{\theta}{2}\, e^{-i\phi/2}\, e^{ik^+x}\, e^{-i(E-\Delta E/2)\,t/\hbar} \end{pmatrix} \sin\left(\frac{\mu B_1 l}{\hbar\, v}\right) \qquad (16)$$

In this notation θ and ϕ may be interpreted as the polar angles of the spinor in the coordinate system related to the B field, which is chosen as z-direction. $k^\pm$ are given by the index of refraction $k^\pm = k\, n^\pm = k\,(1 \mp |\mu|B_o/E)^{1/2}$, where k_o is the wave vector of the neutron outside the magnetic field region and $E = \hbar^2 k^2/2\,m$. If the resonance condition (equ. 14) is fulfilled, only the inelastic contributions remain and during the exit of the neutron from the B field an additional change of the kinetic energy occurs and we write the wave functions behind the whole field arrangement as

$$\Psi = -i \begin{pmatrix} \sin\frac{\theta}{2}\, e^{i\phi/2}\, e^{ik^{--}x}\, e^{-i(E+\Delta E/2)\,t/\hbar} \\ \cos\frac{\theta}{2}\, e^{-i\phi/2}\, e^{ik^{++}x}\, e^{-i(E-\Delta E/2)\,t/\hbar} \end{pmatrix} \qquad (17)$$

where $k^{\pm\pm} = k^\pm\, n^\pm \approx k\,(1 \mp \frac{|\mu|B_o}{E})$. Note that different k-vectors belong to the + and − component and therefore no overlap between the + and − state exist for a strictly monochromatic neutron beam.

In practice a wave packet representation with an amplitude function $f(k-k_o)$ centered around k_o has to be used for the incident wave function. These amplitude functions are transformed to $f^\pm\,(k - k_o^{\pm\pm})$ accordingly. No momentum overlap exist if /25/

$$\gamma = \frac{\left|\int f^+(k-k_o^{++})\;\; f^-(k-k_o^{--})\, dk\right|}{\left(\int |f^+(k-k_o^{++})|^2\, dk \int |f^-(k-k_o^{--})|^2\, dk\right)^{1/2}} \qquad (18)$$

is much smaller than 1. This condition is satisfied if the half width of the incident beam ΔE_o is smaller than the Zeeman splitting $\Delta E_o \leq 2\Delta E = 4\,|\mu|B_o$ (or $4\,n \cdot |\mu|B_o$).

These beams split in momentum and energy space enter the spin overlapping system (Fig. 5) which acts as one half of a spin echo system.

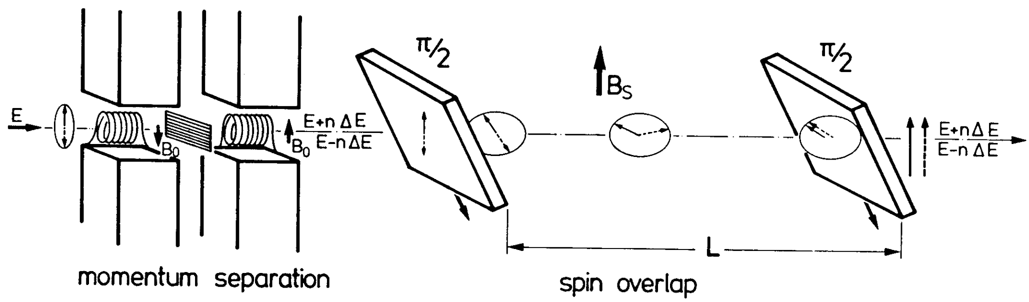

Fig. 5: Proposed arrangement for dynamical neutron polarization consisting of a momentum separation (left) and spin superposition part (right)

The behaviour of the + and - states within the $\pi/2$ turners and the precession field are well described by the rotation operator and are known from spin echo systems /21/. According to the different velocities associated with both states $v \pm \Delta v$ ($\Delta v = n\Delta E/mv$) a condition can be formulated where the spinor points in one direction only. This condition is fulfilled if the difference in the rotational angle of the two states within the precession field B_s reaches a value of π

$$\gamma\, B_s L \left(\frac{1}{v+\Delta v} - \frac{1}{v-\Delta v}\right) = \pi \qquad (19)$$

where L is the length of the precession field. From equ. (19) we get the condition for dynamic neutron polarisation

$$\frac{8\,n\,\mu^2}{m\,\hbar}\,\frac{B_o\,B_s\,L}{v^3} = \pi \qquad (20)$$

which is shown graphically in Fig. 6. It is seen that a multistage Zeeman splitting part allows the polarisation of a broader wave length band and simplifies the spin overlapping part.

It should be mentioned that equ. (20) can be fulfilled for a broad incident spectrum if a defined relation of direction and energy exists and if the precession field is

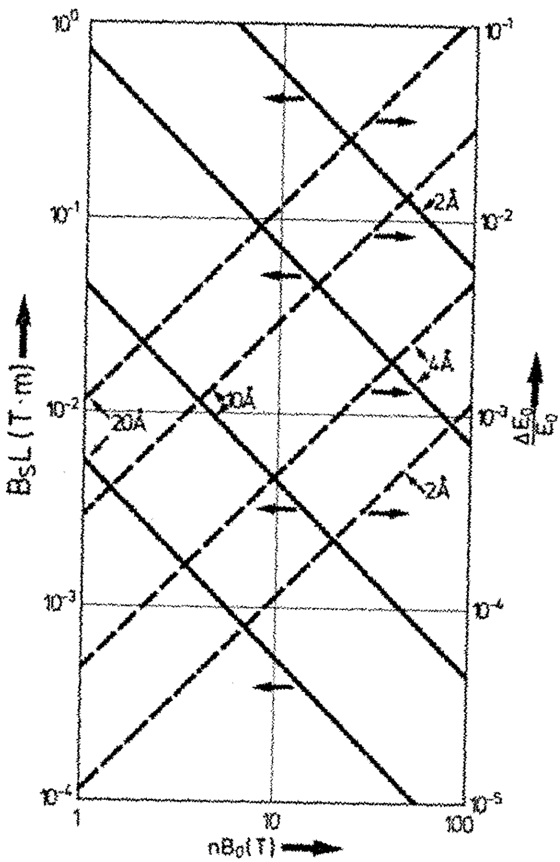

Fig. 6: Condition between the momentum separation and the spin overlapping part for dynamical neutron polarization

shaped to fulfil equ. (20) for any direction. This situation exists for example if perfect crystal reflections are used.

Neutron guide tubes can be used for both parts of the arrangement because the h.f. flippers as well as the $\pi/2$ spin turners are rather insensitive to the neutron velocity or to the effective path length through these devices.

The method described allows a complete polarisation of an incident unpolarised neutron beam with a certain energy width using only electromagnetic interaction and without material in the beam. The method is especially suited for long wave length neutrons and provides an optimal use of available neutrons.

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