I.1. In the lecture, we argued that for a form of Einstein Eqs. as

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

with $G_{\mu\nu} = C_1 R_{\mu\nu} + C_2\, g_{\mu\nu} R$

Use • Bianchi Identities (= differential Id. on $R^\rho{}_{\mu\sigma\nu}$)

$$\nabla_\eta R_{\lambda\mu\nu\kappa} + \nabla_\kappa R_{\lambda\mu\eta\nu} + \nabla_\nu R_{\lambda\mu\kappa\eta} = 0$$

to find a relationship between C_1 & C_2

• find C_1 such that non-relat., weak field limit follows

recall $g_{00} = 1 + 2\phi$ [$|G_{ij}| \ll |G_{00}|$ in non-relat.]

I.2. Geodesic eqn. from action principle

Action of freely falling particle from $A \to B$:

$$S_{AB} = \int_A^B ds = \int_A^B \frac{ds}{d\sigma}\, d\sigma = \int \sqrt{g_{\mu\nu}\frac{dx^\mu}{d\sigma}\frac{dx^\nu}{d\sigma}}\, d\sigma$$

proper time $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$

parameter describing path $x^\mu(\sigma)$

Consider variation of $x^\mu(\sigma) + \delta x^\mu(\sigma)$ with $\delta x^\mu\big|_{A,B} = 0$

Show that $\delta S_{AB} = 0$ yields geodesic eqn

I.3. Show that under coord. trafo $x'^\mu = x^\mu + \xi^\mu$ (ξ^μ small)

the metric transforms as $g^{\mu\nu}(x) \to g^{\mu\nu}(x) + \delta g^{\mu\nu}(x)$

with $\delta g^{\mu\nu}(x) = \nabla^\mu \xi^\nu(x) + \nabla^\nu \xi^\mu(x)$