III. 1. Gauge invariance & scalar perturbations

In the lecture we stated that there are 4 scalar perturbations to the metric

$$ds^2 = a^2(\eta)\left\{(1+2A)\,d\eta^2 - 2(\partial_i B)\,dx^i d\eta - \left[(1+2C)\delta_{ij} + 2\partial_i\partial_j E\right]dx^i dx^j\right\}$$

Consider a coord. trafo $x^\mu \to x^\mu + \xi^\mu(x)$ for which we identified the gauge transformation $h_{\mu\nu}(x) \to h_{\mu\nu}(x) + \Delta h_{\mu\nu}(x)$.

a) find Δh_{00}, Δh_{0i}, Δh_{ij}

b) Use this to write down the gauge transformations for A, B, C, E.
For this, decompose the spatial part of ξ^μ into scalar & vector part
$\xi^i = \partial^i \xi + \xi_\perp^i$ with $\partial_i \xi_\perp^i = 0$.
Show that 2 of the perturbations can be eliminated by picking appropriate coordinates.

c) verify the transformation properties by checking that the following variables are gauge-invariant

$$\Phi = A - \frac{1}{a}\left[a(E' - B)\right]' \quad , \quad \Psi = -C + \mathcal{H}(E' - B) \qquad (' = \partial_\eta)$$

[if Φ & Ψ vanish in one coord. system, they vanish in any coord. syst
$\Rightarrow$ a way to distinguish a real perturb. from fictitious one]