

II.6. Consider a massless scalar field ϕ with $V(\phi) = 0$. Suppose this field is rolling, $\dot{\phi} \neq 0$, and that it dominates the energy density of the universe. From $T_{\mu\nu}^{(\phi)}$, find the equation of state for ϕ . What does it imply for the redshifting of ρ_ϕ ?

II.7. Slow roll inflation:

a) Express the slow roll conditions $\epsilon \equiv -\frac{\dot{H}}{H^2} \ll 1$ & $\eta \equiv \frac{\dot{\epsilon}}{H\epsilon} \ll 1$ which guarantee an inflationary expansion & long enough e-folding period of inflation, respectively, solely in terms of (derivatives of) $V(\phi)$

b) Consider an inflationary potential $V(\phi) = \lambda\phi^4$. Assume field rolls from some value ϕ_i towards 0. At what values of ϕ does $\epsilon \ll 1$, $\eta \ll 1$ break down?

Assuming that inflation ends when $\epsilon = 1$, calculate the number of e-foldings of inflation, given a starting value of ϕ_i .