Calculating Byzantium?

Social Network Analysis and Complexity Sciences as tools for the exploration of medieval social dynamics

Johannes Preiser-Kapeller
(Institute for Byzantine Studies, Austrian Academy of Sciences)*

"Numbers exert a magic spell on human minds, particularly if they are presented with the help of computers."¹

In the last decade, historical and social studies have been confronted with a new kind of scientific research on natural as well as social, economic and historical phenomena based on the concept of complex systems; one may even speak of a “complexity turn”.² Quantitative and mathematical methods and models are used to analyse social processes and structures, and it is suggested that these models capture dynamics of real-world phenomena and even have some predictive value.³ At the same time, these new methods claim to be more adequate for the analysis of social and historical dynamics than earlier attempts at the “calculation” of history which were based on the mechanistic thinking of 19th century natural sciences; one prominent researcher in the field of “historical dynamics”, Peter Turchin, even claims that only with these methods “historical sociology (will) become a theoretical, mature science”.⁴

Especially the theses of Peter Turchin, Professor of Ecology and Evolutionary Biology, found favour (but also critics) among various scholars in the fields of socio-historical research – but less so among specialist historians. One cause for the appeal of Turchin’s model may be his usage of relatively comprehensible mathematics; a reviewer gratefully wrote “the author eschews the hyper-parameterized, computer-dependent approach found in many contemporary modelling efforts, and instead focuses on variants of three basic and

*Email: Johannes.Preiser-Kapeller@oeaw.ac.at | http://www.oeaw.ac.at/byzanz/historicaldynamics.htm.
An abridged version of this paper has been presented at the International Medieval Congress in Leeds (UK) in July 2010.

well understood differential equation models: the exponential, logistic, and predator-prey."\(^5\) The reviewer may have in mind models such as those of Wolfgang Weidlich in his book on "Sociodynamics"; for his model for “the rise and fall of interacting social groups” Weidlich used 15 Key-variables and 30 “Trend-functions” and at the same time stated: “we restrict the model to one sector or one dimension of social life”.\(^6\) Peter Turchin on the contrary claimed to be able to explain in his book “Historical Dynamics” on 240 pages “Why States Rise and Fall”.\(^7\)

But can these models really provide any further insight into the development of a medieval society such as Byzantium? One limiting factor is the necessity to provide statistical material in order to evaluate the appropriateness of a quantitative model – material which in most cases for the Byzantine period simply does not exist. To evaluate for instance Weidlich’s model of “the rise and fall of interacting social groups” we would need figures for the numbers of followers of the various political and ecclesiastical factions in 14\(^{th}\) century Byzantium. Turchin’s simpler models demand less specific data, but still rely mostly on historical statistics to a degree which we are not able to provide on the basis of Byzantine sources.

The core of Turchin’s framework is his "demographical-structural theory", for which he developed further concepts which he borrowed from the 14\(^{th}\) century Arab historian Ibn Khaldun and the American sociologist Jack A. Goldstone.\(^8\) According to this theory, the internal stability of a premodern state depends primarily on the dynamics of its population, more precisely on the number of producers and of members of the elite.\(^9\) The growth of the producing population depends not only on the “Malthusian” constraints of premodern agriculture, but also on the demands of the elite; high population numbers and especially high numbers of elite members lead to socioeconomic instability, to higher intra-elite competition and finally to civil war or even state breakdown which in turn effect population decline until balance within the system is restored. Turchin’s model produces cycles of the rise and decline of population number and state power with a duration of 200-300 years; therefore, he calls them

---

\(^7\) Turchin, Historical Dynamics (Subtitle).
\(^9\) Turchin, Historical Dynamics 118–149.
“secular cycles”. In order to test his model, Turchin needed data on the development of population, the size of the state’s elite and the occurrence of instability within a premodern state. As everybody who has ever taken a look on the statistical source basis for premodern Europe would expect, Turchin made use of what he considered “the best data set: the population history of England and Wales between 1080 and 2000 C.E.”, augmented by data on the number of English aristocrats and a so-called “instability index”, which measured the years of socio-political instability per decade through the centuries. Turchin claimed to be able to identify “secular cycles” in the development of pre-modern England. Subsequently, Turchin and his colleague Sergey A. Nefedov – of course successfully – looked for “secular cycles” in medieval and early modern France, but also in statistically less well documented periods such as late medieval and early modern Russia or Republican and Imperial Rome; in order to have a statistical basis, they used other data such as the temporal distribution of temple building activity or the number of coin hoards per decade as indicators for instability – but as one reviewer has stated, this data is clearly of “questionable reliability”. And even “the best data set” for medieval England should have been used with great caution, since there is a relatively wide range of alternative population estimates. Still, the overall trends in data could be interpreted in a way which supports the validity of the Turchin-Nefedov-model.

But what of Byzantium? Various scholars provide some very rough estimates for population figures, but since we possess no evidence of an empire-wide census and documents only for some villages in 14th century Macedonia for instance, “it is not possible to procure exact population figures at any time for any territory within the

realm of (...the Byzantine state”, as Johannes Koder has stated. Also on the size of the Byzantine “elite” one finds some estimates; for the 11th and 12th century for instance, the number of aristocratic families was calculated on the basis of prosopography by Alexander Kazhdan and Silvia Ronchey, who also included some statistics on the “duration” of families in their excellent study – but of course they did not provide any information on the quantitative ratio between the elite and the rest of the population. At least it is possible to construct an instability index for Byzantium after Turchin’s example. Helpful lists of rebellions, usurpations, etc. can be found in the works of Friedhelm Winkelmann for the 8th and 9th century and of Jean-Claude Cheynet for the 10th to 13th century, for instance. But one has to ask which instability-events should be interpreted as symptoms of an underlying general crisis dynamics – a rebellion in a small provincial town provoked by a greedy local tax official, for example? In Turchin’s studies, this question is not answered. We introduced some kind of threshold in order to include only over-regional and instability phenomena of wider significance in our list. Our instability index (see fig. 1) for Byzantium begins where Turchin’s index for Rome ends in 280 AD and ends in the year 1400, since the small size of the Byzantine polity after the middle of the 14th century raises some doubts on the applicability of the model which Turchin developed for “large scale agrarian empires”. As a gaze at the graph shows, of course we could identify several of Turchin’s secular cycles for the Byzantine instability trajectory if we choose various 200–300 years periods, for the time from 1150 to 1400 AD for instance (see fig. 2). But since we do not have any corresponding data on the population trajectory or other elements of the demographic-structural framework, an evaluation of the model as it has been done for England, France, etc. seems impossible. More interesting seems the observation (also indicated with the trend line, see fig. 1) that the intensity of instability grew during these more than 1000 years;

---

but this phenomenon may as well depend on the larger number of sources which we
possess for the later centuries of Byzantine history (an aspect which Turchin also did
not take into consideration). A primary weakness of Turchin’s demographic-structural
model with regard to the Byzantine case is also that it completely ignores external
influences on the system such as foreign invasions, climate changes or plagues,
phenomena, which seem especially relevant for the development of Byzantium.
Therefore we modified the basic demographic-structural model and introduced into
the equations randomized influences on the reproduction rate of the population (to
“simulate” external invasions, plagues, etc.), on the constraints of population because of
the productive area (territorial losses or gains) and on the limitations of the ability of
the state to exploit the population (strong and weak regimes). If we start with a
population of 0.5 (meaning that the population has half the size of the maximum in
relation to the carrying capacity of the state’s territory) and a state income of 0.2
(meaning that 20% of the total production are absorbed by the state and its elite) for
instance, we find all kinds of trajectories (in the model, one timestep equals one
generation): some look like Turchin’s secular cycles (see fig. 3), but others show long
periods of depression (see fig. 4) or have a stable downward trend (see fig. 5) (the same
holds true if we choose other initial values for the parameters). So does our modified
model explain anything? As the renowned French historical demographer Noël Bonneuil
has stated in his review of Turchin’s first book: “it is one thing to create possible
scenarios; it is another to believe that such scenarios, dependent on arbitrary values of
five parameters, have more general explanatory power.” One could say that we
contaminated Turchin’s model with our randomisations of these parameters. But to cite
Bonneuil once more: “When simulations fit observed data reasonably well, it is wise to
wonder about the extent to which the number and significance of the parameters made
it possible.” As anyone only slightly familiar with complex dynamic systems knows,
logistic models such as the one of Turchin always show cyclic trajectories within a
certain range of values of the basic parameters, such as the reproduction rate (r) of the
population for instance (see fig. 6 and 7); but with just a little increase, the trajectory

---

21 For the unmodified equations see TURCHIN, Historical Dynamics 123 and 208–211.
22 BONNEUIL, Review 265–266.
23 BONNEUIL, Review 270.
becomes much more unpredictable (see fig. 8), even “chaotic” (see fig. 9) and sometimes leads to collapse after a few time steps (see fig. 10).  

Therefore, it may be more profitable to look at some other properties of the model than to attempt to fit its trajectory to one constructed on the basis of historical data and estimates of uncertain reliability; as Bonneuil wrote: “the theory of dynamical systems can bring much more to history than the application of equations yielding cycles.” We can learn from a complex system based on relatively simple equations such as the one we have inspected that even small changes in environmental, demographic or socio-economic parameters can have an unexpected, sometimes even dramatic effect on a polity. Complex systems are understood as large networks of individual components, whose interactions at the microscopic level produce “complex” changing patterns of behaviour of the whole system on the macroscopic level, which are hard or almost impossibly to predict. These systems show a nonlinear character, which means that they answer to certain stimuli (actions of individuals on different scales or external influences, for instance) not in a linear way (which would mean that the output is proportional to its input), but because of the interactions between the parts of the system these stimuli can be reinforced (or weakened) through feedback mechanisms in an unexpected way, as we have seen. Change within complex systems is described as transition between alternative (more or less) stable states or “attractors”. Chaos in terms of complexity theory means that small differences in the initial state expand exponentially with time, such that the system’s long-term behaviour is fundamentally unpredictable, but this “unpredictable behaviour can result from completely deterministic rules”. At the same times, a chaotic attractor “is always bounded. One can draw a box around it from which it will not escape. (...) the precise dynamics cannot be predicted, (but) the range of behavior can be described”. These bounds we can also observe in the simple randomized Turchin-model; one interesting phenomenon is that

---

25 BONNEUIL, Review 269.
26 Or to speak with Bonneuil once more, we observe “broken threads, sudden switches from one regime to another, temporary stagnations, discontinuity, unexpected futures.”, cf. BONNEUIL, Review 269.
the trajectory for state income never much exceeds the value of 0.2 (which means, as already mentioned, that 20% of the total production are absorbed by the state and its elite), no matter which values we choose for the initial parameters (see fig. 11–14 for examples). That there is a limit for the proportion of total production which the state is able to claim we would have assumed before; but the same holds true for the model. The value of 20% is interestingly very well within the range which Jacques Lefort and Angeliki Laiou proposed in their model calculations in the volumes of the Economic History of Byzantium and also Peter Fibinger Bang did in his recent book on “The Roman Bazaar” for imperial Rome.

There is also “real world” data from Byzantine sources which indicates that complex dynamic processes are relevant for the understanding of Byzantium. One example which hints at the patterns of distribution of demographic and economic potential within the regions of the Late Byzantine empire is the analysis of a list of contributions from 33 bishoprics to the Patriarchate of Constantinople in the year 1324, which we executed (see fig. 15). This contribution list is part of a document in the Register of the Patriarchate of Constantinople. As we analysed this distribution of values with statistical means, we detected that the model which fitted the data best is a “power law”. Power law distributions have been identified for many phenomena for which underlying complex dynamics are assumed; one of the most prominent of these distributions is the so called “Zipf-Law”, named after the linguist George Kingsley Zipf, who first observed it for the frequency of words in English texts, but later also for the distribution of the populations of settlements. According to the classical Zipf-distribution, the second largest city in a country or region would have one half of the

---


30 J. PREISER-KAPELLE – E. MITSIOU, Hierarchies and Fractals: Ecclesiastical Revenues as Indicator for the Distribution of relative demographic and economic Potential within the Cities and Regions of the Late Byzantine Empire in the early 14th Century (under review).


32 R² = 0.926, which means that the model can explain 92% of the variations within the list.


population of the largest city, the third largest city one third of the population of the largest city, et cetera. This can be expressed with the formula:

\[ P(r) = \frac{P(1)}{r^Z} \]

where \( P(r) \) is the population of the city of the \( r \)-ranked city within the totality of the sample, \( P(1) \) the population of the largest city, \( r \) the rank of the city (1, 2, 3, ...) and \( Z \) is a constant in the order of magnitude of 1.\(^{35}\) This rank-size rule has been empirically studied in many regions throughout the globe for various time periods; many cases satisfy Zipf’s law very closely with values for \( Z \) around 1, whereas in other cases rank-size distributions of populations of cities obey power-law behaviour, but have a different power exponent \( Z \) (see the example of England in 1377, fig. 16).\(^{36}\) The distribution from Byzantium in 1324 (see fig. 15) yields a value of \( Z = 0.93 \) and shows the same patterns as other quantities which have been used for the formation of a settlement rank-size hierarchy; therefore it can be connected with the pattern of distribution of demographic and economic potential within the Byzantine Empire of this time.\(^{37}\) The working of the process which generates these patterns is still under discussion; but most probably they result from the complex interactions within the network of settlements and their hinterland which produce an uneven distribution of demographic and economic potential and a hierarchy of cities.\(^{38}\) This hierarchy can also be connected with the classic model of central place hierarchies developed by Walter Christaller in 1933, which Johannes Koder has used for the analysis of the Byzantine urban system.\(^{39}\)

---


\(^{37}\) For the entire argument see PREISER-KAPELLE – MITSIOU, Hierarchies and fractals.


So, as already mentioned, the interactions and connections between components of a system – regions, cities, but also groups, cliques or individuals – produce the complex patterns we observe. A already well established method for the recording, analysis and visualisation of such connections is network analysis. It has been used in historical studies for decades, also by scholars of the Western middle ages, much less so by Byzantinists – with the exception of Margaret Mullett’s book on Theophylact of Ochrid and Giovanni Ruffini’s recent study on 6th century Egypt. What may make this method more acceptable for the sceptic historian is the fact that it allows him or her to directly construct the nodes and ties of a network on the basis of the sources and than to analyse the outcome; one has not to attempt to find evidence for the explanatory value of a certain model which has been constructed in advance. As an example we re-constructed the network of the Byzantine village priest Basileios Aroules from Radolibos in Macedonia (see fig. 17) on the basis of a register of dues for the Athos monastery of Iviron from the year 1316; there the following entrance can be found: “The priest Basileios Aroules, has (a woman) Helene, a son, the priest Konstantinos, from him a daughter-in-law Anna, another son Chalkos, a daughter Maria, from her a son-in-law Ioannes, one house, one ox, a vineyard of nine modioi; the total tax amount is two hyperpyra. Ioannes, the shoemaker, his son, has (a woman) Zoë, the sons Daniel und Basileios, a house, a vineyard of three modioi; the total tax amount is one hyperpyron. Michael, the shoemaker, his other son, has (a woman) Eirene, a daughter Anna, a house, a vineyard of three modioi; the total tax amount is one hyperpyron.” One can recognize the variety of ties which we find in this (and other) document(s) and which connected Basileios Aroules with the members of his family as well as with the members of his

parish and the village, but also with the local bishop (as his superior), the representatives of the monastery of Iviron (as the lord of the manor) and the local officials of the state (who composed the register of dues)\textsuperscript{44}; through the medium of these actors Basileios Aroules could – hypothetically – even get linked with the highest authorities of the Patriarch and the Emperor in Constantinople (see fig. 17); the famous dictum of “only six degrees” which separate an individual from any other\textsuperscript{45} is also valid for Byzantium.

In historical study, network analysis until now mainly has been used to evaluate the position of individuals within the network, their relevance and influence on the basis of the number of their links (“degree”) or the number of other nodes for which they could function as a bridge (“betweenness”).\textsuperscript{46} But other aspects of recent research in the field of what is now called complex network analysis are of equal interest for historians. As we have already seen in the case of Basileios Aroules, there exist different kinds of ties connecting him with his social environment. Michael Szell, Renaud Lambiotte and Stefan Thurner from the Vienna Complex Systems Research Group\textsuperscript{47} argued in their recently published paper on „Multi-relational Organization of Large-scale Social Networks“:

“Human societies can be regarded as large numbers of locally interacting agents, connected by a broad range of social and economic relationships. (...) Each type of relation spans a social network of its own. A systemic understanding of a whole society can only be achieved by understanding these individual networks and how they influence and co-construct each other (...) A society is therefore characterized by the superposition of its constitutive socio-economic networks, all defined on the same set of nodes. This superposition is usually called multiplex, multi-relational or multivariate network.”\textsuperscript{48}

This multiplexity of networks “defined on the same set of nodes” provides a framework not only for the conceptualisation of social, economic, religious, political, etc. ties within, but also beyond the borders of Byzantium; would Aroules have lived in a village in Bithynia in the 1340s for instance (see fig. 18), we could still observe a similar family

\textsuperscript{44} This and other networks in this proposal were created with the software packages Pajek, cf. W. DE NOOY – A. MRVAR – V. BATAGELJ, Exploratory Social Network Analysis with Pajek (Structural Analysis in the Social Sciences). Cambridge 2005, and ORA, cf. http://www.casos.cs.cmu.edu/projects/ora/.
\textsuperscript{46} On these parameter cf. WASSERMANN – FAUST, Social Network Analysis 100–107, 189–191; JACKSON, Social and economic Networks 38–39, 59–65.
\textsuperscript{47} See http://www.complex-systems.meduniwien.ac.at/about/.
and social network, but the landlord would now be an Ottoman sipahi as holder of a timar which he had received from the Emir Orhan Bey.⁴⁹ On the other side, if the Ottoman authorities allowed it, a local priest could still have a bishop or a metropolitan as superior, who would have been elected by the Patriarch and the synod in Constantinople, with the consent of the Emperor.⁵⁰ As Daniel H. Nexon in his recent study on “The Struggle for Power in Early Modern Europe” has demonstrated, this concept of network allows us to analyse relations between different political and religious communities and authorities in medieval and premodern times in a better way than within the framework of classic international relations which is based on the modern concept of the nation state.⁵¹

A second relevant aspect of recent network studies is once more connected with the phenomena of complex systems; the combination of individual components into larger scale networks results in the emergence of new patterns and characteristics. Therefore, networks should not only be analysed at the level of individual actors or smaller groups, but also at the level of the total network.⁵² We re-constructed for instance on the basis of the Prosopography of the Palaiologean Era the network of the “dynatoi”, the powerful noblemen and state functionaries, for the period between 1310 and 1341, with 187 nodes (see fig. 19).⁵³ One of the most striking characteristics of this network is of course that we recognise two centres of gravity; these are the two emperors who were competing for power in the 1320s, Andronikos II Palaiologos and his grandson Andronikos III Palaiologos. We also can identify various clusters which represent the

---

most important noble families whose members all possessed important positions in the state; the largest cluster is that of the imperial clan of the Palaiologoi itself. In order to evaluate the network in its totality, we constructed a random network with the same basic parameters as a benchmark so that we could identify some special features. We compared the two networks with regard to the “three robust measures of network topology” (see fig. 20): average path length (or average distance between two nodes; the path length between two directly connected actors is 1), the clustering coefficient (a measure of the likelihood that two associates of a node are associates themselves. A higher clustering coefficient indicates a greater “cliquishness”) and the average degree (the average number of nodes an actor is directly connected with). Although we used the same average degree for the random network, the differences between the two networks with regard to path length and especially clustering coefficient are significant. This indicates that the distances between the nodes in the network of Byzantine dynatoi were smaller than what could be expected for a network of this size (the “small world” phenomenon which has also been observed for many real world modern networks), at the same time, the “cliquishness” among the Byzantine aristocracy at this time is high, a characteristic which supports a more decentralized flow of power and influence within the network. This result of network analysis converges with the internal situation in Byzantium – civil war, intra-aristocratic competition – at this time. Yet of course influence and power were not equally distributed among the dynatoi; this becomes evident if we compare the distribution of degree (the number of nodes every node is connected with) of the dynatoi-network with the random network (see fig. 21 and 22). While in the later case the degree is very equally distributed around its average value, the degree-distribution for the dynatoi-network is much more unequal and shows similarities with the power law-distribution we observed for the cities in 1324. Again, this is a phenomenon now well known from many studies on modern day larger scale networks; it has been connected with a preferential attachment process, which means that emerging ties between nodes are distributed among individuals according to how much they already have, so that those who already have receive more than those who have not. This would also be very much the case within a stratified elite such as we

54 As Réka Albert and Albert-László Barabási have called them, cf. the citation in fn. 52; cf. also JACKSON, Social and Economic Networks 56–65.
55 Cf. WATTS, Small Worlds; ALBERT – BARABÁSI, Statistical Mechanics of Complex Networks.
56 ALBERT – BARABÁSI, Statistical Mechanics of Complex Networks (also for further literature); JACKSON, Social and Economic Networks 130–134.
observe it in later Byzantium; also a cumulative distribution of the “points of nobility” determined by Kazhdan and Ronchey for the aristocratic families in the 11th and 12th century which we calculated suggests a similar dynamic process for the unequal distribution of power and influence within the elite (see fig. 23).\textsuperscript{57}

Finally, we also compared the network of the Byzantine *dynatoi* with the network of the “powerful” in a neighbouring emerging polity, that is the network of Osman Bey which Karen Barkey constructed for her book “Empire of Difference” (see fig. 24) (we are well aware of the problems of any such reconstruction on the basis of early Ottoman chronicles).\textsuperscript{58} The network of the then significantly smaller incipient Ottoman state is of course also smaller than the Byzantine one and consists of 43 nodes. As the comparison of some key figures demonstrates (see fig. 25), this smaller size results in a lower average distance between the actors and a higher average potential speed for the flow of information, resources etc. The network of the *dynatoi* is significantly more stratified (11 network levels vs. 3 network levels); but although the Ottoman network is more than four time smaller, the Clustering Coefficient of the Byzantine network is higher, which once again indicates the high “cliquishness” within the Byzantine aristocracy. That the network of Osman Bey in contrast is more centralized with regard to the potential flow of influence, resources etc. is indicated by the higher centralization values for closeness (which measures the average distance between one node and all other nodes) and degree (cf. also the degree distribution of the Osman Bey-network, see fig. 26), whereas the higher betweenness centralization of the *dynatoi*-network hints again at a higher de-centralisation of cliques and influence which gave actors the opportunity to establish themselves as brokers. Thus, our analysis shows a highly stratified, more de-centralised network of the Byzantine “powerful” confronted with a smaller, flexible Ottoman network, in which the potential flows of power and resources are more centralised in the hand of the ruler.

Finally, Alexander Kazhdan and Giles Constable shall be cited once more: “It is (...) necessary to be very careful with Byzantine numerical data and to keep constantly in mind how meager they are and how strongly their evaluation depends upon subjective principles, which underlie all statistical investigations. Neither simple human calculations


nor the more intricate and sophisticated work of a computer can provide a completely objective picture or create a secondary source of unshakable significance. But despite its many limitations and restrictions, statistical evidence provides better, clearer, and more reliable conclusions than the accumulation of occasional and separate examples."\(^5\)\(^9\)

The combination of “separate examples” from the historical sources into models and networks allows us to observe the emergence of new patterns which hint at complex dynamics behind various aspects of late Byzantine society, economy and politics. As complexity studies give us an opportunity to understand better interrelations and mechanisms which form our reality, they may as well provide new insights into the emergence of the past reality of Byzantium.

\(^5\)\(^9\) Kazhdan – Constable, People and Power in Byzantium 177.
Fig. 1: Instability Index for Byzantium, 280–1400 AD
Fig. 2: Instability Index of Byzantium, 1150–1400 AD

![Fig. 2: Instability Index of Byzantium, 1150–1400 AD](image)

Fig. 3: A „Secular cycles“ – trajectory of the randomized Turchin-model

![Fig. 3: A „Secular cycles“ – trajectory of the randomized Turchin-model](image)
Fig. 4: A „Dark Ages“- trajectory of the randomized Turchin-model

![Figure 4: A „Dark Ages“- trajectory of the randomized Turchin-model](image)

Fig. 5: A Gibbon („1000 years of decline“) – trajectory of the randomized Turchin-model

![Figure 5: A Gibbon („1000 years of decline“) – trajectory of the randomized Turchin-model](image)
Fig. 6: A population trajectory in the basic Turchin model (r = 3.5)

Fig. 7: A population trajectory in the basic Turchin model (r = 3.7)

Fig. 8: A population trajectory in the basic Turchin model (r = 3.9)
Fig. 9: A population trajectory in the basic Turchin model \((r = 4)\)

![Population in the basic Turchin model \((r = 4)\)](image)

Fig. 10: A population trajectory in the basic Turchin model \((r = 4.1)\)

![Population in the basic Turchin model \((r = 4.1)\)](image)

Fig. 11: A trajectory of state income in the randomized Turchin-Model \((S_0 = 0.2)\)

![State income for the randomized Turchin-model \((S_0 = 0.2)\)](image)
Fig. 12: A trajectory of state income in the randomized Turchin-Model ($S_0 = 0.0$)

Fig. 13: A trajectory of state income in the randomized Turchin-Model ($S_0 = 0.5$)

Fig. 14: A trajectory of state income in the randomized Turchin-Model ($S_0 = 0.9$)
Fig. 15: The distribution of contributions from 33 bishoprics to the Patriarchate of Constantinople, 1324 (in hyperpyra)

Distribution of annual contributions from 33 bishoprics to the Patriarchate of Constantinople, September 1324 (PRK I, Nr. 88)

\[ y = 706x^{-0.9309} \]
\[ R^2 = 0.9264 \]

Fig. 16: The distribution of taxpaying population among towns in England in 1377 AD

Towns with recorded taxpaying population of over 1,000 in England in 1377 AD

\[ y = 14920x^{-0.706} \]
\[ R^2 = 0.9636 \]
Fig. 17: The network of the Byzantine village priest Basileios Aroules in Rabolibos, Macedonia (1316)
Fig. 18: A hypothetical "Aroules-network" in a village in Bithynia in the 1340s
Fig. 19: Visualisation of the network of the dynatoi (the powerful), 1310-1341 (n = 187), with Emperor Andronikos II Palaiologos and Emperor Andronikos III Palaiologos as two centres of gravity

Fig. 20: A comparison between the network of the dynatoi and a random network with the same basic parameters

<table>
<thead>
<tr>
<th>Network of the dynatoi (n = 187)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average path length</td>
<td>2.9254</td>
<td></td>
</tr>
<tr>
<td>Clustering Coefficient</td>
<td>0.2933</td>
<td></td>
</tr>
<tr>
<td>Average Degree</td>
<td>3.0214</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Random network (n = 187)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average path length</td>
<td>4.4706</td>
<td></td>
</tr>
<tr>
<td>Clustering Coefficient</td>
<td>0.0146</td>
<td></td>
</tr>
<tr>
<td>Average Degree</td>
<td>3.0214</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 21: Cumulative degree distribution for the random network

![Cumulative degree distribution for the random dynatoi-network](n = 187)

Fig. 22: Cumulative degree distribution of the network of *dynatoi*, 1310–1341

![Cumulative degree distribution for the Network of dynatoi, 1310-1341](y = 38.648x^{-0.9543}, R^2 = 0.6537)

Fig. 23: Cumulative distribution of “points of nobility” among noble families in Byzantium, 976–1204 AD (data from: KAZHDAN – RONCHEY, L’aristocrazia 239–246)

![Cumulative distribution of “points of nobility” within 257 noble families in Byzantium, 976-1204 AD (after Kazhdan and Ronchey, 1999)](y = 50.018e^{-0.1899x}, R^2 = 0.8107)
Fig. 24: The network of Osman Bey, 1324 (after: K. BARKEY, Empire of Difference. The Ottomans in Comparative Perspective. Cambridge 2008, 49).

Fig. 25: A comparison between the Byzantine dynatoi-network and the Osman Bey - network

<table>
<thead>
<tr>
<th>Byzantine dynatoi - network</th>
<th>Osman Bey - network</th>
</tr>
</thead>
<tbody>
<tr>
<td>n = 187</td>
<td>n = 43</td>
</tr>
<tr>
<td>av. path length = 2.9254</td>
<td>av. path length = 1.3095</td>
</tr>
<tr>
<td>Clust. Coeff. = 0.2933</td>
<td>Clust. Coeff. = 0.2004</td>
</tr>
<tr>
<td>Centr. Betweenness = 0.0113</td>
<td>Centr. Betweenness = 0.0074</td>
</tr>
<tr>
<td>Central. Closeness = 0.0028</td>
<td>Central. Closeness = 1.7231</td>
</tr>
<tr>
<td>Central. Degree = 0.3220</td>
<td>Central. Degree = 0.4123</td>
</tr>
<tr>
<td>av. speed = 0.3418</td>
<td>av. speed = 0.7636</td>
</tr>
<tr>
<td>network levels: 11</td>
<td>network levels: 3</td>
</tr>
</tbody>
</table>
Fig. 26: Cumulative degree distribution of the Osman Bey - network

Cumulative degree distribution of the Osman Bey - network

$y = 11.528x^{-0.8554}$

$R^2 = 0.6028$