Optimal Investment Strategies With Demand-Side and Cost-Side Risks

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Motivation

- Firms operating in different industries are exposed to
  - different sources of risks, and
  - alternative market structures in industries.

- This paper studies the consequences of two different types of risks on the investment behavior of a company in two alternative industry structures: monopoly and perfect competition.

- Firms face demand-side (exogenous shifts in the demand curve or prices) and cost-side risks (changes in wages).

- The sources of risks together with the industry structure determine the investment strategies and hence firm values and industry risk dynamics.
This Paper

- This paper addresses a couple of important questions in industry analysis.

- Are the structural consequences of stochastic shifts in the demand curve on a firm’s value fundamentally different to stochastic changes in input prices (cost levels)?

- How do alternative market structures influence the decision to invest in cost reductions?

- What are the consequences of operating flexibility (the opportunity to adjust output downwards) and investments in cost reductions on firm values?

- How do optimal investment decisions influence firm own and industry risk?
Related Literature


- Dockner, Mæland, and Miltersen (2009) study optimal capital structure in a model with investments in cost reductions.


• Aguerrevere (2009) evaluates the asset pricing implications of industry competition in a symmetric homogenous product oligopoly with demand shocks.

• Novy-Marx (2008) studies equilibrium investment decisions in a dynamic Cournot market with demand side risks.

• Pawlina and Kort (2006) study equilibrium investment strategies when firms face a single expansion option and strategically interact.
Summary of Results

- The value of both types of firms varies nonlinearly with demand and cost shocks and consists of the value of the technology in place and the option value to invest in cost reductions.

- The option to invest in cost reductions generates a positive value which is smaller for the monopoly than for the competitive firm.

- Investment in cost reductions corresponds to an American call option that is exercised depending on the ratio of demand to cost side risks.

- If marginal costs are very high will the investment trigger of the monopoly firm always be larger than that of the competitive firm.
Summary of Results (cont’d)

- If demand is very elastic and marginal costs are small will the monopoly firm exercise its investment option earlier than the competitive firm.

- Investment in cost reductions is later the higher are the investment costs or the smaller the cost reduction.

- Systematic risk of the firm consists of the revenue beta and option risk associated with cost reductions.

- Risk of a competitive firm can be larger or smaller than that of the monopolist (monopolies can earn a lower return than firms in a competitive industry).
The Model

- Monopolistic or competitive industry with a representative firm.

- In case of a monopolist price $P_t$ is inversely related to output $Q_t$:

  \[ P_t = \alpha_t(Q_t)^{-\gamma}, \]

  where $0 \leq \gamma < 1$.

- In case of a competitive firm it is exogenously given by $P_t = \alpha_t$.

- Firms operate with convex production costs $C_0C_tQ_t^\kappa$, with $\kappa > 1$ in which $C_0C_t$ is the current level of costs for one unit of production, and choose optimal quantities $\max_Q \left( \pi_i = \alpha_t(Q_t)^{1-\gamma} - C_0C_tQ_t^\kappa \right)$ in the short-run.
The Model (cont’d)

• In the long-run firms can invest IC to decrease unit costs from the level $C_0 \rightarrow C_1$ with $C_0 > C_1$.

• Investment decisions depend on demand and cost-side risks.

• Let $\alpha_t$ and $C_t$ be two dynamic stochastic process given by

$$
\begin{align*}
    d\alpha_t &= \mu_{\alpha} \alpha_t dt + \sigma_{\alpha} \alpha_t dW_{\alpha,t}, \\
    dC_t &= \mu_{C} C_t dt + \sigma_{C} C_t dW_{C,t},
\end{align*}
$$

with $(dW_{\alpha,t}, dW_{C,t})$ two correlated Wiener processes with $E(dW_{\alpha,t}, dW_{C,t}) = \rho dt$.

• $\alpha_t$ represents demand shocks and $C_t$ cost shocks.
Optimal Output Choices

- $0 < \gamma < 1$ corresponds to the monopoly case with a price elasticity given by $\eta = \frac{-1}{\gamma}$.

- The case of perfect competition is a special case given by $\gamma = 0$.

- Optimal output decisions in case of a monopolistic industry are given by

$$Q_M(C_0, \alpha_t, C_t) = \left[ \frac{(1 - \gamma)\alpha_t}{\kappa C_0 C_t} \right]^{\frac{1}{\kappa + \gamma - 1}}.$$

- Optimal output decisions in case of a competitive industry are given by

$$Q_C(C_0, \alpha_t, c_t) = \left[ \frac{\alpha_t}{\kappa C_0 C_t} \right]^{\frac{1}{\kappa - 1}}.$$
Firm Values

- Total firm value is the expected sum of discounted futures profits:

\[ V(C_0, \alpha, C) = \mathbb{E}_0 \left\{ \int_0^\infty e^{-rt} \Pi_0(C_0, \alpha_t, C_t) dt \mid \alpha_0 = \alpha, C_0 = C \right\} \]

\[ \Pi_0 = \left( 1 - \frac{1 - \gamma}{\kappa} \right) \left( \frac{1 - \gamma}{\kappa} \right)^{\frac{1-\gamma}{\kappa+\gamma-1}} C_0^{\frac{\gamma-1}{\kappa+\gamma-1}} (\alpha_t)^{\frac{\kappa}{\kappa+\gamma-1}} (C_t)^{\frac{\gamma-1}{\kappa+\gamma-1}} \]

- We introduce the following state variable transformation

\[ Y_t \equiv (\alpha_t)^{\delta} (c_t)^{1-\delta} \]

with \( \delta \equiv \frac{\kappa}{\kappa+\gamma-1} > 1 \) which implies

\[ dY_t = \mu Y_t dt + \sigma Y_t dW_t, \quad r > \mu > 0, \sigma > 0. \]
Firm Values cont’d

- The continuation value of the firm (no exercise of the technology adjustment) is given by

$$V(C_0, Y) = \mathbb{E}_0 \left\{ \int_0^\infty e^{-rt} AC_0^{1-\delta} Y_t dt \mid Y_0 = Y \right\}$$

which satisfies the arbitrage condition:

$$rV(Y) = \underbrace{AC_0^{1-\delta} Y} + \underbrace{\mathbb{E}(dV(Y))}.$$  

Total Return  Dividends  Capital Gains
Firm Values cont’d

- The arbitrage condition results in the standard valuation equation:

\[
0 = AC^{1-\delta}_0Y - rV(Y) + \mu Y V'(Y) + \frac{1}{2}\sigma^2 Y^2 V''(Y),
\]

with the general solution

\[
V(Y) = \frac{AC^{1-\delta}_0}{\rho - \mu Y}Y + DY^\lambda + EY^\epsilon
\]

where \( D \) and \( E \) are integration constants and \( \lambda > 1 \) is the positive root of the characteristic equation and \( \epsilon < 0 \) the negative one.
Choice of Technology (Costs)

- Technology choice corresponds to the exercise of a real option:

\[
\max V(C_0, Y) = \mathbb{E}_0 \left\{ \int_0^{t(Y)} e^{-rt} AC_0^{1-\delta} Y_t dt \mid Y_0 = Y \right\} \\
+ \mathbb{E}_0 \left\{ \int_{t(Y)}^{\infty} e^{-r(t-t(Y))} AC_1^{1-\delta} Y_t dt \mid Y_0 = Y \right\}
\]

- We derive the trigger levels that characterize the optimal investment strategies in case of monopolistic \((0 < \gamma < 1)\) and perfect competition \((\gamma = 0)\).
Theorem 1. When the firm optimally invests in cost reductions the firm value is given by

\[ V(Y) = \begin{cases} 
\frac{AC_0^{1-\delta}}{r-\mu} Y + \frac{IC}{\lambda-1} \left( \frac{Y}{\bar{Y}} \right)^\lambda & Y < \hat{Y} \\
\frac{AC_1^{1-\delta}}{r-\mu} Y & \text{else}
\end{cases} \]

with the exercise trigger and the positive characteristic root:

\[ \hat{Y} = \frac{\lambda}{\lambda - 1} \frac{r - \mu}{A(C_1^{1-\delta} - C_0^{1-\delta})} IC, \]

\[ \lambda \equiv \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left( \frac{1}{2} - \frac{\mu}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}} > 1. \]
Lemma 1. The firm’s option value is strictly positive and

1. decreasing in the level of irreversible investment costs $IC$,

2. increasing in the level of cost reductions $C_1$, and

3. larger for the competitive firm $\gamma = 0$, than for the monopoly $\gamma > 0$.

Lemma 2. The option trigger in a monopolistic industry is

1. always higher than the competitive level if $\kappa$ is large;

2. is smaller than the competitive level if $\gamma$ close to 0 and $\kappa$ close to 1.
\[ \hat{Y} \ (\kappa = 5) \]

\[ \hat{Y} \ (\kappa = 2.5) \]

\[ \hat{Y} \ (\kappa = 2) \]

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Results (cont’d)

• The value of the firm consist of the *value of the technology in place*

\[ V_A(Y) \equiv \frac{AC_0^{1-\delta}}{r - \mu} Y, \]

and the *option value* to reduce production costs,

\[ V_O(Y) \equiv \frac{IC}{\lambda - 1} \left( \frac{Y}{\hat{Y}} \right)^\lambda > 0. \]

• The option to switch to a more productive technology is value increasing:

\[ V(Y) = V_A(Y) + V_O(Y) > V_A(Y). \]
Risk Implications

- The risk of the firm (the firm beta) is defined by

\[ \beta(Y) \equiv \frac{V'(Y)Y}{V(Y)} \]

- Prior to the investment in the new technology the firm’s risk is given by

\[ \beta(C_0, Y) = \frac{1}{\text{Revenue Beta}} + (\lambda - 1) \frac{V_0(Y)}{V(Y)}. \]
Conclusions

- We explored the relationship between industry structure and risk dynamics arising from different sources of risks for rival firms competing in two alternative market structures (monopoly and perfect competition).

- Demand and cost shocks have identical value and risk implications.

- Individual firm risk can substantially be influenced by the option to reduce unit costs.

- Firm values are driven by flexibility, cost reductions and two interacting shocks.