The optimal prevention of epidemics

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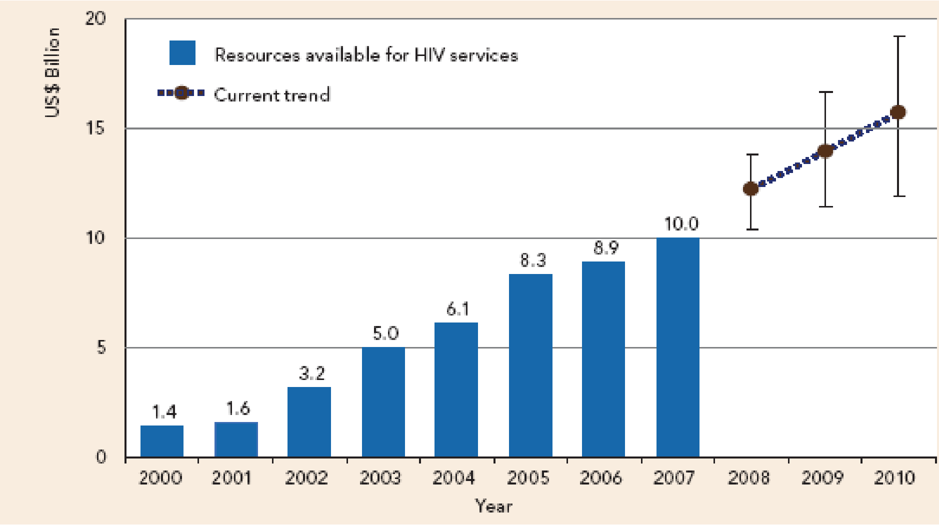
Is this dynamics optimal?

Annual resources for HIV *(Data: UNAIDS)*



Introduction

The problem at stake

- How should we allocate resources in the presence of an epidemic disease?

  $=>$ The optimal trade-off between wealth and health?

  A "modified" problem on the optimal size of the population.

  $=>$ The optimal timing?

  Present generations pay for the size of the tomorrow population?

- Our approach:

  $=>$ The tradition of optimal planning.

  A large consensus for government intervention.

  $=>$ Utilitarism.

Introduction: litterature

The mathematical epidemiology tradition (since Bernoulli, 1760)

- In most studies, the **objective of the planner** is to minimize a convex combination of:

  - the sum over time of the costs of the control,
  - the number of infected individuals.

$$\min_{h_t} \int_0^T \theta C(h_t) + (1-\theta) I(h_t)\, dt$$

- Moreover, in most studies:

  - the planner horizon is finite with no discounting,
  - the problem is (often) linear with respect to the control,
  - the control has a fixed upper limit $h_t \in [0, \bar{h}]$.

Introduction

Overview of our paper

- We propose a general framework for studying first-best allocations that relies on individual preferences.
- We analyze the optimal trajectories for the case of prevention.
- Some results:

  $=>$ Trajectories that converges to a SS display a monotonic increase of prevention over time.

  $=>$ These trajectories are not necessarily optimal. Other trajectories yielding to *zero intervention* or *eradication* may be optimal.

  $=>$ On the latter trajectory, prevention decreases with time.

Plan of the talk

1. A framework for the analysis

   - The epidemiological model.
   - The social planner problem.
     - The social welfare function.
     - The optimization program.

2. Three normative issues:

   - Is prevention optimal in the long run?
   - Prevention: the sooner, the better?
   - Should the epidemics be eradicated?

The epidemic dynamics

Two main components

- "Biological" law of motion. Possible dynamics:

  - **Stabilizes within the population** (HIV/AIDS, Smallpox).

  - Spreads till the whole population is contaminated (Plague).

  - Disappears after a while (Spanish Flu, Ebola).

- Controlling the epidemic. Policies:

  - **Prevention**: affects behaviors and contamination coefficient (as in marketing/advertising models).

  - Quarantine (or vaccination) $\rightarrow$ a third class: individuals who are (not) infected and are not susceptible to infect (be infected by) the others.

The epidemic dynamics

General assumptions

- Two classes in the population: the susceptible and the infected:

$$P_t = S_t + I_t.$$

- Consider the ratio: $a_t = I_t / S_t$ ($\simeq$ the **prevalence rate**).
- Assume that:

$$\dot{a}_t = g\left(h_t, a_t\right) a_t,$$

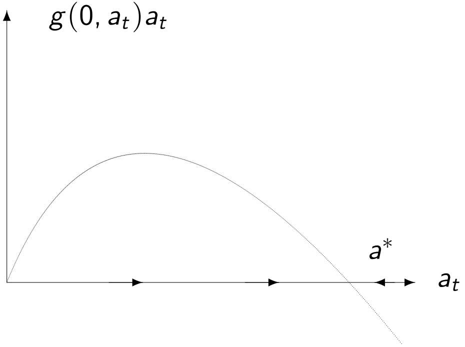
  where h_t is the **control** (the prevention campaign).

- There exists an **endemic state**: $a^* > 0$ such that $g\left(0, a^*\right) = 0$.
- Moreover:

$$g_1'\left(h_t, a_t\right) < 0, \text{ and } g_2'\left(h_t, a_t\right) < 0.$$

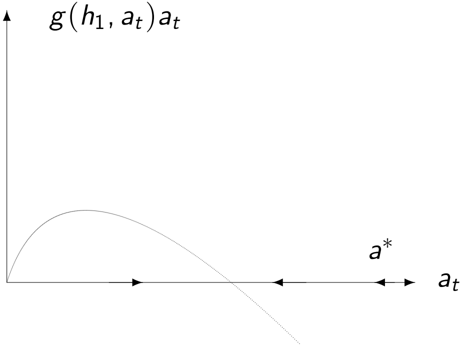
The dynamics with no prevention

If $h_t = 0$, the prevalence rate converges to the endemic steady-state.



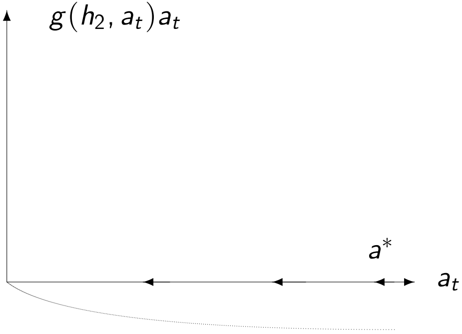
The dynamics with small prevention

If $h_t = h_1 < h^*$, the prevalence rate stabilizes at a lower level.



The dynamics with important prevention

If $h_t = h_2 > h^*$, the prevalence rate converges to zero.



The epidemic dynamics

A more restrictive assumption

- The population growth rate is independent of h_t:

$$\frac{\dot{P}_t}{P_t} = n\left(a_t\right),$$

- The population growth rate **decreases** with the prevalence rate:

$$n'\left(a_t\right) < 0,$$

- The population growth rate is bounded:

$$\bar{n} < n\left(a_t\right) < n\left(0\right).$$

The resource constraint

A yeoman farmer economy

- There is one material good produced using labor.
- The productivity of an individual is **lower** if infected.

  Denote production per capita by:

  $$\alpha f\left(a_t\right) \text{ where } f'\left(a_t\right) \leq 0,$$

  and $\alpha > 0$ is a measure of the **productivity of labor**.
- The produced good can be used for consumption or for prevention.

  The resource constraint per capita is then:

  $$c_t + h_t \leq \alpha f\left(a_t\right).$$

The social welfare function

The preferences of a dynasty

- Consider a dynasty of altruistic individuals, whose growth rate without epidemic is $n(0)$.

- At each point of time, the epidemic may kill the dynasty.
- Let λ_t be the probability at time 0, that the dynasty is still alive at time t.

- Let the individual utility be:

  - $u(c_t)$ if alive,

  - 0 if "not alive".

- Assume: $u' > 0$, $u'' < 0$, $\lim_{c \to 0} u'(c) = +\infty$, <u>and</u> $u(0) \geq 0$.

The social welfare function

- The expected utility of a dynasty writes (under discounted total utilitarism):

$$\int_0^\infty e^{-[\rho - n(0)]t} \lambda_t u\left(c_t\right) dt,$$

where: $\rho > n\left(0\right)$ is the **pure** discount rate.

- At the aggregate level, the probability is:

$$\lambda_t = \frac{P_t}{P_0 e^{n(0)t}} = e^{\int_0^t [n(a_s) - n(0)] ds}.$$

A utilitarist criteria

- The social welfare function is then:

$$P_0 \int_0^\infty e^{-\int_0^t [\rho - n(a_s)] ds} u\left(c_t\right) dt = \int_0^\infty e^{-\rho t} P_t u\left(c_t\right) dt.$$

The problem

$$\max_{h_t} \int_0^\infty e^{-\int_0^t \theta(a_u)du} u\left(\alpha f\left(a_t\right) - h_t\right) dt$$

$$s.t. \left| \begin{array}{l} \dot{a}_t = g\left(h_t, a_t\right) a_t, \\ \\ 0 \leq h_t \leq \alpha f\left(a_t\right), \\ \\ a_t \geq 0, a_0 > 0 \text{ given.} \end{array} \right.$$

with $\theta\left(a_t\right) \equiv \rho - n\left(a_t\right)$.

- For $a_0 = 0$, the solution is: $h_t = 0$ and $c_t = \alpha f\left(0\right)$.
- For $a_0 > 0$: an "endogenous discounting" problem.

The social planner

The intertemporal trade-off

An increase in h_t:

- reduces immediate consumption c_t
- reduces the prevalence rate at time $t + \varepsilon$: $a_{t+\varepsilon}$.

Hence, at time $t + \varepsilon$, it:

- increases the production per capita.
- reduces the spread between the pure discount rate and the population growth rate:

$$\theta(a_t) = \rho - n(a_t).$$

A reduction of the spread can be interpreted as a **more equal treatment** between generations.

The optimal solution

Solving the model

- Use the virtual time method, Uzawa (1968):

- There exists an optimal solution. The optimal path is a solution of the following system:

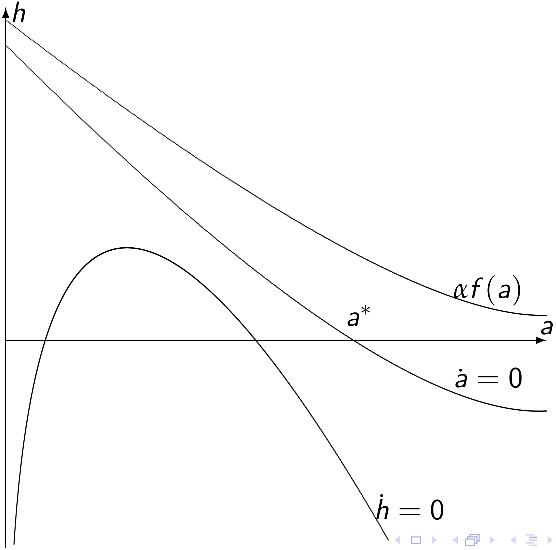
$$\begin{cases} \dot{a}_t = g\left(h_t, a_t\right) a_t \\ \dot{h}_t = \phi\left(h_t, a_t\right) \end{cases} \qquad \text{if } h_t > 0,$$

$$\dot{a}_t = g\left(0, a_t\right) a_t \qquad \text{if } h_t = 0,$$

- The system has two dynamics: an interior and a corner one.
- Note: $\phi\left(h_t, a_t\right)$ is ugly.

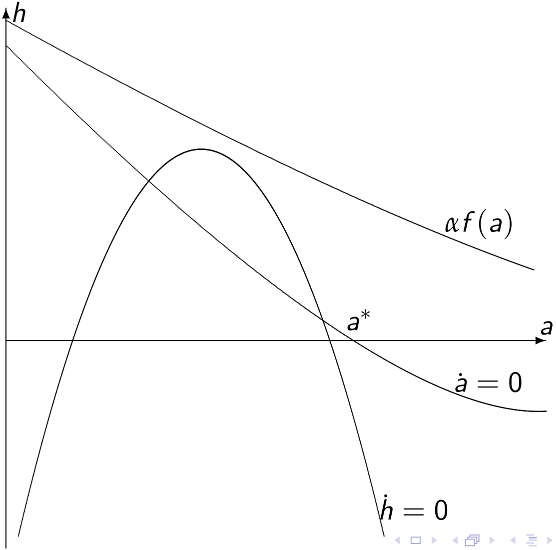
Is prevention optimal in the long run?

Graphical representation (low α)



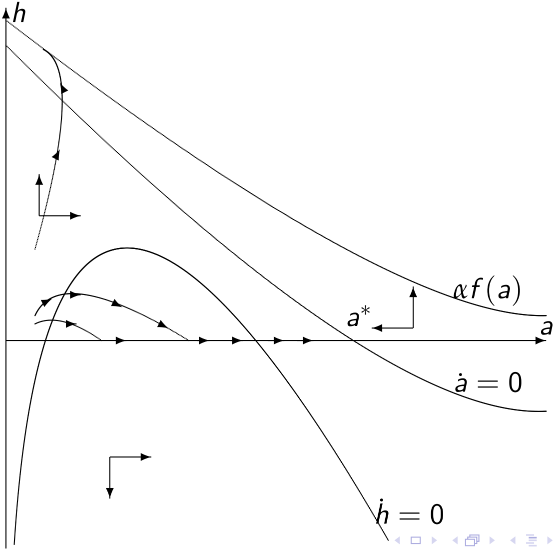
Is prevention optimal in the long run?

Graphical representation (high α)



Prevention: the sooner, the better?

Phase diagram with no interior steady-state



Prevention: the sooner, the better?

Optimal paths

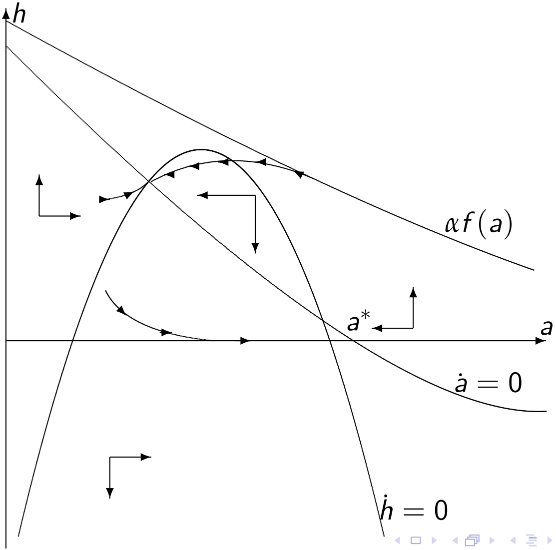
- A family of candidates whose date T at which the prevention stops is different. A larger T slows down the epidemics but is costly.
- It is a problem with free terminal conditions (a_T, T).

$$\max_{h_t, a_T, T} \int_0^T e^{-\int_0^t \theta(a_u) du} u\left(\alpha f\left(a_t\right) - h_t\right) dt + G\left(a_T, T\right),$$

$$s.t. \left| \begin{array}{l} \dot{a}_t = g\left(h_t, a_t\right) a_t, \\ 0 \leq h_t \leq \alpha f\left(a_t\right), \ a_t \geq 0 \text{ and } a_0 > 0 \text{ given}, \\ h_T = 0. \end{array} \right.$$

Prevention: the sooner, the better?

Phase diagram with interior steady-state



Conclusions

- If the economy cannot afford long run prevention. *The sooner is not always the better*. Prevention campaigns may never be launched.
- If the economy can afford long run prevention. *The later, the better.* Prevention grows over time on the stable arm.
- **Robustness**: let us look at paths that do not converge to a steady-state.

Eradication of the epidemic

A slight modification of the initial assumptions

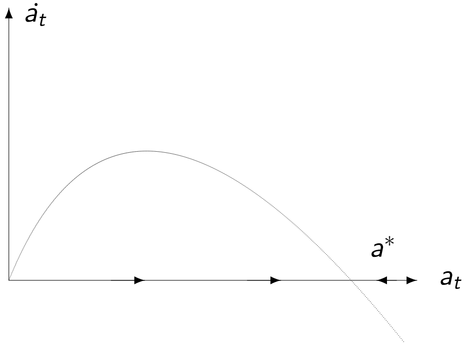
- Consider the following dynamics for the epidemic:

$$\dot{a}_t = g\left(h_t, a_t - a_{\min}\right)\left(a_t - a_{\min}\right)$$

- $a_{\min} \in \left(0, a^*\right)$ is a threshold below which the epidemic disappears in a finite time.
- A **minimal prevalence rate** is thus necessary for the epidemic to survive.
- This threshold may be reach by e.g. the immigration of infected individuals.

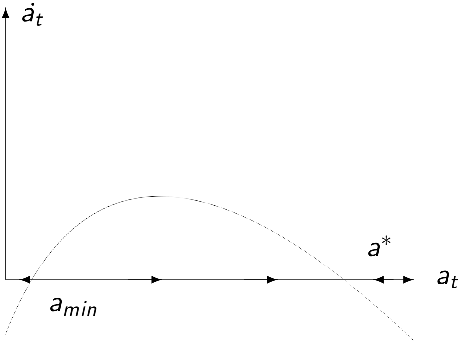
Eradication of the epidemic

The initial dynamic without intervention...



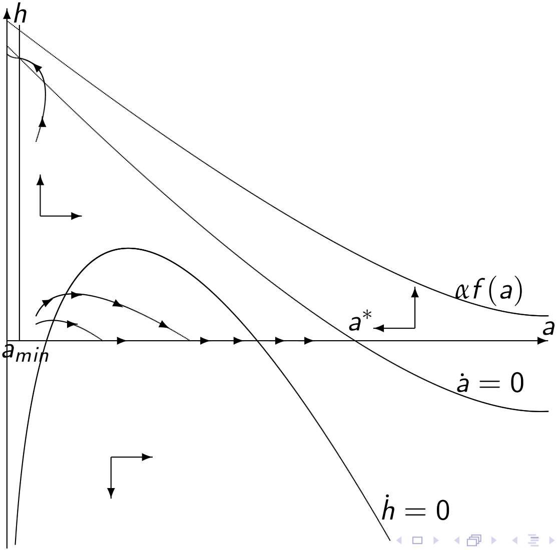
Eradication of the epidemic

... is replaced by this new one:

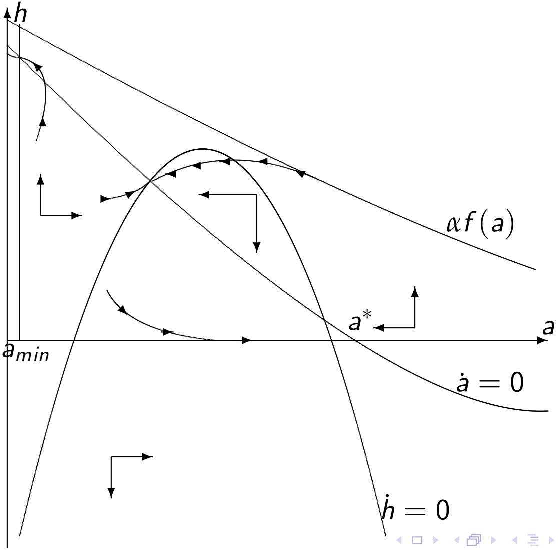


Eradication of the epidemic

A path leading to eradication (no interior steady-state)



A path leading to eradication (two interior steady-states)



Eradication of the epidemic

Optimality of eradication

Lemma

Suppose there exists a path leading to the eradication of the epidemic. There exists a threshold for the pure discount rate, denoted $\bar{\rho}$, such that for $\rho < \bar{\rho}$, this path is optimal.

- After eradication, the discounted utility of generation t is the highest possible.
- For $\theta(0) \longrightarrow 0$ (the discount rate equals the maximal population growth rate), the discounted utility $\longrightarrow +\infty$.

Conclusion

- An epidemic like HIV can be eradicated in finite time.
- If we agree there is no ethical ground for pure discounting (Ramsey 1928, among others).
- Then: "the sooner, the better" hold in general provided that resources are abundant.
- If the Social Planner represents the United Nations, available resources should be sufficient.
- The dynamics of resources allocated for HIV is not optimal (or not fair).

Conclusion

- Many extensions are possible:

  - Human and physical capital accumulation.

  - Demographic structure and delays.

  - Uncertainty and stochastic dynamics.

- Next step: integrate individual behaviors and decentralize of the optimum.