Report on Kuhn & Wrzaczek: On optimal fertility and mortality in economies with age structured population

OFAS - Vienna

Hippolyte d'Albis

TSE

Dec 9, MMX

A dynamic issue?

Optimal dynamics of distributions are difficult to study.

The authors:
- Define a problem that is bounded:

$$\max \int_0^T (...) \, dt$$

- Write the optimal conditions
- Study the steady-state solution??

I think that the relevant assumption is:

$$T \longrightarrow +\infty$$

Writing differently the problem

If consumption for a given age is constant

$$\max \int_0^\infty e^{-\rho t} N_t^\epsilon \left(\int_0^\omega u\left(c_a\right) \frac{S_a N_{t-a}}{N_t} da \right) dt$$

or equivalently:

$$\max_n \left(\int_0^\omega u\left(c_a\right) S_a e^{-na} da \right) \int_0^\infty e^{-(\rho - \epsilon n)t} dt$$

Then
- $\left(\int_0^\omega u\left(c_a\right) S_a e^{-na} da \right)$ decreases with n but is finite for $n \in \left[-\frac{\rho}{1-\epsilon}, \frac{\rho}{\epsilon} \right]$
- $\int_0^\infty e^{-(\rho - \epsilon n)t} dt$ increase with n and is infinite for $n \longrightarrow \frac{\rho}{\epsilon}$

Conclusion: there is no interior solution
- The maximal n is always preferred if $\epsilon > 0$ (as in Parfit, 1984)
- The minimal n is always preferred if $\epsilon = 0$

A conjecture

Introduce a utility that (positively) depends on birth rate:

$$\max_{n}\left(\int_0^{\omega} u\left(c_a, n\right) S_a e^{-na} da\right) \int_0^{\infty} e^{-(\rho-\epsilon n)t} dt$$

Then:

- The maximal n is always preferred if $\epsilon > 0$
- One may have a interior n if $\epsilon = 0$ (no need for a satiation assumption)

A remark on "dynamic efficiency"

The authors use this term for characterizing the sign or $f'(k) - n$

A intertemporal allocation is dynamically efficient if it is Pareto efficient

With exogenous population, $f'(k) - n$ is relevant

With endogenous population: it is not

How dealing with those who are not born? How computing a transfer for those who might be born toward those who were born?

Some recent papers on that.

A remark on the utility function

On should be careful

Let us define a population of size 1 and a cake of size c

Suppose you may choose the survival probability p

$$\max_p pu\left(\frac{c}{p}\right) + (1-p)\, UID$$

The FOC is

$$u\left(\frac{c}{p}\right) - \frac{c}{p}u'\left(\frac{c}{p}\right) - UID = 0$$

By the concavity of u one has:

$$u\left(\frac{c}{p}\right) - u(0) \geq \frac{c}{p}u'\left(\frac{c}{p}\right)$$

Then, if $u(0) \geq UID$, there is no interior solution

Advice

1. Use natality and mortality as two distinct variables

2. Better relate to the recent literature
VSL and genuine savings: Arrow, Bensoussan, Feng, Sethi (2010)
The dynamics and the extinction issue Boucekkine, Fabbri, Gozzi (2010)

3. Go beyond: Millian vs Benthamite