On Optimal Fertility and Mortality in Economies with Age-Structured Populations

Michael Kuhn

Vienna Institute of Demography

Stefan Wrzaczek

Institute of Mathematical Methods in Economics, TU Vienna

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1. MOTIVATION

Models of Optimal Population Growth:

- 2-period OLG:

  Samuelson (1975, IER): optimal fertility
  De la Croix et al. (2009): optimal fertility, mortality (and labour supply).

- Continuous time - age-structured:

  Arthur & McNicoll (1977, RES): optimal fertility
  Feichtinger et al. (2004, TPB): migration

1. MOTIVATION

Our objectives:

(i) Amend the continuous time – age-structured model to account for mortality...

...and provide optimal ‘population’ rules a la Arthur & McNicoll (1978)

(ii) Examine the role of preferences for population (Benthamite vs. Millian) and, thereby, extend Kuhn et al. (2010, EL) to (a) include fertility; (b) a proper macro-setting.

(iii) Arrive at numerical assessment of how preferences and technology shape optimal population (growth and distribution)

2. THE MODEL

Planner's objective:

Period utility from consumption c(a,t) and fertility control m(a,t)

Survival function

$$\int_0^T \int_0^\omega e^{-\rho t} u(c(a,t), m(a,t)) S(a,t) B^\varepsilon(t-a) \; da \; dt$$

life cycle utility

size of birth cohort

Benthamite preferences: ε = 1
Millian preferences: ε=0

Note: u(c,m) is assumed to have a peak in m(a,t)

2. THE MODEL

Mortality:

McKendrick: age-groups N(a,t)=S(a,t)B(t-a) die off at age-specific rate μ(a,h)

$$N_a + N_t = -\mu(a, h(a,t))N(a,t) \qquad N(0,t) = B(t), N(a,0) = N_0$$

Mortality rate: can be reduced by health care h(a,t)

$$\begin{aligned} \mu(a, h(a,t)) &\in (0, \bar{\mu}(a)] \ (\forall a); \quad \mu(a,0) = \bar{\mu}(a), \ \mu(a,\infty) \geq 0 \ (\forall a) \\ \mu_h(\cdot) &< 0, \ \mu_{hh}(\cdot) > 0; \quad \mu_h(a,0) = -\infty, \ \mu_h(a,\infty) = 0 \ (\forall a) \end{aligned}$$

2. THE MODEL

Fertility:

Total number of births: at time t due to age-specific fertility v(a,t)

$$B(t) = \int_0^{\omega} \nu(a, m(a,t)) N(a,t) da$$

Fertility rate: can be controlled within an age-specific band

$$\begin{aligned} \nu(a, m(a,t)) &\in \left[\tilde{v}(a) - \varepsilon^-(a), \tilde{v}(a) + \varepsilon^+(a)\right] \quad (\forall a); \\ \nu(a, 0) &= \tilde{v}(a) - \varepsilon^-(a), \ \nu(a, \infty) = \tilde{v}(a) + \varepsilon^+(a) \ (\forall a) \\ \nu_m(\cdot) &> 0. \end{aligned}$$

2. THE MODEL

Closed economy:

Capital accumulation: at time t due to age-specific fertility v(a,t)

$$K_t = Y(K(t), L(t)) - C(t) - H(t)$$

with… Supply of effective labour:

$$L(t) = \int_0^{\omega} l(a,t) N(a,t) da$$

Aggregate consumption:

$$C(t) = \int_0^{\omega} c(a,t) N(a,t) da$$

Aggregate health expenditure:

$$H(t) = \int_0^{\omega} h(a,t) N(a,t) da$$

2. THE MODEL

Model Summary:

$$\max_{c(a,t),h(a,t),\nu(a,t)} \int_0^T \int_0^\omega e^{-\rho t} u(c(a,t),\nu(a,t)) S^{1-\epsilon}(a,t) N^{\epsilon}(a,t) \; da \; dt$$

$$N_a + N_t = -\mu(a, h(a,t)) N(a,t), \quad N(0,t) = B(t), N(a,0) = N_0$$

$$B(t) = \int_0^\omega \nu(a,t) N(a,t) \; da$$

$$S_a + S_t = -\mu(a,t,h(a,t)) S(a,t), \quad S(0,t) = 1, S(a,0) = S_0$$

$$K_t = Y(K(t), L(t)) - C(t) - H(t)$$

$$K(0) = K_0, \quad K(T) = K_T$$

$$L(t) = \int_0^\omega l(a,t) N(a,t) \; da$$

$$C(t) = \int_0^\omega c(a,t) N(a,t) \; da$$

$$H(t) = \int_0^\omega h(a,t) N(a,t) \; da.$$

Note: We substitute $B(t-a)=N(t,a)S(t,a)^{-1}$

3. OPTIMAL ALLOCATION

(Generalised) value of life: The decision maker's marginal willingness to pay for survival of an (a,t) individual.

$$\psi^S(a,t) = \int_a^\omega e^{-\int_a^s [Y_K(K,L)+\mu(h)]ds'} \left(\frac{u(c,m)}{u_c(c,m)} + Y_L(K,L)l - c - h \right) ds + \int_a^\omega e^{-\int_a^s [Y_K(K,L)+\mu(h)]ds'} \psi^N(a,t,s)\nu(m)\ ds$$

Conventional value of life as in Shepard and Zeckhauser (1984):

(Discounted stream of) consumer surplus + change in net wealth over the remaining (expected) life time

Value of progeny:

(Discounted) value to the decision-maker (aged a and at time t) of a new birth at time t+(s-a).

3. OPTIMAL ALLOCATION

The value of progeny (VOP):

Conversion of valuation by a newborn (age=0) into valuation by s-year old parent.

$$\psi^N(a,t,s) = \left(\frac{B(t-a+s)}{B(t-a)}\right)^{\epsilon-1} \frac{u_c(c(0,t-a+s),m(0,t-a+s))}{u_c(c(s,t-a+s),m(s,t-a+s))} \int_0^{\omega} e^{-\int_0^{s'}[Y_K(K,L)+\mu(h)]ds''} \times \left[\epsilon \frac{u(c(s',t-a+s+s'),m(s',t-a+s+s'))}{u_c(c(s',t-a+s+s'),m(s',t-a+s+s'))} + Y_L(K,L)l - c - h + \psi^N(0,t-a+s,s')\nu(m)\right] ds'. \quad (8)$$

Weight of future population:

=1 for ε=1 or sationary population
<1 for ε<1 and ↑ population
>1 for ε<1 and ↓ population

Valuation of the newborn:

Weighted utility stream
+ net contribution to wealth
+ VOP from the newborn's perspective

3. OPTIMAL ALLOCATION

Optimal Allocation:

$$\frac{u_c(c(a,t),m(a,t))}{u_c(c(s,t-a+s),m(s,t-a+s))e^{-\rho(s-a)}} = \exp\left[\int_a^s Y_K(K(s'),L(s'))ds'\right]$$ Euler

$$-\frac{1}{\mu_h(a,h(a,t))} = \psi^S(a,t)$$ health

$$-\frac{u_m(c,m)}{u_c(c,m)\nu_m(m)} = \psi^N(a,t,a)$$ birth control

Value of life = effective marginal cost of life saving ≥ 0

Value of progeny = monteray (dis-)benefit of birth control ≥ or < 0.

4. STABLE GROWTH

- Stable population growth => Lotka equation:

$$1 = \int_0^{\omega} e^{-na} S(a, h(a)) \nu(a, m(a)) \; da$$

=> h(a) and m(a) imply rate of population growth n.

- Stable economic growth => constant capital intensity k=K/L

  aggregates B(t), N(a,t), L(a,t), C(a,t), H(a,t), K(t) ↑ at rate n

  per-capita values k(t), c(a,t), h(a,t), m(a,t) and marginal products Y_K, Y_L = constant

- Y(K,L) is (a) neo-classical => stable growth at any n
  (b) classical => stable growth at n=0.

4. STABLE GROWTH

Modified Golden Rule: $Y_K = \rho + (1-\epsilon)n$

- Dynamic efficiency if and only if $\rho \geq \varepsilon n$
- With a growing (shrinking) population a Millian planner tends to discount more (less) than a Benthamite planner.
- For a neo-classical production function:

  (i) Millian planner implements a lower (higher) capital intensity if the population grows (shrinks)

  (ii) Millian allocation supports lower (higher) total expenditure under population growth (decline) if the Milian support ratio d=L(t)/N(t) is smaller (larger) than the Benthamite one.

4. STABLE GROWTH

VOP:

$$\psi^N(a,t,s) = \frac{\int_0^\omega e^{-[\rho+(1-\epsilon)n]s'} S(s')\left(\epsilon\frac{u(c,m)}{u_c(c,m)} + Y_L l - c - h\right)ds'}{1 - \int_0^\omega e^{-[\rho+(1-\epsilon)n]s'} \nu(m) S(s')\, ds'}$$

- depends no longer on parent's age
- adjustment factors cancel out

$$\left(\frac{B(t-a+s)}{B(t-a)}\right)^{\epsilon-1} = e^{(\epsilon-1)ns} \qquad \frac{u_c(c(0,t-a+s),m(0,t-a+s))}{u_c(c(s,t-a+s),m(s,t-a+s))} = e^{(Y_K-\rho)s} = e^{(1-\epsilon)ns}$$

- For n>(<)0: Millian planner discounts by more (less)

4. STABLE GROWTH

Birth multiplier:

$$b(n) := \left[1 - \int_0^{\omega} e^{-[\rho+(1-\epsilon)n]s'} \nu(m) S(s')\, ds'\right]^{-1}$$

- positive > 1 if and only if $\rho \geq \varepsilon n$, i.e. if and only if the allocation is dynamically efficient.

- **Note:** To see this substitute from Lotka equation.

4. STABLE GROWTH

Time paths:

Consumption:

$$c_a + c_t = \frac{u_c}{u_{cc}}(\epsilon - 1)n$$

Millian planner: c ↑ with age iff n>0

Health:

$$\begin{aligned} h_a + h_t &= -\frac{\mu_{ha}}{\mu_{hh}} - \frac{\mu_h}{\mu_{hh}} \frac{\psi_a^S + \psi_t^S}{\psi^S} \\ &= -\frac{\mu_{ha}}{\mu_{hh}} - \frac{\mu_h}{\mu_{hh}} \Big[Y_K + \mu \\ & \quad + \mu_h \Big(\frac{u(c,m)}{u_c} + Y_L l - c - h + \psi^N \nu(m) \Big) \Big] \end{aligned}$$

Changing effectiveness of h with age (> (<) 0 for young (old) ages .

↓ in value of the life year passed

Birth control:

$$m_a + m_t = \frac{u_m}{u_{mm}\nu_m - \nu_{mm}u_m} \left[\nu_{ma} + (\epsilon - 1)n\nu_m \right]$$

effectiveness ↓ with age

↓ control with pop. growth

4. STABLE GROWTH

Existence of equilibrium without preferences for m:

- Should allow a sharper characterisation of v(a) and n…

$$\psi^N = b(n)\int_0^\omega e^{-r(n)a} S(a,h)\left(\epsilon\frac{u(c)}{u_c(c)} + Y_L l - c - h\right)da = 0$$

- …but may not exist.

- For a maximum :

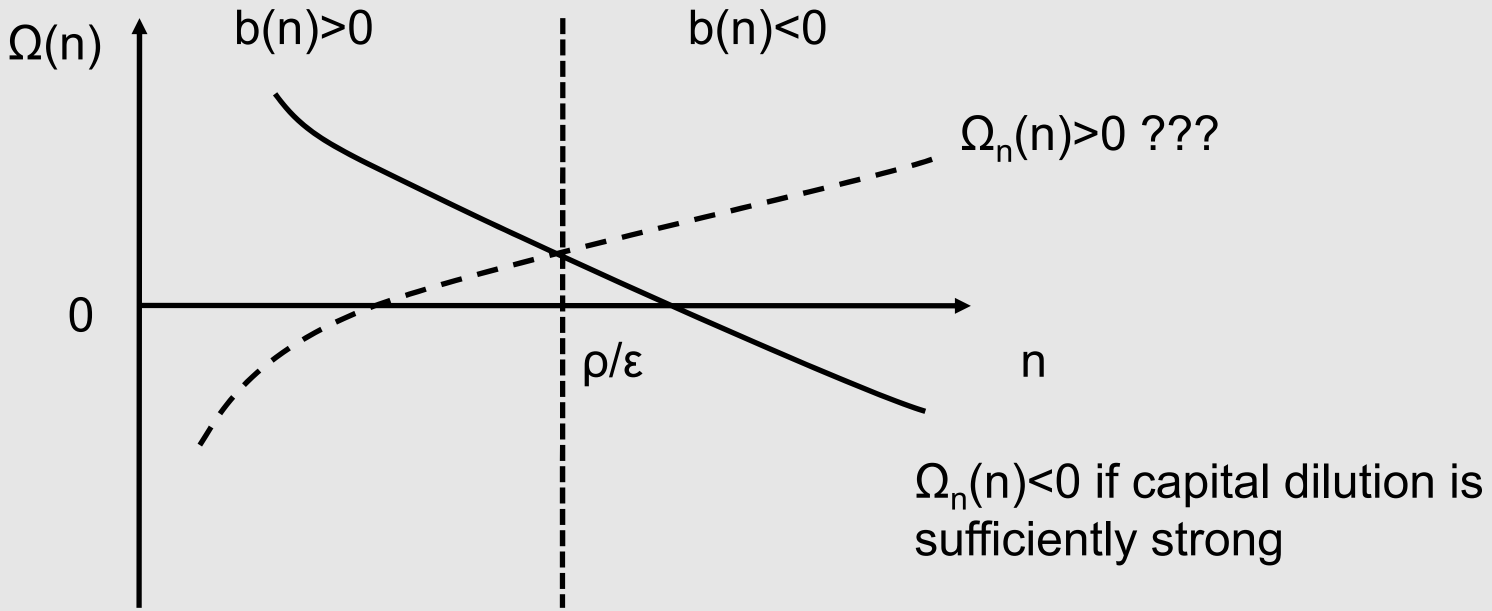
$$\Omega(n) \; : = \int_0^\omega e^{-r(n)a} S(a)\left(\epsilon\frac{u(c)}{u_c(c)} + Y_L l - c - h\right)da = 0$$

$$\frac{\partial\psi^N}{\partial n}\Big|_{\Omega(n)=0} = b(n)\,\Omega_r(n) < 0,$$

4. STABLE GROWTH

- But recall that $b(n) \geq 0 \iff \rho \geq \epsilon n.$

- We can then depict



4. STABLE GROWTH

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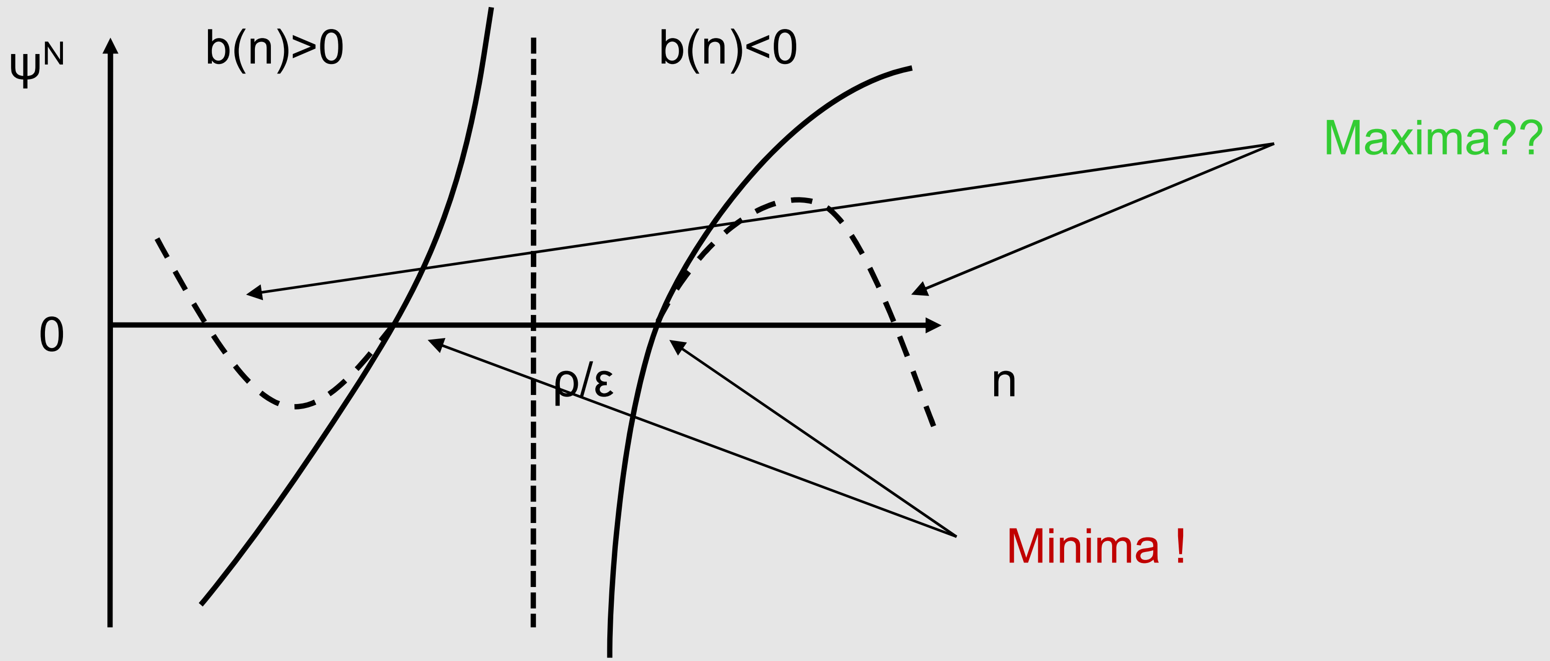

5. CONCLUSION AND OUTLOOK

- An age-structured model of population ‘choice’ including both mortality and fertility (besides consumption)

- Derive a fertility rule a la Arthur & McNicoll (1977) and amend this by a mortality rule.

- Characterise the rules and the stable growth path depending on preferences for population size.

- Problem (as is usual for these models): Existence and uniqueness

- Question: Can we pull out from population rules insights about what preferences (and technology) imply for (i) rates of population growth; (ii) fertility and mortality patterns; and (iii) resulting stable age-structures (e.g. dependency ratios, etc.)?